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POWER CORRECTIONS AND NONLOCAL OPERATORS IN QCD

(PT as a window to NP physics)

Work in Progress
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I. FROM PT TO THE OPE

II. METHOD OF SUBSTITUTION

III. e^+e^- ANNIHILATION IN THE TWO-JET LIMIT (2JET)

IV. IR ANALYSIS OF 2JET

V. LEADING POWER CORRECTIONS FOR 2JET

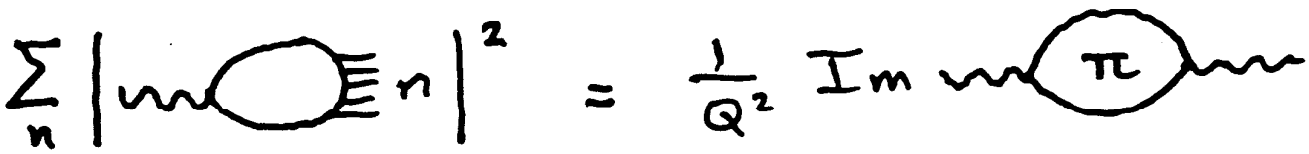
VI. OUTLOOK

I. FROM PT TO THE OPE

1.1. $\sigma_{\rm tot}$, Unitarity and IR Safety

$$\sigma_{\rm tot}^{e^+e^-} = \frac{(4\pi\alpha)^2}{Q^2} \mathrm{Im}\, \pi(Q^2)$$

$$\pi(Q^2) = \left(\frac{-i}{3Q^2}\right) \int d^4x\, e^{-iq\cdot x} \langle 0|T j^\mu(0) j_\mu(x)|0\rangle\,,$$



- OPE:

$$\begin{aligned}\langle 0|j^\mu(0) j_\mu(x)|0\rangle &= \frac{1}{x^6}\tilde{C}_0(x^2\mu^2) \\ &\quad + \frac{1}{x^2}\tilde{C}_{F^2}(x^2\mu^2)\alpha_s(\mu^2)\langle 0|F_{\mu\nu}F^{\mu\nu}(0)|0\rangle + \ldots .\end{aligned}$$

- *$C_0(Q^2/\mu^2)$ from PT*
- *IR Safety ...*

- *No solutions to Landau equations:*

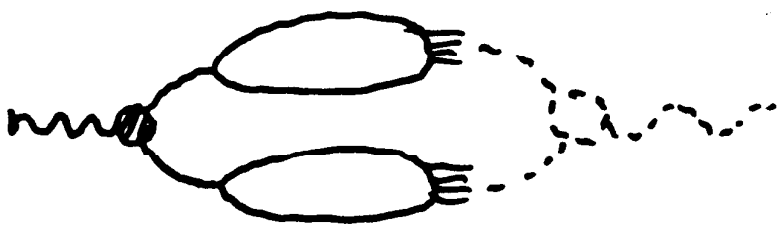
$$\sum \sum_{i \text{ in loop } j} \alpha_i \epsilon_{ij} p_i^\mu = 0$$

$$\updownarrow$$

$$\sum_{i \text{ in loop } j} \tau_i \epsilon_{ij} v_i^\mu = 0$$

$$\updownarrow$$

- *No classical process:*



$$\updownarrow$$

- *Infrared Safety:*

$$C_0(Q^2/\mu^2, \alpha_s(\mu)) = \sum_{n=0}^{\infty} c_0^{(0)}(Q^2/\mu^2)\ \alpha_s^n(\mu))$$

- *C_0 IR and CO finite for $m_q = 0$ in $D = 4$*
- *But ... series for C_0 nonconvergent*

1.2 Perturbative Nonconvergence of C_0

Mueller (1985,1992)

- *Generic pinched integration region:*

$$
\begin{aligned}
C_0^{(0)} &\sim (1/Q^4) \int_0^{Q^2} dk^2 k^2 \, \ln^n(k^2/\mu^2) \\
&\sim (1/2)\,(1/2)^n \; n! \,.
\end{aligned}
$$

- *RG + Gauge invariance* $\rightarrow$

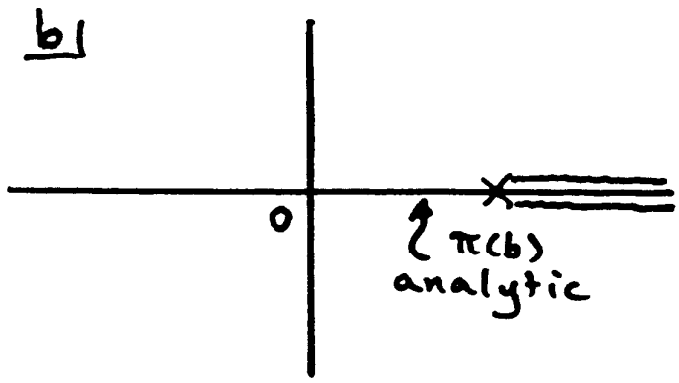
$$
C_0 = C_0^{(\mathrm{reg})} + C_0^{(\mathrm{pinch})} + \mathcal{O}(Q^{-6})
$$

$$
\begin{aligned}
C_0^{\mathrm{pinch}}\left(Q^2/\mu^2, \kappa/Q, \alpha_s(Q)\right) &= H(Q) \int_0^{\kappa^2} dk^2 k^2 \, \alpha_s(k^2) + \ldots \\
&= H(Q) \int_0^{\kappa^2} dk^2 k^2 \\
&\quad \times \frac{\alpha_s(Q^2)}{1 + \left(\frac{\alpha_s(Q^2)}{4\pi}\right) b_2 \ln(k^2/Q^2)} + \ldots \\
&= H(Q) \int_0^{\kappa^2} dk^2 k^2 \, \frac{4\pi}{b_2 \ln(k^2/\Lambda^2)} + \ldots
\end{aligned}
$$

