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DIS97 16. IV. 97

QCD analysis of ZEUS 1994 F_2 data

– Preliminary results –

- Outline.
- Results.
- Few systematic investigations.
- Systematic ~~error~~ propagation.

Extrapolation of high x (fixed tgt) structure functions to large Q^2

Summary

ZEUS 1994 QCD analysis – outline

- $F_2 = F_2^{uds} + F_2^{c}$

  F_2^{uds}: Obeys NLO evolution with $f = 3$ light flavours

  F_2^{c}: Calculated in NLO cf Riemersma et al. ($F_2^b \lesssim 3\% \rightarrow$ neglect)

- Parametrise $f(x, Q_0^2 = 7\,\mathrm{GeV}^2) = A\, x^{\delta}(1-x)^{\eta} \cdot P(x)$

  - xg : $P(x) = (1+\gamma x)$
  - $x\Sigma = \sum_{i=1}^{3} x(q_i + \bar{q}_i)$: $P(x) = (1+\varepsilon\sqrt{x}+\gamma x)$
  - $x\Delta_{ud} = x\{(u+\bar{u})\ (d+\bar{d})\}$: $P(x) = 1$
  - $xs^+ = xS - \frac{1}{3}x\Sigma$

    $\hookrightarrow 0.2 * \mathrm{sea} = 0.2 * (\Sigma - \mathrm{valence})$

    $\hookrightarrow$ MRSD$^-$

  Mom. sumrule $\int_0^1 dx\,(x\Sigma + xg) = 1 \Rightarrow 11$ free param.

- Input: $\alpha_s(M_Z^2) = 0.113 \pm 0.005$ (SLAC/BCDMS)

- Data: ZEUSNVG4 – Nominal vertex

  ZEUSSVG4 – Shifted vertex

  ZEUSISRG4 – Init. state radiation

  NMC p, d

  $\hookrightarrow$ – Constrain fit @ high x

  – Constrain $\Delta_{ud} \propto F_2^p - F_2^d$

F_2^c (and F_L^c) calculation in NLO

Riemersma, Smith, van Neerven, Phys. Lett B347 (1995) 143

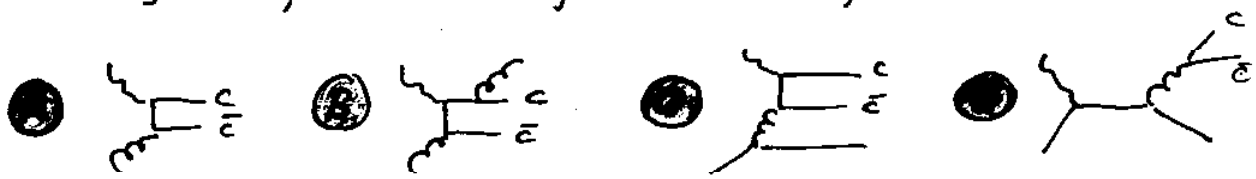
$$F_2^c = \frac{Q^2 \alpha_s}{\pi^2 m^2} \Big\{ e_c^2 \, \frac{1}{4} \left(g \otimes C_{2,g}^{(0)} \right) +$$

$$\alpha_s e_c^2 \left[g \otimes \left\{ C_{2,g}^{(1)} + \bar{C}_{2,g}^{(1)} \ln \frac{\mu^2}{m^2} \right\} \right] +$$

$$\alpha_s e_c^2 \left[\Sigma \otimes \left\{ C_{2,q}^{(1)} + \bar{C}_{2,q}^{(1)} \ln \frac{\mu^2}{m^2} \right\} \right] +$$

$$\alpha_s \left[q^P \otimes \left\{ d_{2,q}^{(1)} + \bar{d}_{2,q}^{(1)} \ln \frac{\mu^2}{m^2} \right\} \right] \Big\}$$

$$q^P = \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s})$$



Charm threshold:

$$W^2 = M^2 + Q^2(1-x)/x > M^2 + 4 m_c^2$$

Coefficient functions:

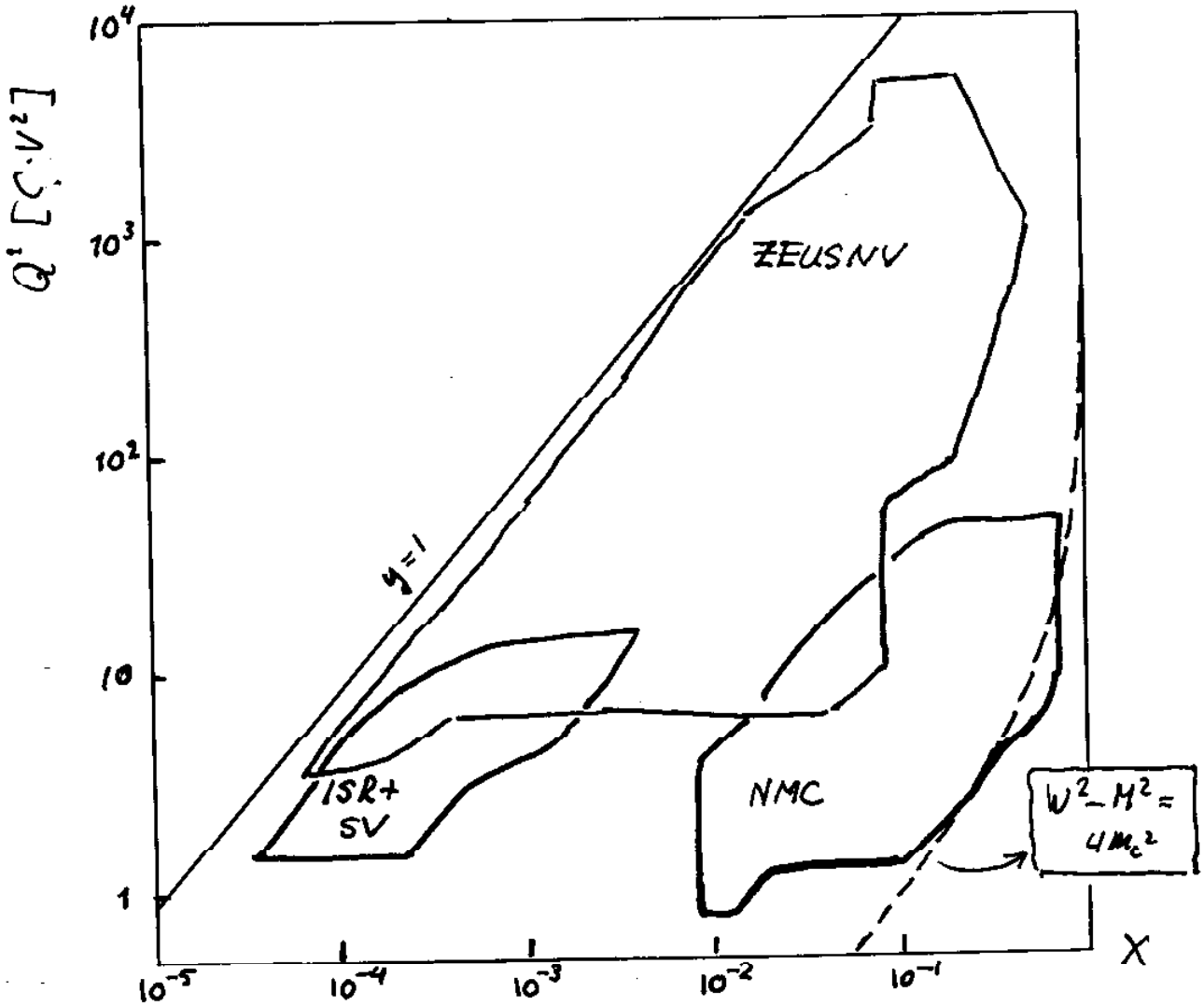
$$C(z, Q^2) = C^{asymp} + C^{threshold}$$

analytical expressions — only available as interpolation tables

Code by S. Riemersma
Coefficient functions (with recent update) incorporated in ZEUS QCD evolution program(s)

Datasets

	x_{min}	x_{max}	Q^2_{min}	Q^2_{max}	# points
● ZEUS NV	6.3×10^{-5}	0.5	3.5	5000	188
○ ZEUS SV	3.5×10^{-5}	2.6×10^{-3}	1.5	15	23
Δ ZEUS ISR	1×10^{-4}	4×10^{-3}	1.5	15	13
✱ NMC p, d	8×10^{-3}	0.5	0.75	48	2×132



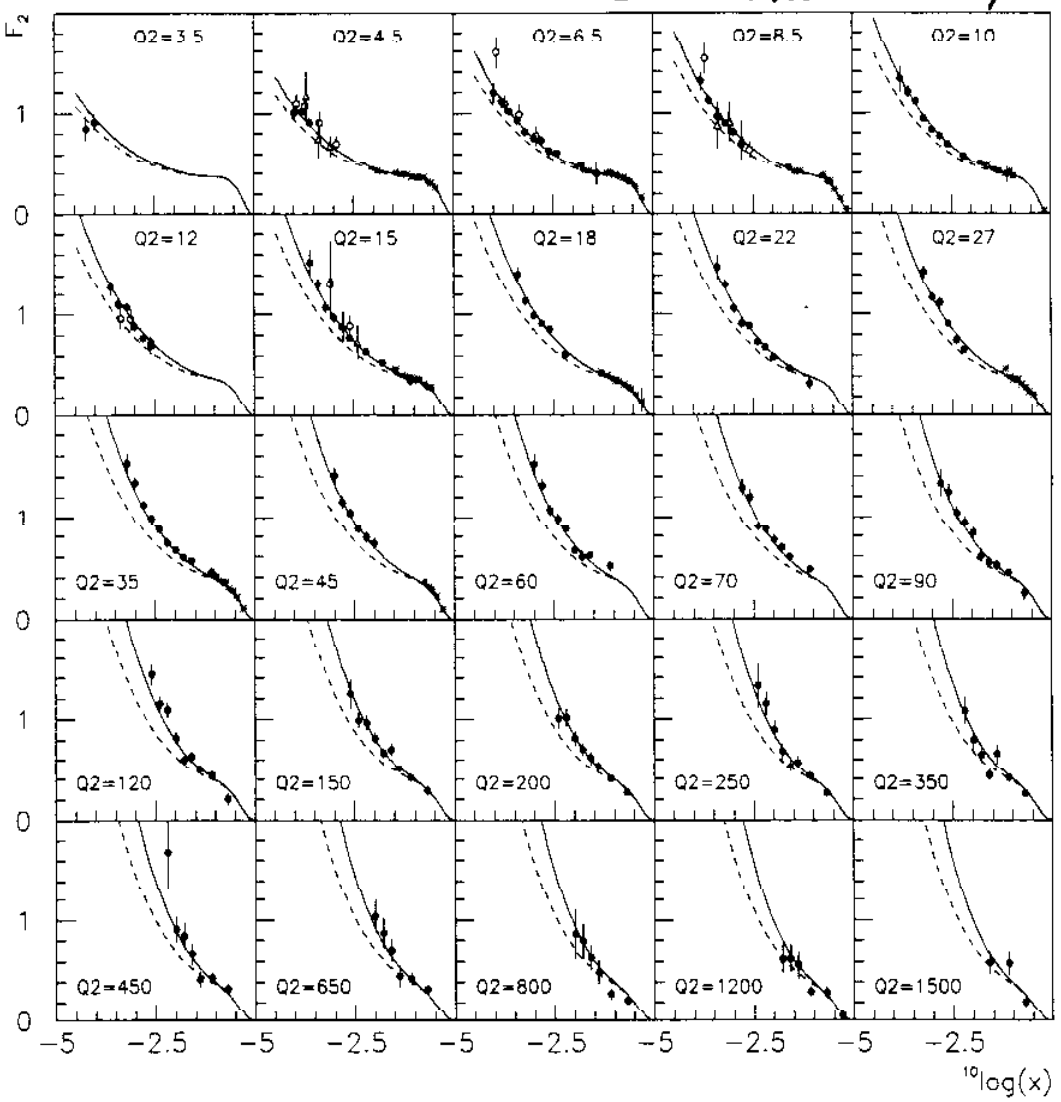
- NMC $Q^2 > 4\ \mathrm{GeV}^2$ (remove possible higher twist effects)
  - Constrain fit at high x
  - constrain $(u-d) \propto F_2^p - F_2^d$

F_2 vs x for the lowest 25 Q^2 bins

QCD fit: —— $F_2 = F_2^{uds} + F_2^{c}$

- - - - F_2^{uds}

ZEUS PRELIMINARY



● ZEUSNV94

○ ZEUSSV94

Δ ZEUS1SR94

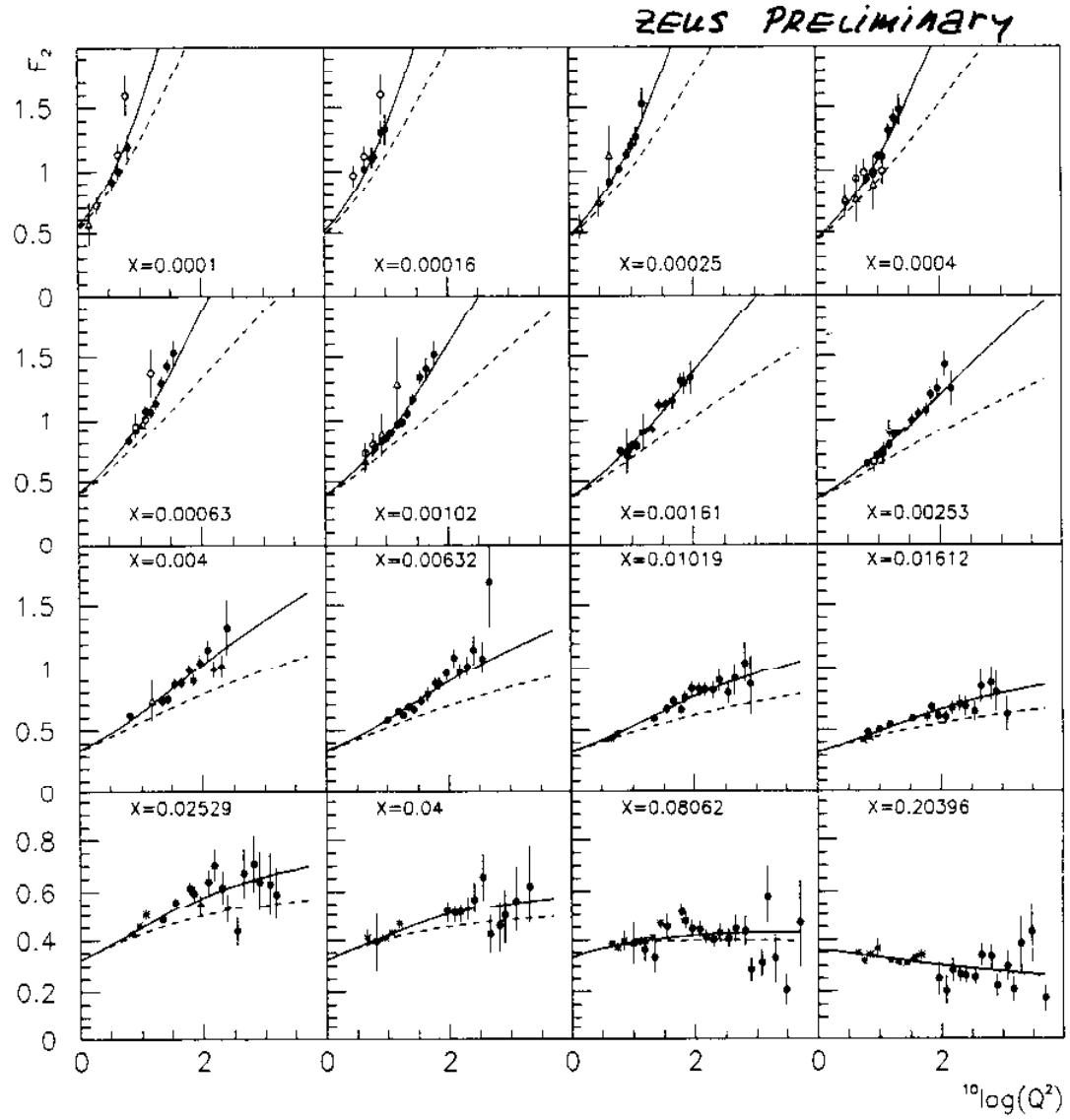
✳ NMC

χ^2(stat⊕syst) = 463/408-14

F_2 vs Q^2

QCD fit: —— $F_2 = F_2^{uds} + F_2^{c}$

- - - - F_2^{uds}



● ZEUSNVG4

○ ZEUSSVG4

△ ZEUSISRG4

✳ NMC

$\chi^2(\text{stat} \oplus \text{syst}) = 463/408-14$

χ^2 definition

$$\chi^2 = \sum_{\text{datapoints}} \left(\frac{N F_2^{meas} - F_2^{fit}}{N \sigma_{stat}} \right)^2 + \sum_{\text{datasets}} \left(\frac{N-1}{\Delta N} \right)^2$$

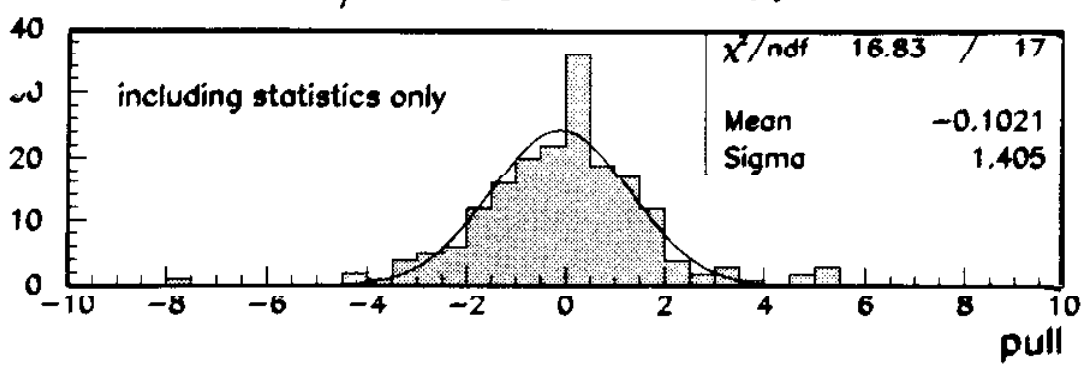
3 normalisation parameters

- ZEUS (NV, SV, ISR) = 97.4 %
  $\Delta N = (2\%, 3\%, 3\%)$
- NMC - 90 GeV = 99.3 %
  $\Delta N = 1.6\%$
- NMC - 280 GeV = 98.9 %
  $\Delta N = 2.6\%$