- *"Borel form"*: $b \equiv 2\alpha_s(Q^2)\ln(Q^2/k^2)$

$$C_0^{\text{pinch}}(1,1,\alpha_s(Q)) = \frac{1}{2\alpha_s(Q)}\, H(Q)Q^4 \int_0^\infty db\, e^{-b/\alpha_s(Q)} \frac{1}{1-\frac{b_2}{8\pi}\, b}$$

- *Pole (= ambiguity) at* $b = 8\pi/b_2 \to Q^{-4}$
- *Borel plane:*



- *In any case ... k soft integrals in* $C_0^{(\text{pinch})}$ *and* $\langle F^2\rangle(\kappa)$ *identical!*

- **Axiom of substitution:** *Behavior of "true" C_0:*

$$\frac{C_{\rm PT}}{Q^4} \to C_{\rm PT}^{\rm reg}\left(Q^2/\mu^2, \kappa/Q, \alpha_s(Q)\right) + \frac{\alpha_s \langle 0|F^2(0)|0\rangle(\kappa)}{Q^4}.$$

- Nonconvergence of PT $\to$ need for new (IR) regularization

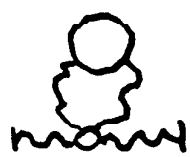
- Cost: new NP parameter ($\langle F^2 \rangle$) implicit in PT
- Benefit: new NP parameter ($\langle F^2 \rangle$) implicit in PT

II. METHOD OF SUBSTITUTION

- For $\pi(Q)$ above not necessary (already had OPE)
- But, semi-inclusive (jet, event shape) σ's, ***power corrections important and not well understood.***
- So propose:

For σ an IR safe cross section at large scale Q:

1. Identify regions R in momentum space where lines are pinched on-shell in σ. (Landau equations)



2. Organize logarithms of momenta ℓ that occur in R into: (a) $\alpha_s(f(\ell))$ and/or (b) explicit kinematic integrals. ((a) RG – example: of $f(\ell) = \ell^2$ above; (b) Resummation – examples below; case-by-case)

$$\int dk^2 k^2 \alpha_s(k^2)$$

3. Introduce a cutoff κ on (some) components: $\ell^\nu < \kappa$, to define contribution σ_R from region R, such that $\alpha_s(f(\ell)) > \alpha_0$, α_0 fixed.

$$\int^{\kappa^2} dk^2 (\ldots)$$

4. With α_s fixed, evaluate: $\sigma_R \sim Q^{-2-m}\kappa^m$. (Power counting)

$$m = 4$$

6 ~~4~~. Find a "universal" matrix element $\langle \mathcal{O} \rangle$, dimension m, whose PT is identical to R with κ as UV cutoff.

$$\langle 0 | F^2 | 0 \rangle$$

6 ~~5~~. Remove σ_R from σ, replace it with $\langle \mathcal{O} \rangle$,

$$\sigma = \sigma^{(\mathrm{reg})}(\kappa) + \frac{\langle \mathcal{O} \rangle(\kappa)}{Q^{-2-m}}$$

Oversubtractions for even higher-power corrections?

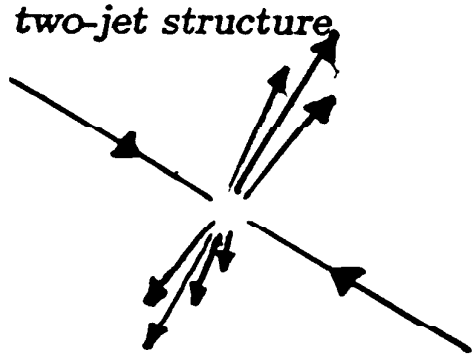
III. e^+e^- ANNIHILATION IN THE TWO-JET LIMIT

3.1. Thrust and Singular Behavior

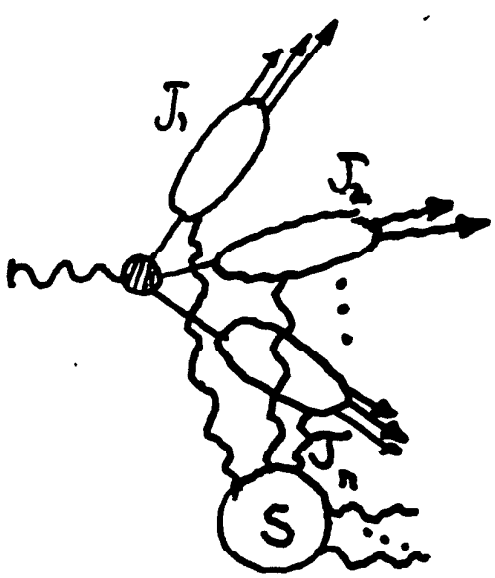
Thrust, *typical IRS event shape:*

$$T = \frac{1}{\sqrt{s}} \max_{\hat{n}} \sum_i |\hat{n} \cdot \vec{p}_i| \,,$$

- $T = 1 \leftrightarrow$ *two-jet structure*



- *Substitution? General pinch surface in* e^+e^-:



- *Complicated ... but one class of corrections singled out:*
- *Singular behavior for $T \to 1$ (LLA):*

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma}{dT} = 2C_F\frac{\alpha_s}{\pi}\left[\frac{\ln(1-T)}{1-T}\exp\left\{-C_F\frac{\alpha_s}{\pi}\ln^2(1-T)\right\}\right]_+$$

- **Simplification.** *Study T for singular behavior in two-jet limit.*

- *Role of moments:*

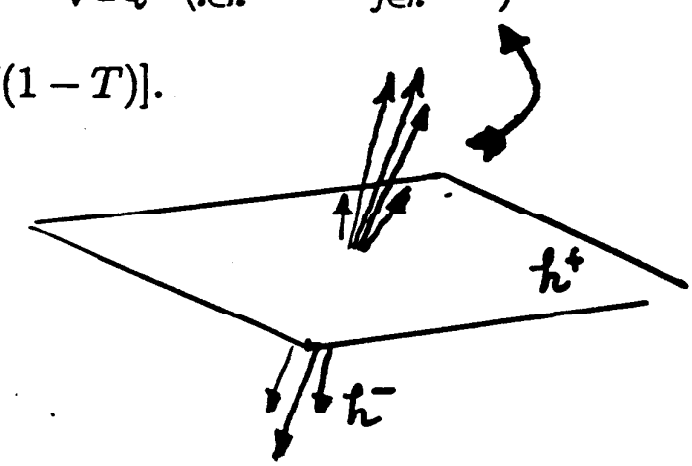
$$\tilde{\sigma}(N) = \frac{1}{\sigma_0}\int_0^1 dT\, T^N \frac{d\sigma}{dT}$$

$$\int_0^1 dT\, \frac{T^N - 1}{1-T}\, \ln^m(1-T) = \frac{(-1)^m}{m+1}\, \ln^{m+1} N + \ldots$$

- *Work to accuracy $(1-T)^0 \leftrightarrow 1/N$.*
- *To this accuracy, thrust axis fixed:*

$$1 - T = \frac{1}{\sqrt{2}Q}\left(\sum_{i\in h^+} p_i^+ + \sum_{j\in h^-} p_i^-\right)$$

- *And $T^N \sim \exp[-N(1-T)]$.*

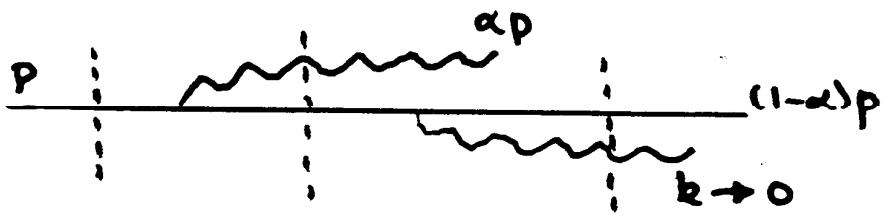


3.2 General Weights

- ***IRS Safe weight "$w(\{p_i\})$"***

$$\frac{d\sigma}{dw} = \sum_n \int_{PS(n)} |M(\{p_i\})|^2 \delta\left(w(\{p_i\}) - w\right)$$

$$w(\ldots p_i \ldots p_{j-1}, \alpha p_i, p_{j+1} \ldots) = w(\ldots (1+\alpha) p_i \ldots p_{j-1}, p_{j+1} \ldots)$$



- *Suppresses long-time evolution*
- *Moments*

$$\tilde{\sigma}(N) = \frac{1}{\sigma_0} \int_0^1 dw\, (1-w)^N \frac{d\sigma}{dw}$$

- $N \to \infty$ enhances long-time behavior.

- ***Consider w such that:***

  1. $w = 0$ in two-jet limit ($w = 1 - T$, *e.g.*).
  2. To order w^0, $w(\{p_i\}) = \Sigma_i\, w(p_i)$.