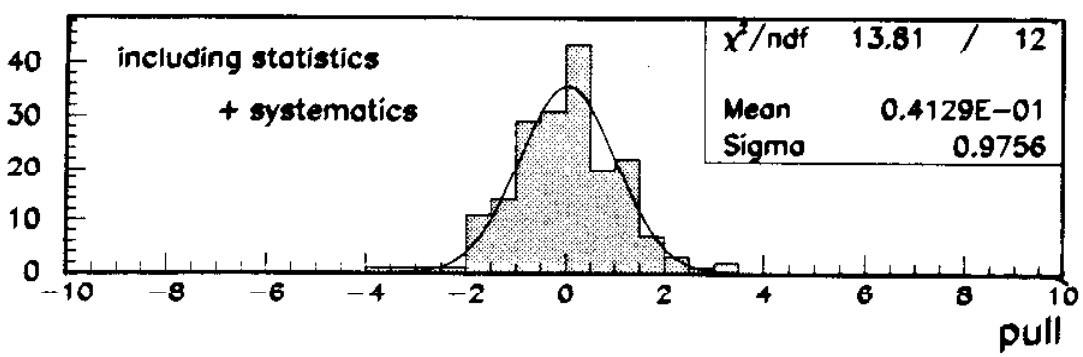
χ^2 (stat) = 986 / 408 - 14 = 2.4

χ^2 (stat $\oplus$ syst) = 463 / 408 - 14 = 1.1

pull $\equiv$ (model - data) / σ



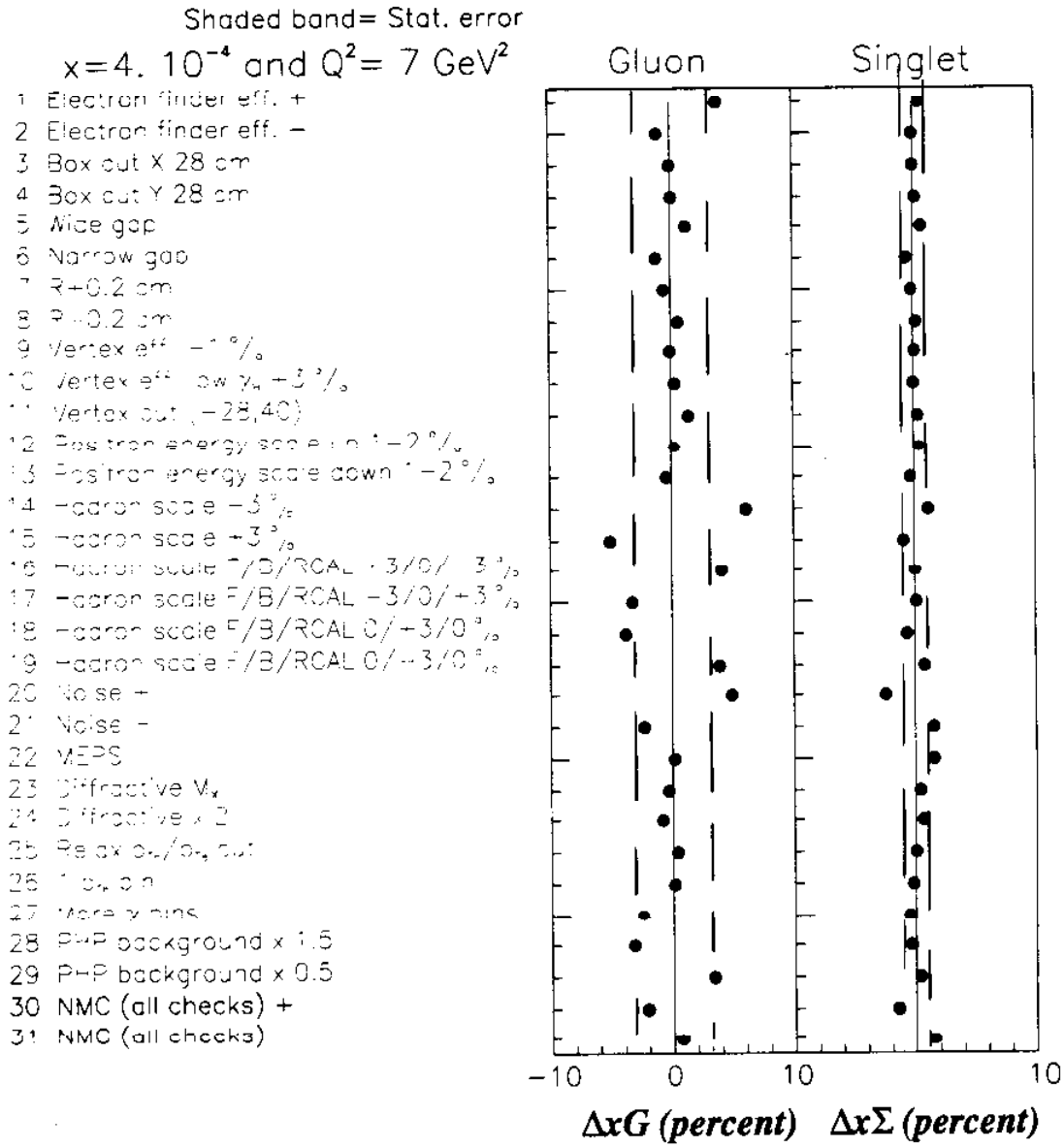
$\sigma = 1.4$



$\sigma = 1.0$

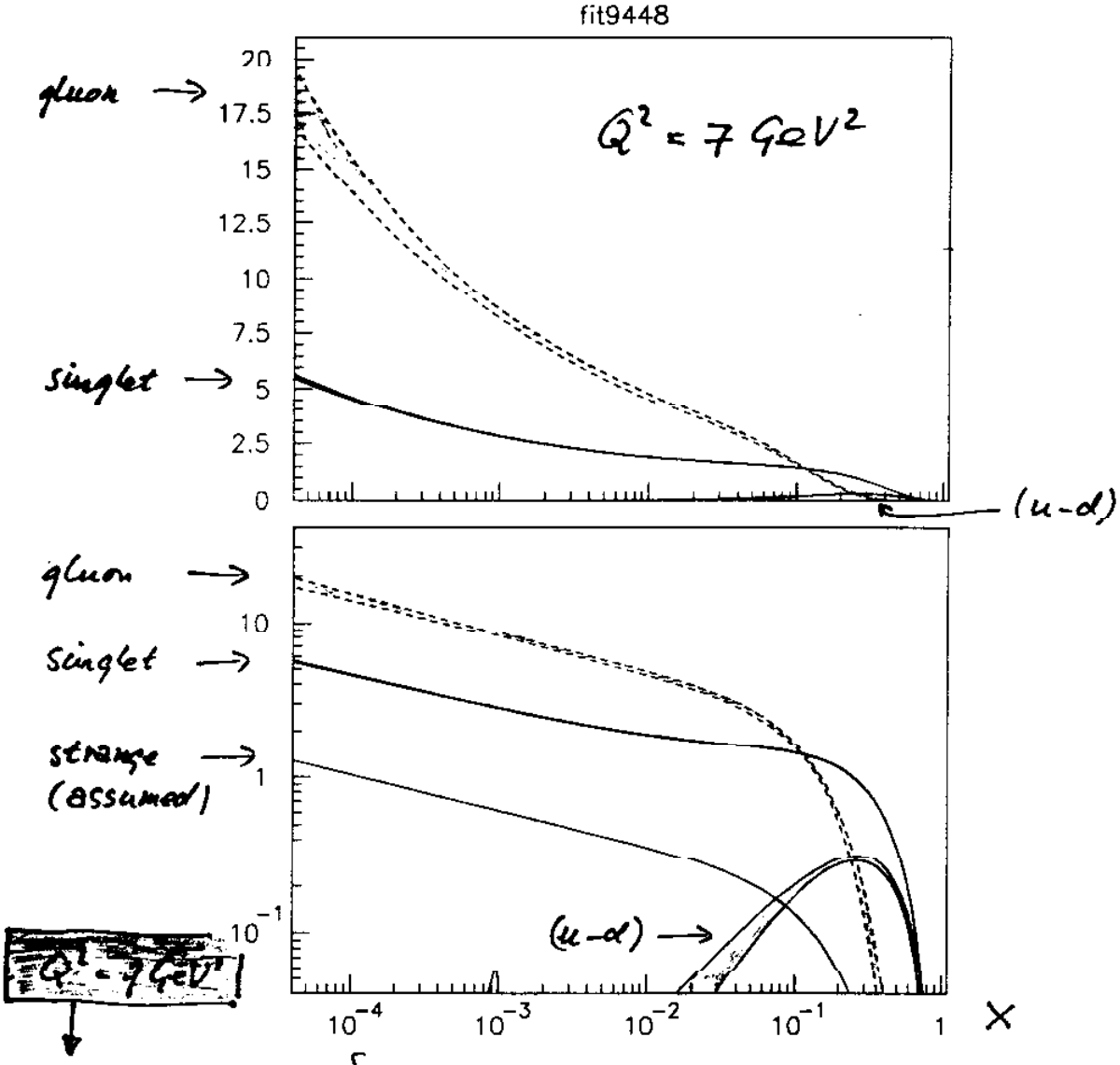
Systematic error estimate (preliminary)

Offset the data by each source of systematic error and redo the fit.



- Add the resulting deviations below (above) the central value in quadrature to give the lower (upper) error band

■ Parton distributions: { ▭ } stat. error band



■ $x\bar{\Sigma}(x) \propto x^{-\delta_S}(\ldots)$: $\delta_S = +0.23 \pm 0.01(\text{stat}) \pm 0.02(\text{syst})$

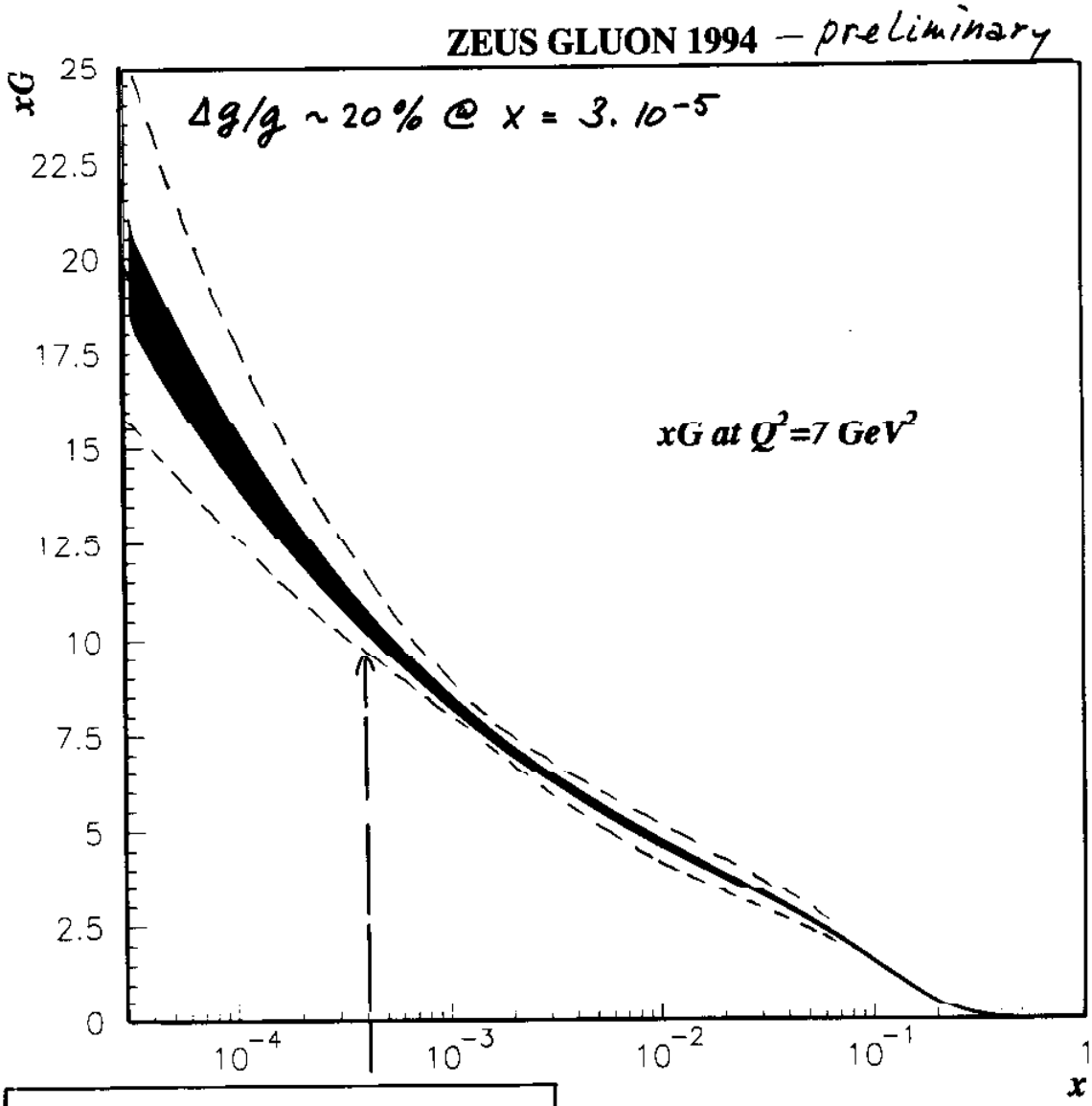
$xg(x) \propto x^{-\delta_g}(\ldots)$: $\delta_g = +0.24 \pm 0.02(\text{stat}) \pm 0.05(\text{syst})$

■ Momentum sum:

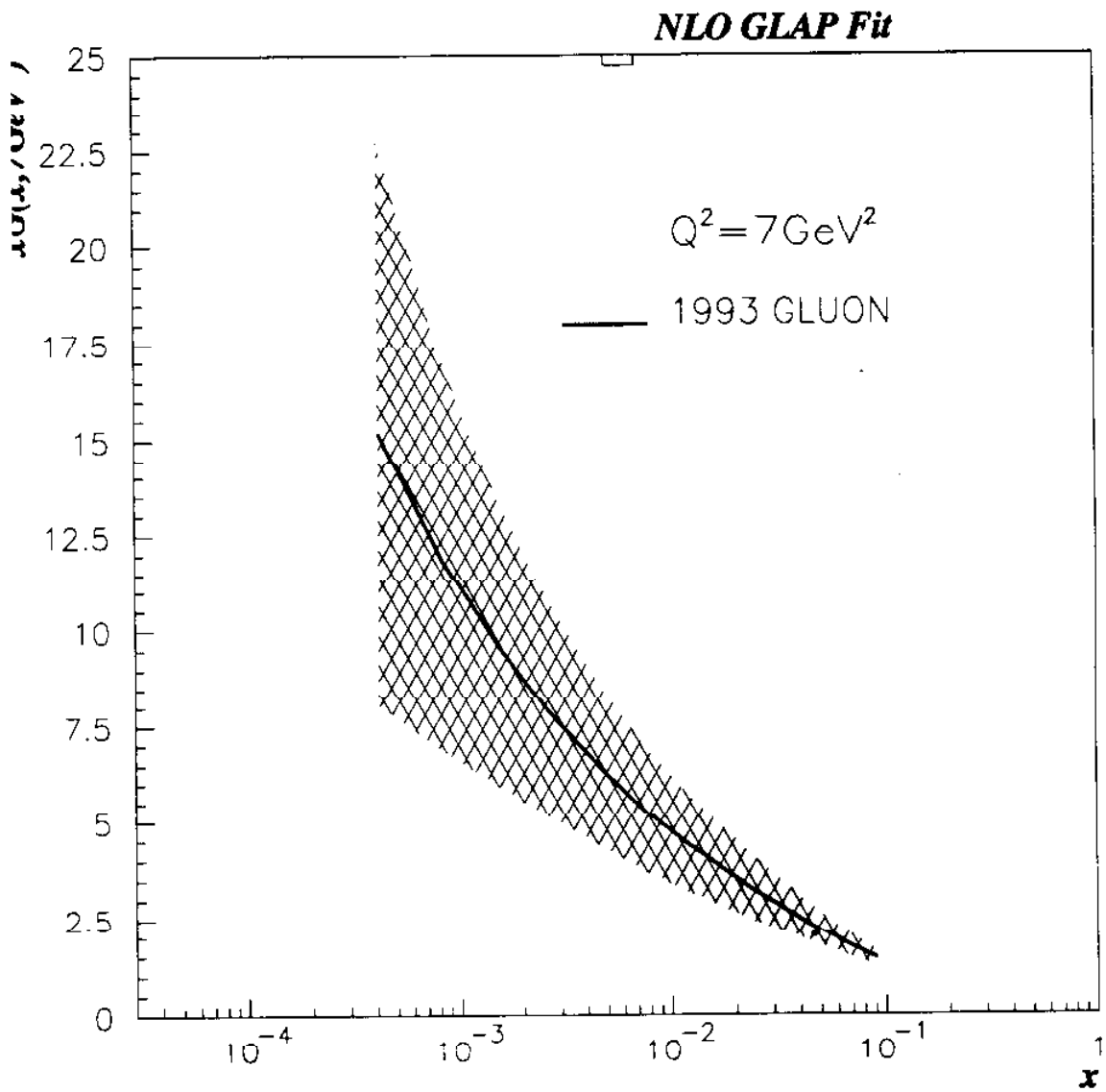
MSR = 0.555 (quarks) + 0.445 (gluons)

$xg(x)$ vs x @ $Q^2 = 7\ GeV^2$

- Blue band: statistical error
- Yellow band: stat ⊕ syst error
- Not shown: $\Delta\alpha_s \rightarrow \Delta xg \sim 10\%$, $x = 4.10^{-4}$