- ***E/θ weights:***

$$w(p_i) = E_i f_w(\theta_i)$$

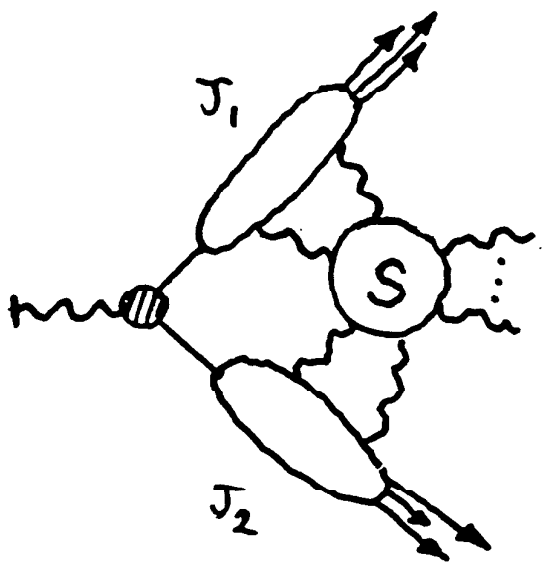
$$f_{1-T}(\theta_i) = \min\left[\frac{1}{\sqrt{2}}(1 \pm cos\theta_i)\right]$$

- *e.g.* $f_n \equiv \min\left[\frac{1}{\sqrt{2}}(1 \pm cos^n\theta_i)\right]$

3.3 Factorization/convolution/moments

- *Factorization/convolution*

$$\frac{d\sigma(w)}{dw\,d\cos\theta} = \sigma_0\, H(p_1/\mu, p_2/\mu,) \int \frac{dw_1}{w_1}\,\frac{dw_2}{w_2}\,\frac{dw_s}{w_s} \times J_1(p_1/\mu, w_1) J_2(p_2/\mu, w_2) \times S(v_i, w_i Q/\mu)\, \delta(w - w_1 - w_2 - w_s) + O(w^0)\,,$$



- *Moments → product:*

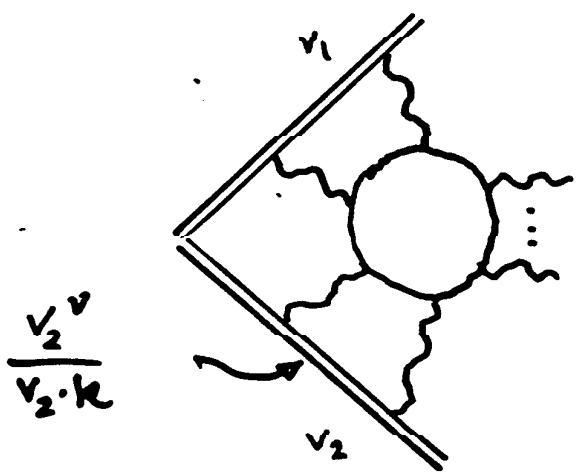
$$\tilde{\sigma}(N) = \int_0^{w_{\max}} dw\, e^{-Nw} \frac{d\sigma(w)}{dw} = \sigma_0\, H(p_1/\mu, p_2/\mu)\, \tilde{S}(v_i, Q/\mu N) \times \tilde{J}_1(p_1/\mu, N)\, \tilde{J}_2(p_2/\mu, N) + O(1/N)\,,$$

3.4 2-Jet Effective Theory (2JET)

- *Soft-gluon function S:*

$$\frac{1}{w_s} S(\{v_i\}, w_s) = \sum_n \delta(w_x - w_s)$$
$$\times \quad \mathrm{Tr}\, \Big\langle 0 \Big| \bar{\mathrm{T}}\Big(\prod_{i=1}^{2} \Phi^\dagger_{v_i}(\infty, 0)\Big) |x\rangle\langle x| \, T\Big(\prod_{j=1}^{2} \Phi_{v_j}(\infty, 0)\Big) \Big| 0 \Big\rangle ,$$

$$\Phi_{v_i}(\lambda, x) = P_\pm \exp\Big(-ig \int_0^\lambda d\lambda' v_i \cdot A(\lambda'\beta + x)\Big)$$



- *Eikonal approximation ...*

Two-Jet Effective Theory for Soft Gluons *(2JET)*

$$\mathcal{L}_{2JET} \sim \sum_{i=1}^{2} \phi_{v_i} \; [i v_i \cdot D(A)] \; \phi_{v_i} - \phi^\dagger_{v_2}\phi_{v_1} - \phi^\dagger_{v_1}\phi_{v_2}$$

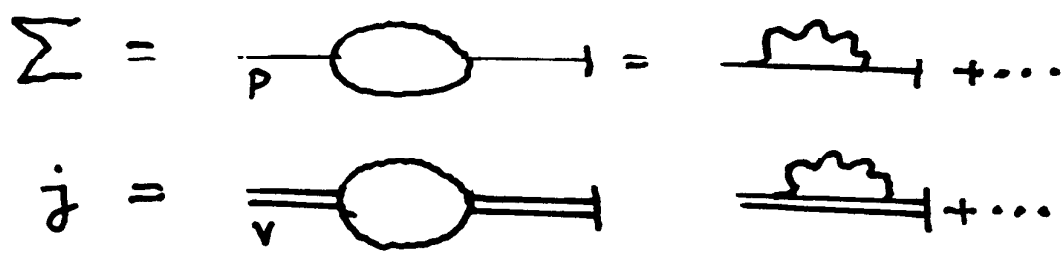
ϕ_v *scalar*

- *Analogous treatment for "jets", defined by*

$$\Sigma(p_i,\zeta) = J(p_i) \otimes_w j(v_i,\zeta)$$

$$\begin{aligned}\Sigma(p_i,\zeta,w_i) &= \frac{1}{\mathcal{N}_q(Q)} \sum_x \delta(w_x - w_i) \int_{-\infty}^{\infty} d\lambda\, e^{-i\lambda p_i\cdot n_i} \\ &\quad \times \langle\, 0 \mid q_a^\dagger(\lambda v_i) \mid x \rangle\, \Gamma_{aa'} \langle x \mid \bar{q}_{a'}(0) \mid 0 \,\rangle_{\zeta\cdot A=0}\end{aligned}$$