$x = 4.10^{-4}$	stat	7 %
	ZEUS	10 %
	NHC	1 %
	total	12 %



$x = 4.10^{-4}$ stat 25%

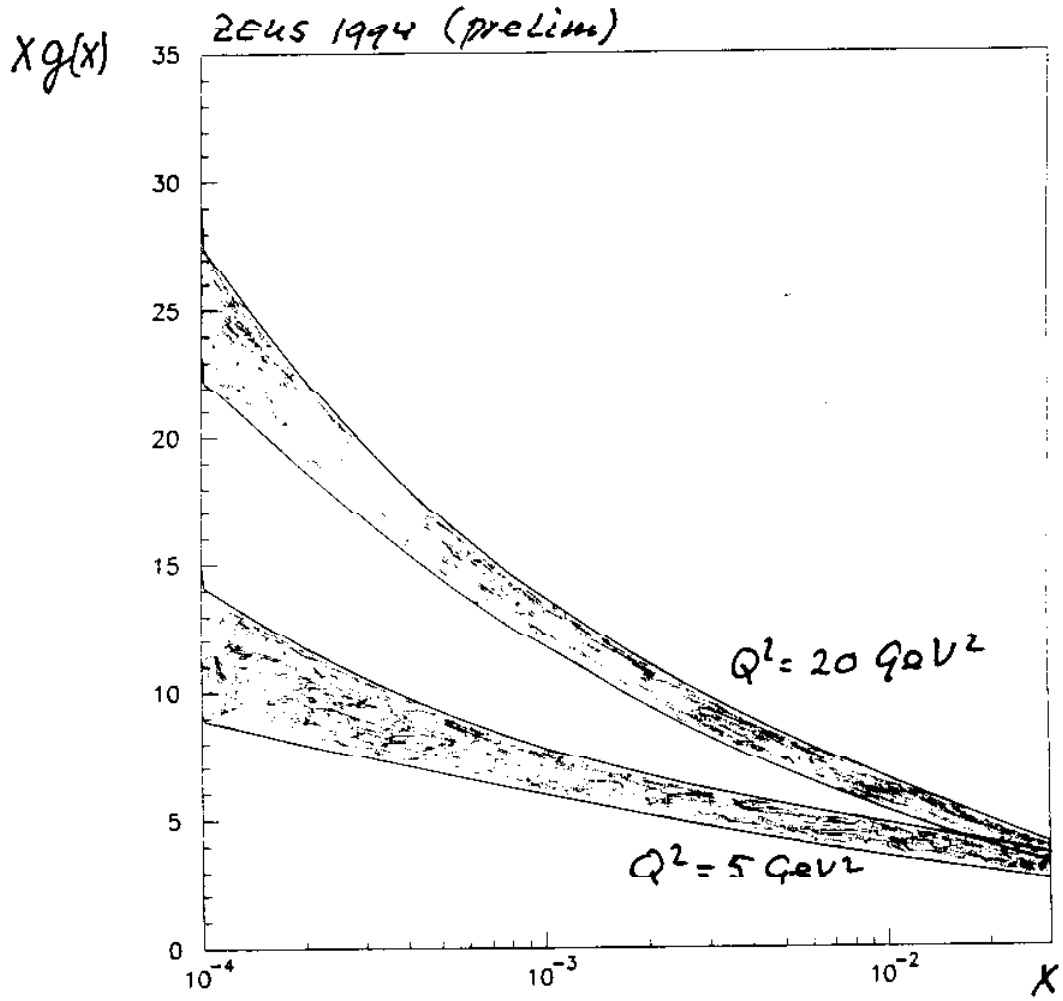
ZEUS 22%

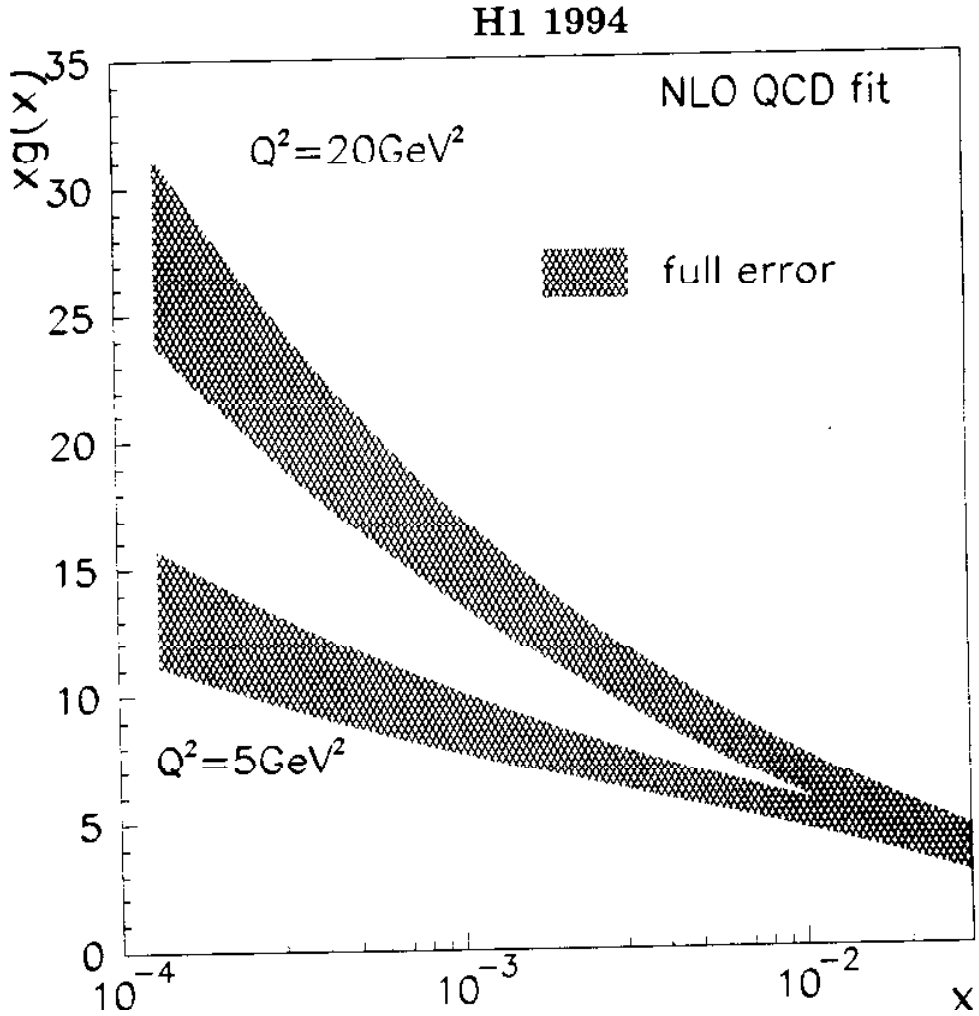
NMC 33%

total 47%

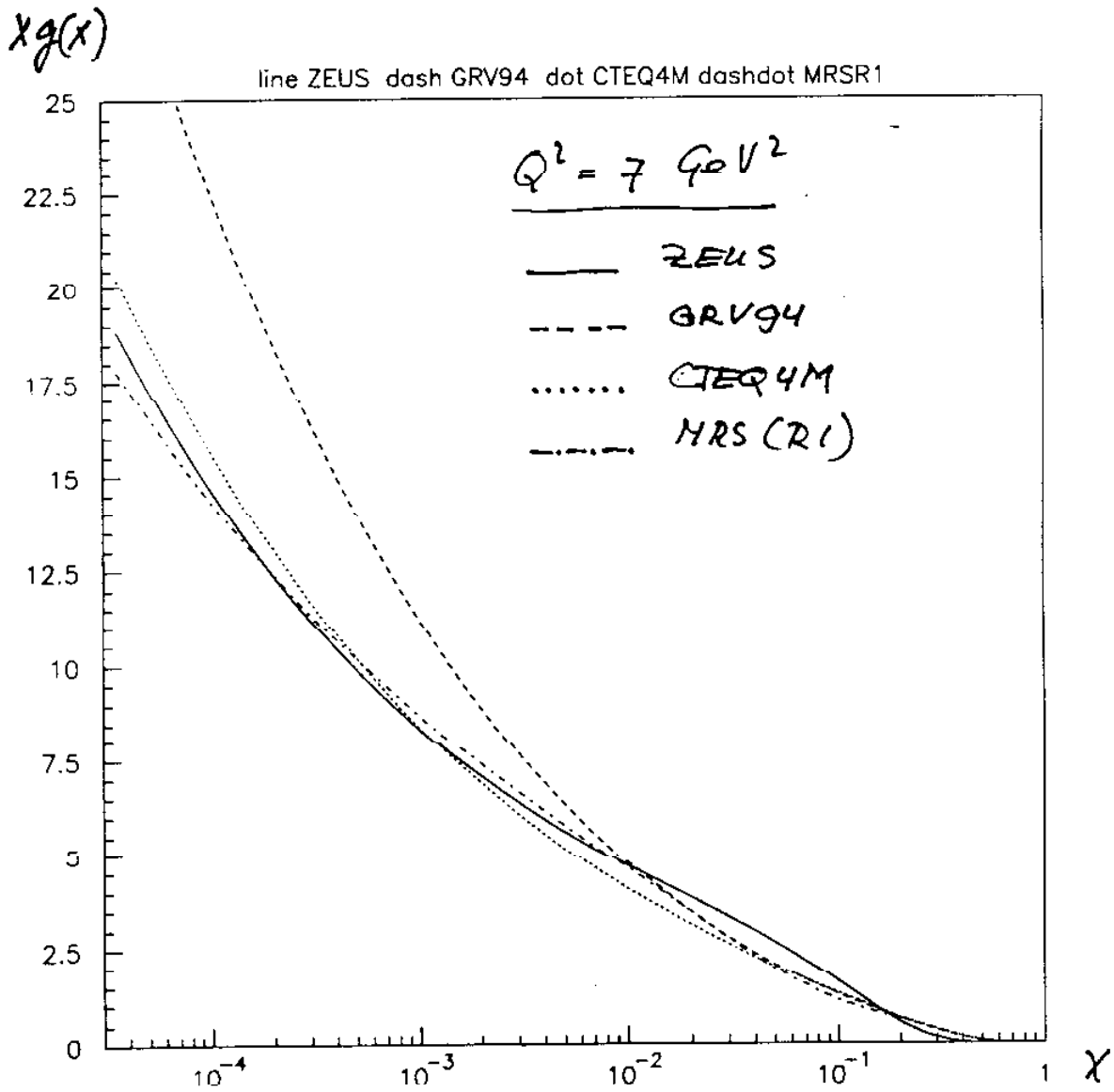
■ $xg(x)$ at $Q^2 = 5$ and $20\ GeV^2$

[redacted] { total error band (incl. $\Delta\alpha_s$).

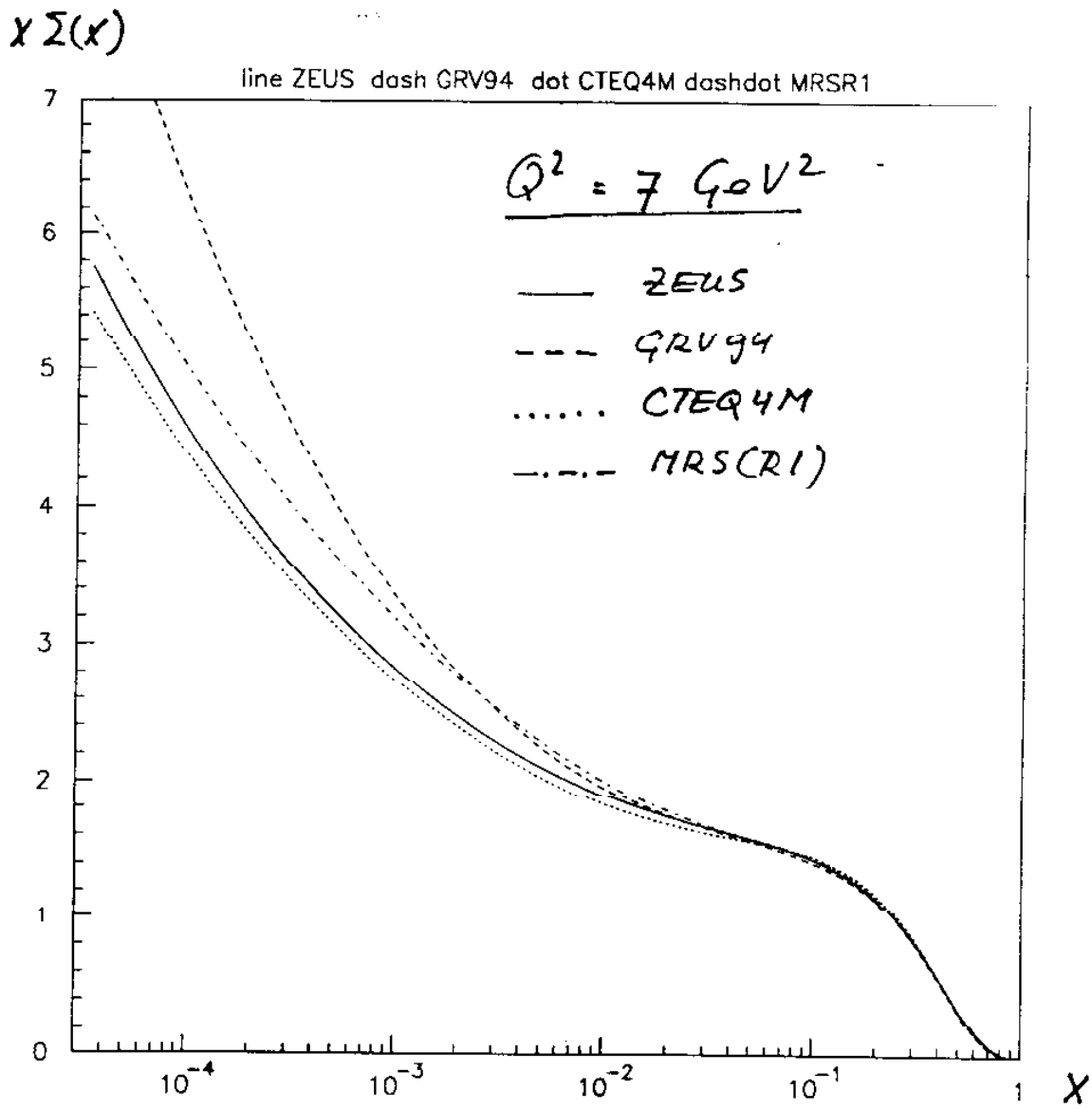



H1 1994
NLO QCD fit
$Q^2=20GeV^2$
full error
$Q^2=5GeV^2$
xg(x)
35
30
25
20
15
10
5
0
10^{-4}
10^{-3}
10^{-2}
x

Comparison $xg(x)$ with existing pdf's

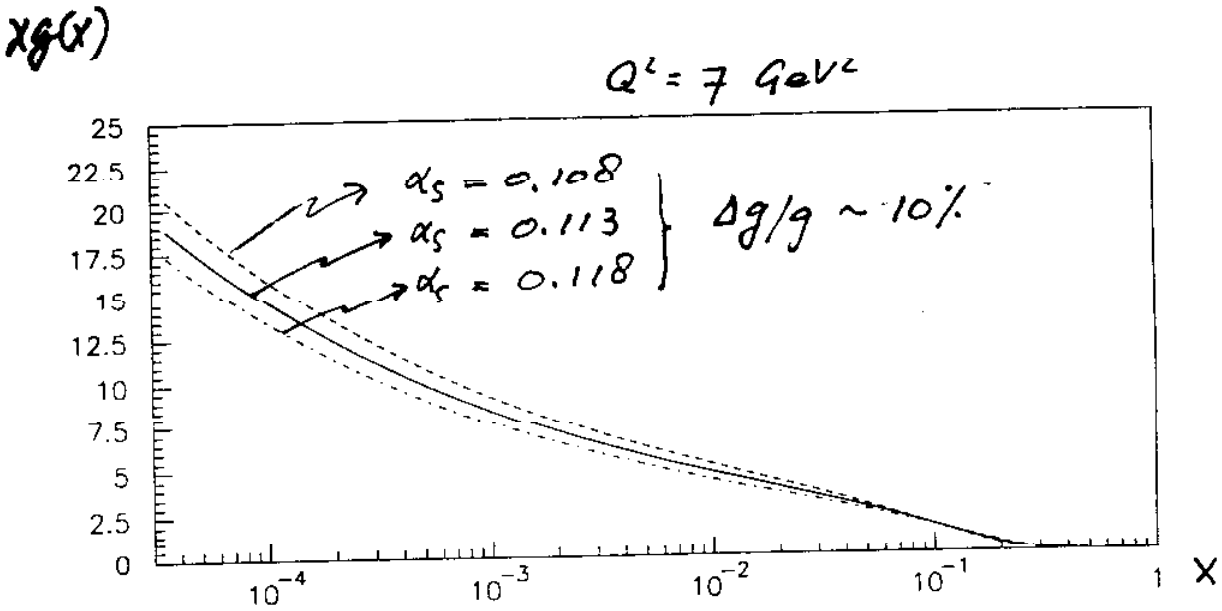


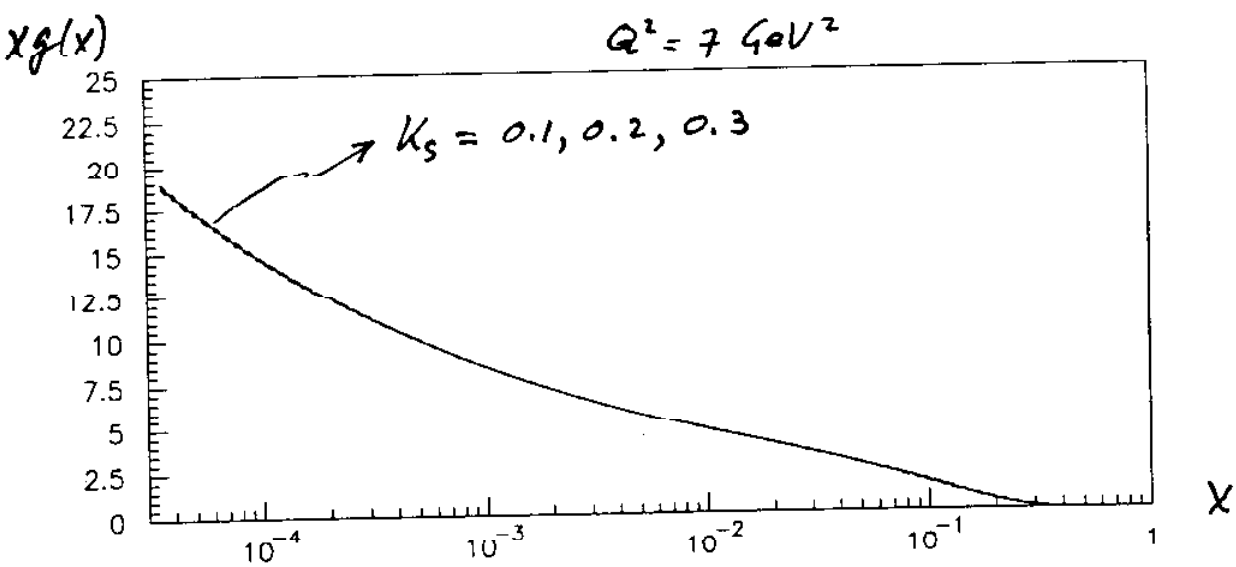
Comparison $x\Sigma(x)$ with existing pdf's



■ Sensitivity of $xg(x)$ to:

- (top) $\alpha_s(M_Z^2) = 0.108, 0.113, 0.118$
- (bottom) strange $= k_s \times$ sea
  $k_s = 0.1, 0.2, 0.3$





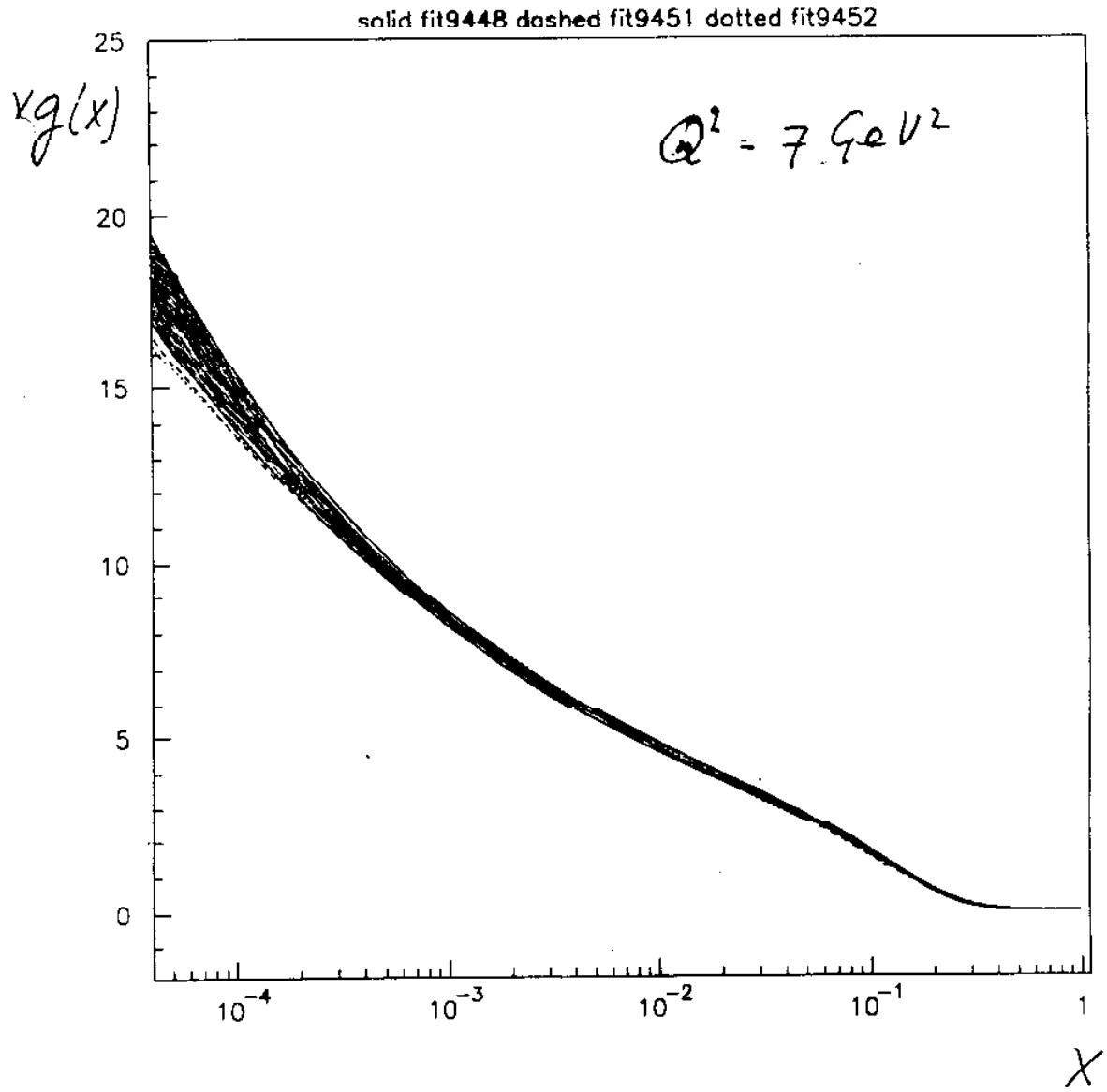
Different parametrisations for $xg(x)$

--- $xg(x) = A x^{\delta} (1-x)^{\eta}$ $\chi^2 = 463/408-13$

stat error { [redacted] $xg(x) = (\ldots) \cdot (1+\gamma x)$ $\chi^2 = 463/408-14$

..... $xg(x) = (\ldots) \cdot (1+\varepsilon\sqrt{x}+\gamma x)$ $\chi^2 = 462/408-15$