$$\begin{aligned}j_i(v_i,\zeta,w_i Q/\mu) &= \frac{1}{d(R)} \sum_x \delta(w_x - w_i) \\ &\quad \times \langle\, 0 \mid \Phi^\dagger_{v_i,da}(0) \mid x \rangle \langle x \mid \Phi_{v_i,cd}(0) \mid 0 \,\rangle_{\zeta\cdot A=0}\end{aligned}$$



IV. IR ANALYSIS OF 2JET

4.1 Web Decompositon and Exponentiation

- *Eikonal cross section at fixed energy:*

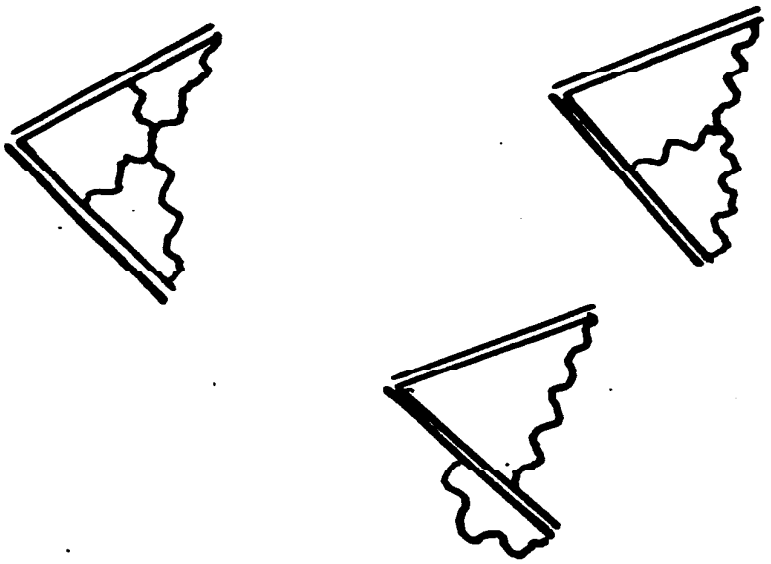
Gatheral, Frenkel and Taylor (1981)

$$\frac{d\sigma_{2\mathrm{JET}}}{dq_0} = \sum_{n=0}^{\infty} \frac{1}{n!} \int dq_0\, \delta(Q - q_0)\ \otimes_{q_0} \prod_{i=1}^{n} S(k)$$

$$S(k) = \sum_{\text{webs } \mathcal{F}} C(\mathcal{F})\, \mathcal{F}(k)$$

- *Web: can't be disconnected by cutting eikonal lines alone.*

Example:



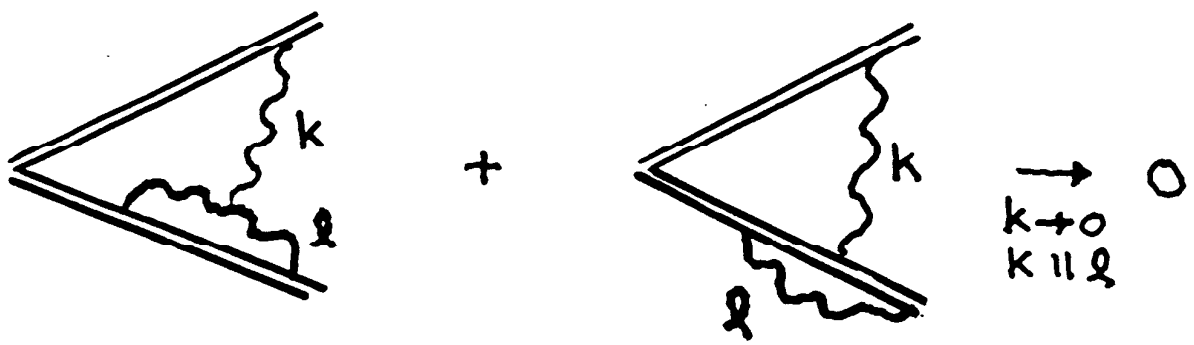
$C(\mathcal{F}) = C_A$

- ***Exponentiation in Moments:***

$$\int_0^\infty e^{-Nq_0} \frac{d\sigma_{2\mathrm{JET}}}{dq_0}\, dq_0 = \exp\left[\int_0^\infty dk_0\, S(k)\, e^{-Nk_0}\right]$$

- *Useful because:*
- **Webs have no (non-RG) IR subdivergences**
- *A form of resummation*