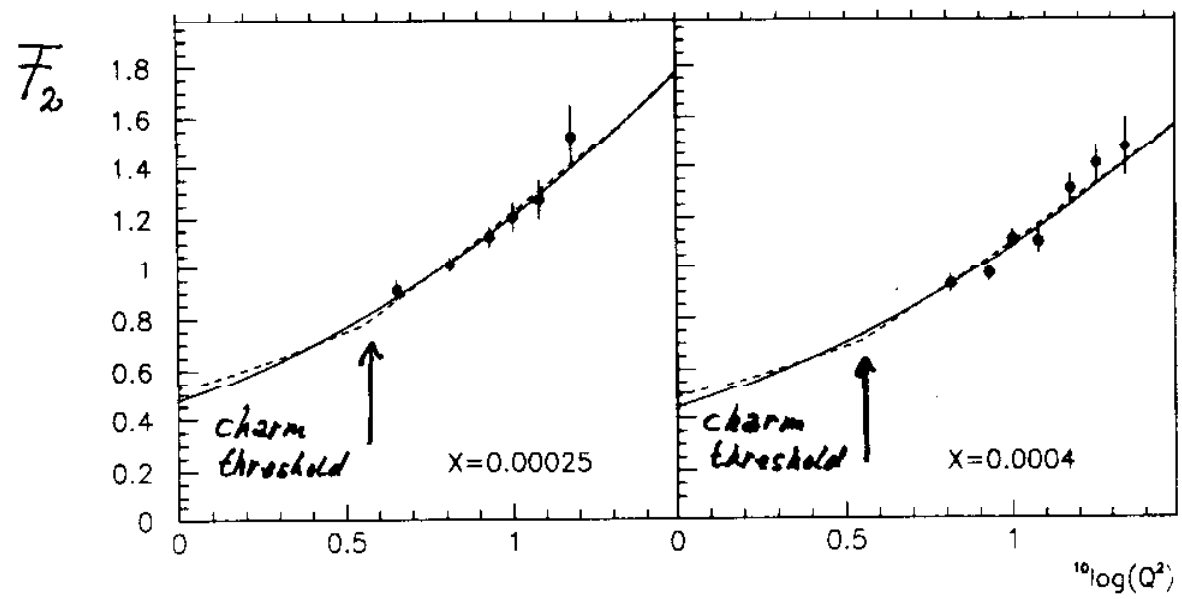
ZEUS preliminary



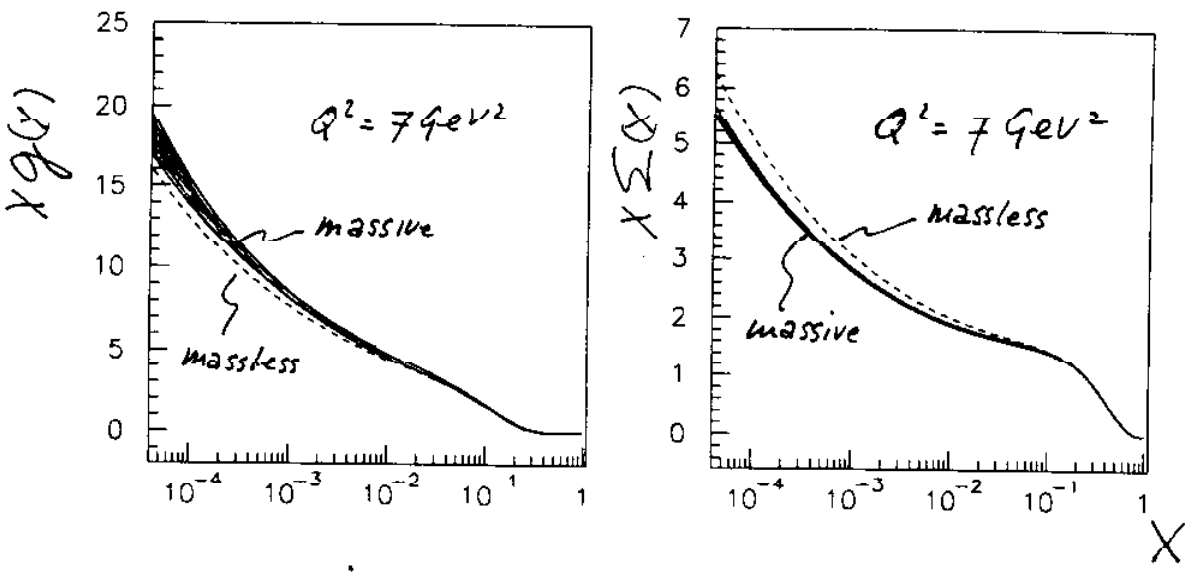
■ Comparison of massive vs massless fit

—— $F_2 = F_2^{uds} + F_2^{c}$
↳ from $f = 3$ evolution : $\chi^2 = 463/408-14$

- - - F_2 from $f = 3, 4, 5$ evolution : $\chi^2 = 472/408-14$



■ stat error { ■ $xg(x)$, $x\Sigma(x)$ from massive fit ($f=3$)
- - - $xg(x)$, $x\Sigma(x)$ from massless fit ($f=4$)
@ $Q^2 = 7\,GeV^2$