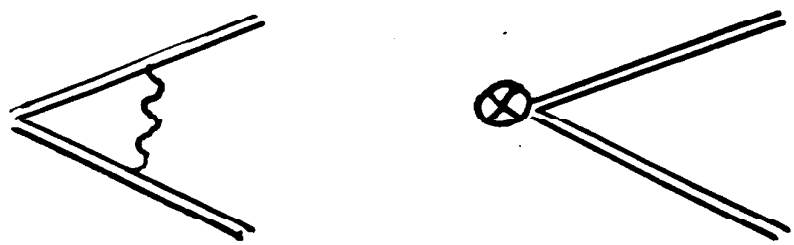
  Example:



4.2 Renormalization Group for the Kernel ($\sigma^{\rm tot}_{\rm 2JET}$)

Polyakov (1979)
Aref'eva (1980)
Brandt & Neri (1981)
Korchemsky & Radyushkin (1986,87)

- Power counting $\rightarrow$ *one overall IR divergence each* $\mathcal{F}$.
- *Web renormalization* $\leftrightarrow$ *renormalization of eikonal vertices*



- Additive Renormalization $\rightarrow$ Generalized "Plus distribution":

$$S_{\rm ren}(k,\mu') = s(k)\theta(\mu' - k^0) - \delta^4(k) \int d^4k' \, \theta(\mu' - k'^0) \, s(k')$$

- Example of momentum subtraction

$$\int d^4k S_{\rm ren}\theta(\mu' - k_0) = 0$$

- *Web integrand with $\mu' = Q$ – make single IR, CO divergences explicit:*

$$\int d^4k\, S_{\rm ren}(k,Q) \;=\; 4\pi \int_0^{Q^2} \frac{dk^2}{k^2} \int_0^{Q^2-k^2} dk_T^2 \frac{1}{k^2+k_T^2} \times \int_{\sqrt{k^2+k_T^2}}^{Q} \frac{dk_0}{\sqrt{k_0^2-k^2-k_T^2}}\, \gamma\left(\frac{k_\lambda}{\mu}, \frac{k_\lambda}{Q}, \alpha_s(\mu)\right)$$

- $\gamma/(k^2[k^2+k_T^2])$ *a plus distribution requires no overall renormalization:*

$$\mu \frac{d}{d\mu} \gamma\left(\frac{k_\lambda}{\mu}, \frac{k_\lambda}{Q}, \alpha_s(\mu)\right) = 0$$

4.4 Moments for $\frac{d\sigma_{2\mathrm{JET}}}{dw}$

$$\frac{d\sigma_{2\mathrm{JET}}(Q,w)}{dw} = \sum_{n=0}^{\infty} \frac{1}{n!} \otimes_w \prod_{i=1}^{n} \int d^4k \, \frac{dS(k,Q,w_i)}{dw_i}$$

- *Web expansion:*

$$\frac{dS(k,Q,w)}{dw_i} = \sum_{\text{webs F}} C(F) \, \frac{d\mathcal{F}(k,Q,w)}{dw}$$

- *Renormalization (virtual diagrams only) the same.*
- *Moments:*

$$\tilde{\sigma}^{w}_{2\mathrm{JET}}(Q,N) = \int_0 dw \, e^{-Nw} \frac{d\sigma_{2\mathrm{JET}}}{dw}$$

- *Assume (dimensionless) weight scaled by Q (see $1-T$):*

$$w(k) = \sum_{\text{particles i}} w(k_i/Q)$$

- *Exponentiation:*

$$\ln \tilde{\sigma}_w(Q,N) = \int d^4k \, dw \, e^{-Nw} S(k,Q,w)$$

- *Representation with overall divergences explicit:*

$$\begin{aligned}\ln \tilde{\sigma}_w(Q,N) \;=\; & 4\pi \int_0^{Q^2} \frac{dk^2}{k^2} \int_0^{Q^2-k^2} dk_T^2 \frac{1}{k^2+k_T^2} \\ & \times \int_{\sqrt{k^2+k_T^2}}^{Q} \frac{dk_0}{\sqrt{k_0^2-k^2-k_T^2}} \, \gamma_w\left(\frac{k_\lambda}{\mu}, \frac{k_\lambda}{Q}, \alpha_s(\mu), N\right)\end{aligned}$$

$$\begin{aligned}\frac{\gamma_w(k,N)}{k^2(k^2+k_T^2)} \;=\; & \sum_{\text{webs F}} C(F) \sum_{\text{f.s. } n} \\ & \times \prod_{\text{lines } i \text{ in } n} \int d^4p_i \, \delta^4(k - \textstyle\sum p_i) \\ & \times \mathcal{F}_n(\{p_i\}) \left(\mathrm{e}^{-N \sum_j w(p_j/Q)} - 1\right)\end{aligned}$$

- $\frac{\gamma_w(k,N)}{k^2(k^2+k_T^2)}$ finite distribution at $k^2, k_T^2 \to 0$
- *Two overall logarithms of N, up to running coupling*
- Power corrections in Q ...

4.5 Power Corrections and Substitution

- *Expand* $\int dk_0\, \gamma$ *in* Q at fixed N:

$$\int_{\sqrt{k^2+k_T^2}}^{Q} \frac{dk_0}{\sqrt{k_0^2 - k^2 - k_T^2}}\, \gamma_w \left(\frac{k_\lambda}{\mu}, \frac{k_\lambda}{Q}, \alpha_s(\mu), N\right)$$
$$= \sum_{n=0} \frac{k_T^n}{Q^n} \Gamma_w^{(n)} \left(N, k^2/\mu^2, k_T^2/\mu^2, \alpha_s(\mu)\right)$$

- *Nonconvergence & substitution? depends on* w*, but: simplified by web structure and IRS-*w

- *Fewer sources of leading power*

- *Leading powers in a few examples ...*

V. LEADING POWER CORRECTIONS FOR 2JET

5.1 k_T imbalance; Minimal Nonlocality

Korchemsky & G.S. (1995)

- *Q_T in Drell-Yan cross section; k_T imbalance in e^+e^-:*

Collins and Soper (1981)
Collins, Soper & G.S. (1985)

$$\ln \tilde{\sigma}_{Q_T}^{(\mathrm{DY})}(b) = \int_0^{Q^2} \frac{dk_T^2}{k_T^2} A(\alpha_s(k_T^2)) \ln(Q^2/k_T^2) \times \left(e^{-ik_T \cdot b} - 1\right) + \dots ,$$

- $bQ \leftrightarrow N$; $k_T/Q \leftrightarrow w(k)$

$$\ln \tilde{P}(b) = \int_{(1/b^*)^2}^{Q^2} \frac{dk_T^2}{k_T^2} A(\alpha_s(k_T^2)) \ln(Q^2/k_T^2) \times \left(e^{-ik_T \cdot b} - 1\right) + g_1 b^2 \ln(Q^2 b^2) + \dots ,$$

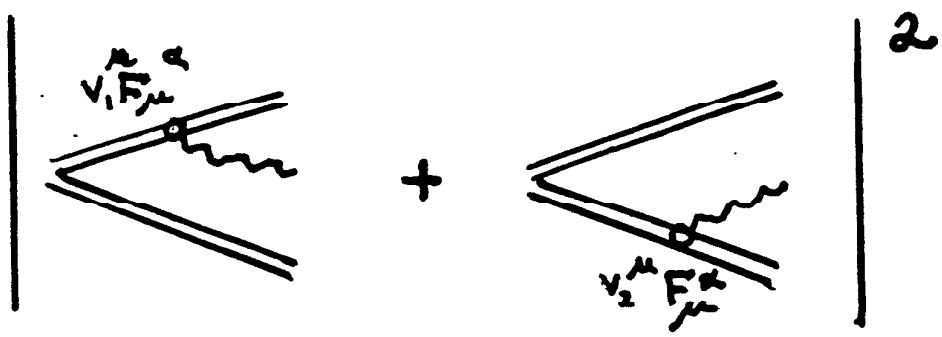
- $1/b^* \leftrightarrow$ "κ"; g_1 *replaces "R":* $(\Gamma_{k_T}^{(2)})$

$$\int_0^{(1/b^*)^2} \frac{dk_T^2}{k_T^2} A(\alpha_s(k_T^2)) \ln(Q^2/k_T^2)(b^2 k_T^2) + \dots ,$$

- The operator?
- *"Nonlocal square of field strengths"*
- *Operators on a one-dimensional manifold:*

$$\langle 0|\left|\Phi^\dagger_{v_2}(0,-\infty)\left(\vec{\mathcal{F}}_{v_1}(0)-\vec{\mathcal{F}}^\dagger_{v_2}(0)\right)\ \Phi_{v_1}(0,-\infty)\right|^2|0\rangle\,,$$

$$\mathcal{F}^\alpha_v(x) = -ig\int_{-\infty}^0 ds\ \Phi_v(x,x+sp)\ v_\mu F^{\mu\alpha}(sp+x)\ \Phi_{-v}(x+ps,x)$$



5.2 E/θ weights; Maximal Nonlocality

- *Class:* $w(k_i) = \Sigma_i(k_i^0/Q) f(\cos\theta_i))$
- *As k_T imbalance, only need consider lowest orders in e^{-Nw}.*
- *Thrust:* $f_{1-T} = 1 - |\cos\theta|$
- *Then at lowest order in N/Q,*

$$\begin{aligned}
\ln \tilde{\sigma}_{1-T}(Q, N)^{(1)} &= 4\pi \int_0^{Q^2} \frac{dk^2}{k^2} \int_0^{Q^2-k^2} dk_T^2 \frac{1}{k^2 + k_T^2} \\
&\quad \times \int_{\sqrt{k^2+k_T^2}}^{Q} \frac{dk_0}{\sqrt{k_0^2 - k^2 - k_T^2}} \\
&\quad \times \left\{ \frac{N}{Q} \gamma_{1-T}^{(1)} \left(k^2/\mu^2, k_T^2/\mu^2, k_0, \alpha_s(\mu) \right) + \ldots \right\}
\end{aligned}$$

- *RG for* γ_{1-T}

$$\Rightarrow \ln\tilde{\sigma}_0(Q,N) = 4\pi\int_0^{Q^2}\frac{dk^2}{k^2}\int_0^{Q^2-k^2}dk_T^2\frac{1}{k^2+k_T^2}$$
$$\times\int_{\sqrt{k^2+k_T^2}}^{Q}\frac{dk_0}{\sqrt{k_0^2-k^2-k_T^2}}$$
$$\times\left\{\frac{N}{Q}\gamma_{1-T}^{(1)}\left(k^2/\mu^2,1,k_0,\alpha_s(\mu)\right)+\ldots\right\}$$

- *IRS of* $1-T \rightarrow$ *no divergence for* $k^2/k_T^2 \rightarrow 0$
- *Expand in* α_s*:*

$$\gamma_{1-T}^{(1)} = \frac{N}{Q}k^2\delta(k^2)\alpha_2(k_T)\left(k_0-\sqrt{k_0^2-k_T^2}\right)+\ldots$$

$$\Rightarrow \ln\tilde{\sigma}_0(Q,N) = \frac{N}{Q}4\pi\int_0^{Q^2}dk_T^2\frac{1}{k_T^2}\,\alpha_s(k_T)$$
$$\times\int_{k_T}^{Q}dk_0\,\frac{k_0-\sqrt{k_0^2-k_T^2}}{\sqrt{k_0^2-k_T^2}}$$

- k_T *integral:*

$$\int_{k_T}^{Q}dk_0\frac{k_0-\sqrt{k_0^2-k_T^2}}{\sqrt{k_0^2-k_T^2}} = \sqrt{Q^2-k_T^2}-(Q-k_T) = k_T+\ldots$$

$$\Rightarrow \ln\tilde{\sigma}_0(Q,N) = \frac{N}{Q}4\pi\int_0^{Q^2}dk_T^2\frac{1}{k_T^2}\,k_T\,\alpha_s(k_T)+\ldots$$

$$\int_0^{Q^2} dk_T^2 \frac{1}{k_T^2}\, k_T\, \alpha_s(k_T)$$

- $k_T \alpha_s(k_T) \rightarrow$ *IRS integral with "infrared renormalon" at* $1/Q$.
- *Nonconvergent* $1/Q$ *corrections in moments of* T.
- *Generalizes from* T *to many* w*'s in two-jet limit*
- *Choice* $f = 1$ *CO divergent but no* $1/Q$ (Beneke and Braun)
- *Invert moments* $\rightarrow \frac{1}{(1-T)Q}$ *corrections, etc.*

  Dokshitzer, Webber & Marchesini
  Manohar and Wise
  Korchemsky and G.S.
  Akhoury and Zakharov
  Nason and Seymour
  Beneke, Braun and Magnea

- *Web structure and IRS:* All *nonconvergent* $1/Q$ *from:*

$$k^2,\ k_T^2 \rightarrow 0$$

- *All logs controlled in* α_s *and explicit* k_T, k^2 *integrals.*

- *Substitution:*

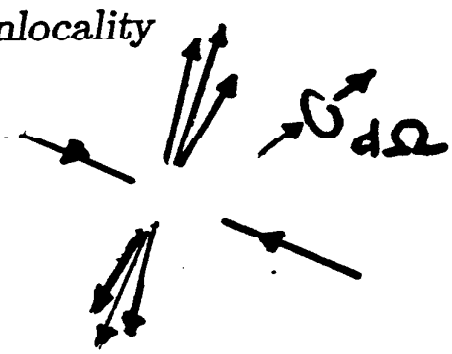
$$\ln \tilde{\sigma}_0(Q,N)^{(1)} \to \frac{N}{Q} \int_{-1}^{1} d\cos\theta \; f(\cos\theta) \; \mathcal{E}(\cos\theta)$$

$$\mathcal{E}(\cos\theta) = \langle 0 \mid W^{\dagger}_{v_1 v_2}(0) \; \Theta(\cos\theta) \; W_{v_1 v_2}(0) \mid 0 \rangle \, .$$

- *W product of Wilson lines*
- *Θ in terms of energy-momentum tensor measures energy flow:* (Basham, Brown, Ellis & Love; Tkachov)

$$\Theta(\cos\theta) = \lim_{|\vec{y}| \to \infty} \int_0^{\infty} dy_0 \; d\Omega'_{ij}(\vec{y}) \; \epsilon_{ijk} \; \theta_{0k}(y^{\mu}) \; \delta(\cos\vartheta' - \cos\theta) \, .$$

- *$1/Q$ corrections for this class of w's from "universal" $\mathcal{E}(\cos\theta)$*
- *"maximal" nonlocality*



- *Relation to* sum of *1PI cross sections:*

$$\sum_{\substack{\text{particle} \\ \text{type } f}} \int_{-1}^{1} \frac{d\sigma_f(z,\theta)}{d\cos\theta} \, z dz \, .$$

VI. OUTLOOK

- NJET
- EFT for Energy Flow

$$\langle 0 \mid \prod_i \Theta(\Omega_i) \mid 0 \rangle$$

- Hadronic Cross Sections
  - Power corrections to PT may be facilitated by the systematic identification of regions in momentum space that are associated with strong coupling. This approach complements the "classic" Borel-transform analysis.
  - This approach may lead to progress in interpretation of operator content of power-suppressed corrections, with theoretical and phenomenological relevance.