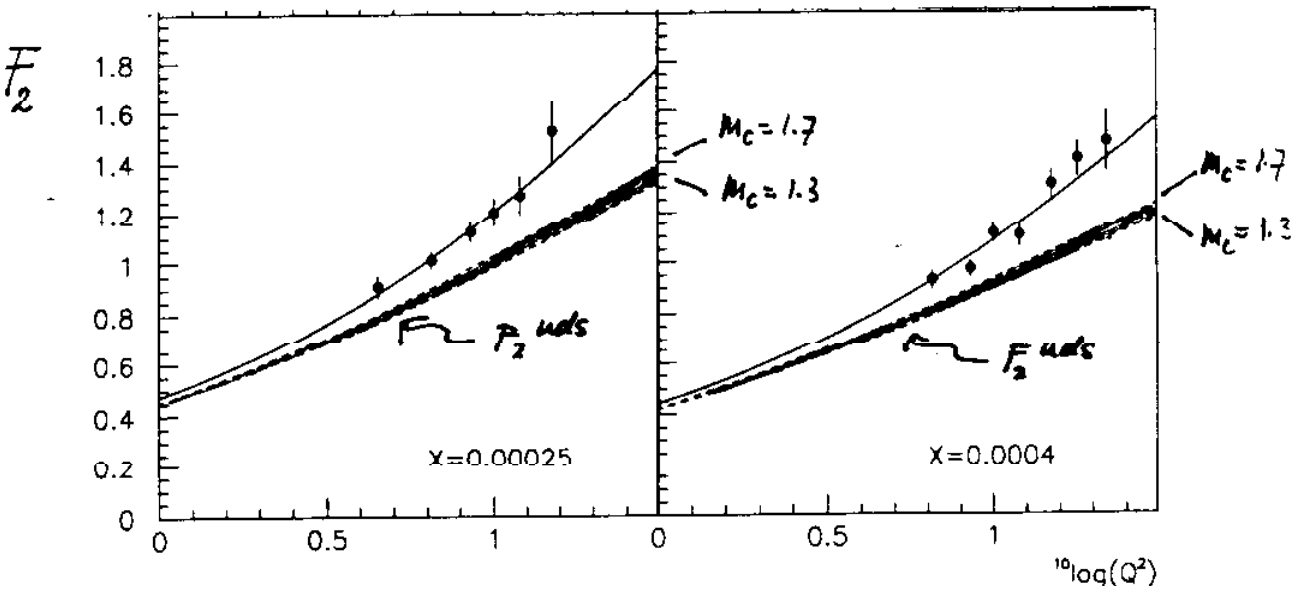


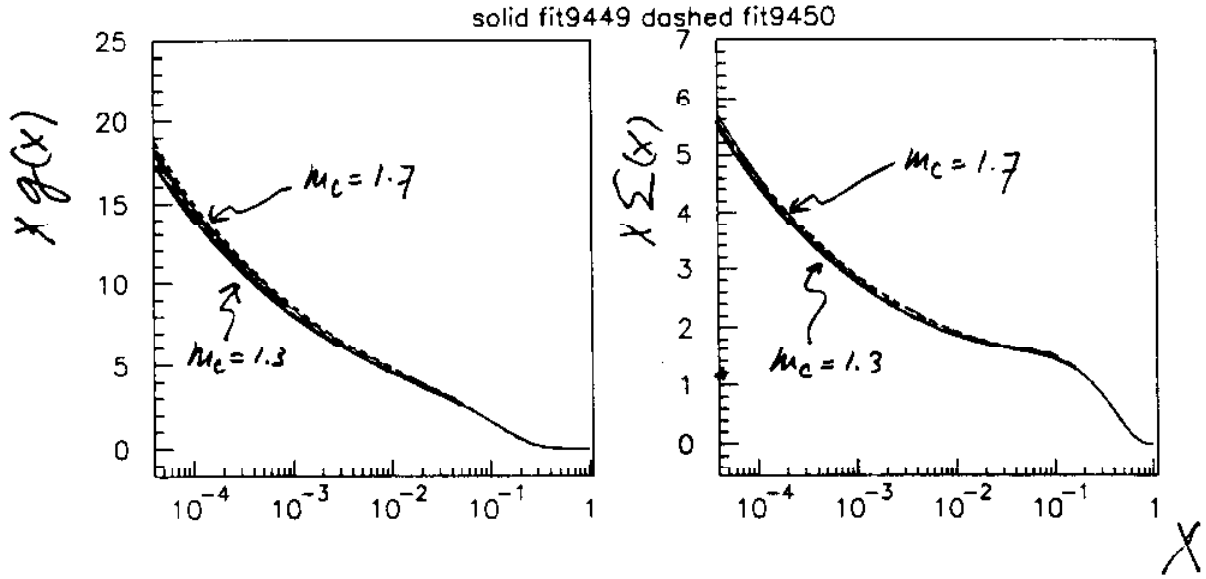
Sensitivity to charm mass

$m_c = 1.3$ GeV $\rightarrow$ $\chi^2 = 459 / 408-14$

$m_c = 1.5$ GeV $\rightarrow$ $\chi^2 = 463 / 408-14$

$m_c = 1.7$ GeV $\rightarrow$ $\chi^2 = 469 / 408-14$

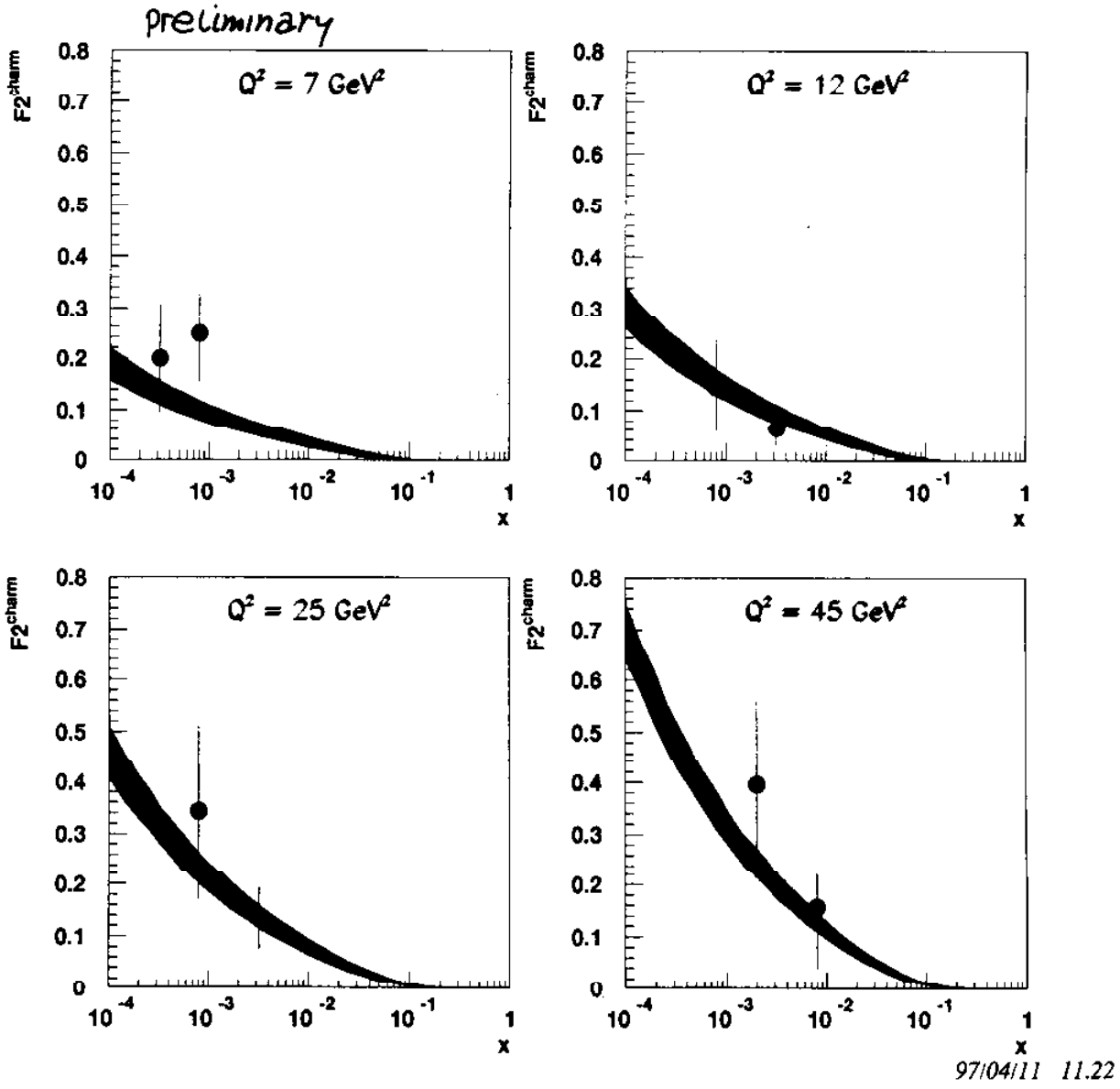




Comparison with ZEUS F_2^c (1994)

- ZEUS F_2^c

$\Rightarrow$ QCD prediction $\pm$ $\begin{cases} \text{stat. error } (+) \\ \text{syst. error } (+) \\ \Delta\alpha_s \ (0.005) \ (+) \\ \Delta m_c \ (0.2) \end{cases}$



* ZEUS F_2^c → presentation by J. Roldan.

Upgrades: (close to final)

- Inclusion of newest NMC data $F_2^{p,d}$ @ 90, 120, 200, 280 GeV.
- Inclusion of NMC F_2^d/F_2^p (?)
- Inclusion of BCDMS F_2^p, F_2^d.
- Further systematic investigations.
- New technique to propagate systematic errors

$$\Rightarrow \Delta F^{stat}(x,Q^2)$$
$$\Delta F^{syst}(x,Q^2)$$

for any function F of the fitted parameters e.g: pdf's, F_2, F_2^c, F_L, ...

Systematic error propagation

(known technique)

$$\chi^2 = \sum_{data} \left(\frac{F^{model}_{(p,s)} - F^{meas}}{\sigma_{stat}} \right)^2 + \sum_{\ell} s_\ell^2$$

$$F^{model}_{(p,s)} = F^{qcd}_{(p)} \left(1 - \sum_{\ell} s_\ell \Delta_\ell \right)$$

- p = fit parameters: $A_g, \delta_g, \ldots, \alpha_s, \ldots$
- Δ_ℓ = 1 sd relative syst. error from source ℓ
- s_ℓ = 'syst parameter': offset in units of s.d.

Minimise χ^2 wrt. p keeping s fixed:

$$s_\ell \equiv 0 \qquad \langle \Delta s_\ell \Delta s_k \rangle = \delta_{\ell k}$$

Calculate <u>full</u> Hessian @ χ^2_{min}:

$$H = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial p \partial s} = \begin{array}{c} \begin{array}{cc} \leftarrow p \rightarrow & \leftarrow s \rightarrow \end{array} \\ \begin{bmatrix} M & C \\ C^T & V_s^{-1} \end{bmatrix} \end{array}$$

One can show that:

$$V_p^{stat} = M^{-1}$$

$$V_p^{syst} = M^{-1} C C^T M^{-1}$$

- $\Rightarrow$ standard error propagation to any $F(p)$:

$$(\Delta F)^2_{stat,\,syst} = \frac{\partial F}{\partial p} \cdot V_p^{stat,\,syst} \cdot \frac{\partial F}{\partial p}$$

- Advantage:
  - H can be calculated with MINUIT
  - V_p^{stat} and V_p^{syst} are obtained by a few matrix inversions and multiplications.
  - Easy to build in correlations between datasets (e.g NMC): define one parameter s_ℓ for two or more datasets.
  - Fast.

- It has been checked that this method yields the same results as the much more elaborate 'offset' method.

- Extrapolation of high x structure functions to large Q^2

- Aim: σ^{NC} and $\Delta\sigma^{NC}$, $x \sim 0.5$ $Q^2 \sim 10^4\ GeV^2$

- - SLAC
  - BCDMS } $F_2^{p,d}$
  - NMC
  - CCFR xF_3

  $x > 0.1$
  $Q^2 > 4\ GeV^2$
  $W^2 > 10\ GeV^2$

- $$\left.\begin{aligned} xg &= A_g (1-x)^{\eta_g} \\ x\Sigma &= A_s (1-x)^{\eta_s} (1 + \varepsilon_s \sqrt{x} + \gamma_s x) \\ x\Delta ud &= A_{ns}\, x^{\delta_{ns}} (1-x)^{\eta_{ns}} \\ xq_v &= A_v\, x^{\delta_v} (1-x)^{\eta_v} \end{aligned}\right\} \quad Q_0^2 = 7\ GeV^2$$

- $f = 3,4,5$ light quark evolution: $Q_c^2 = 4\ GeV^2$
  $Q_b^2 = 25\ GeV^2$

- $\alpha_s(M_Z^2) = 0.118\ (\pm 0.005)$

$$\Rightarrow \quad \sigma^{\pm} \equiv \frac{xQ^4}{2\pi\alpha^2 Y_+} \frac{d^2\sigma^{\pm}}{dx\,dQ^2} = F_2 - \frac{y^2}{Y_+} F_L \mp x F_3$$

for $e^{\pm}p$ scattering (γ^*, $\gamma^* Z$, Z terms included)

- Full analysis of systematic errors taking all correlations into account.
  (24 independent sources of syst. error included)

NC structure functions vs x

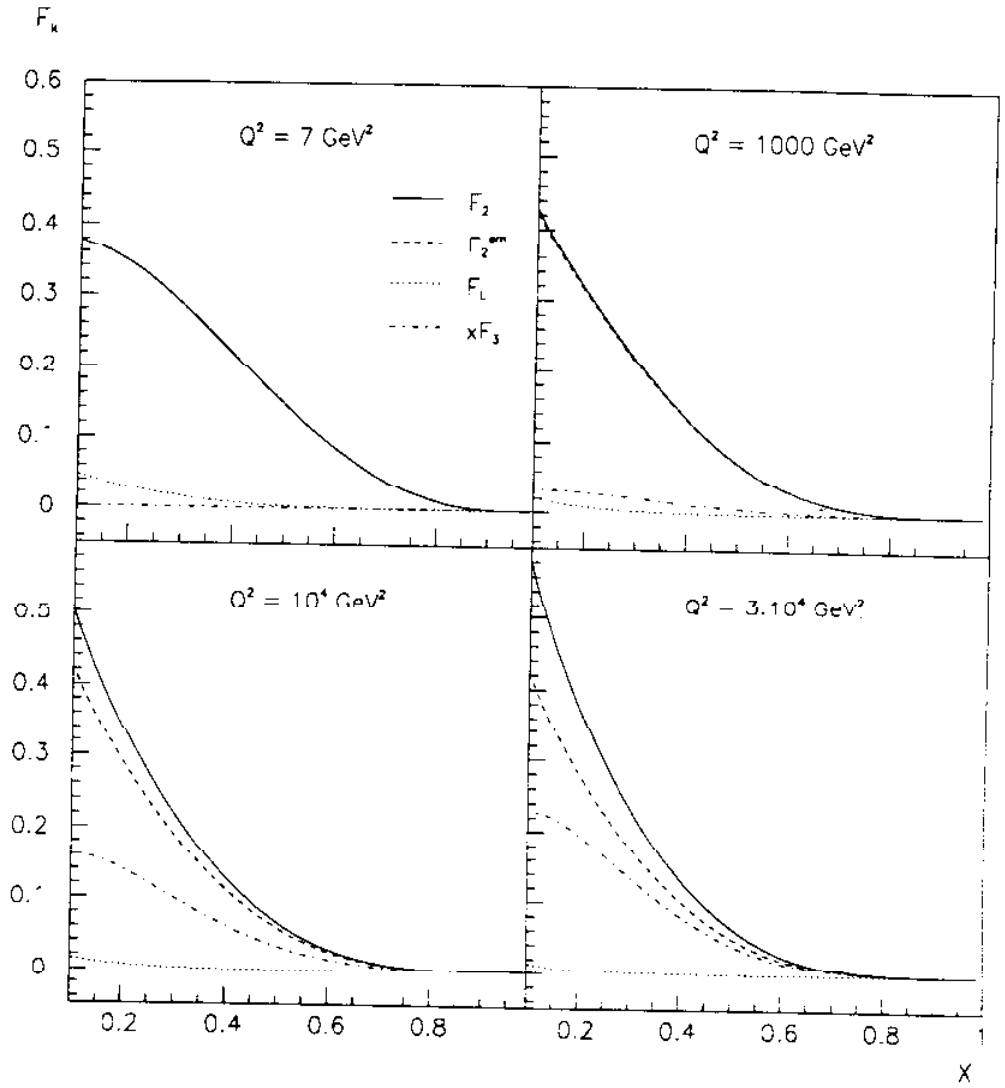


Figure 6: The NC structure functions F_2^{ep} (solid curve), F_2^{em} (γ exchange contribution, dashed curve), F_L^{ep} (dotted curve) and xF_3^{ep} (dashed-dotted curve) versus x for different values of Q^2.

□ F_2 vs Q^2

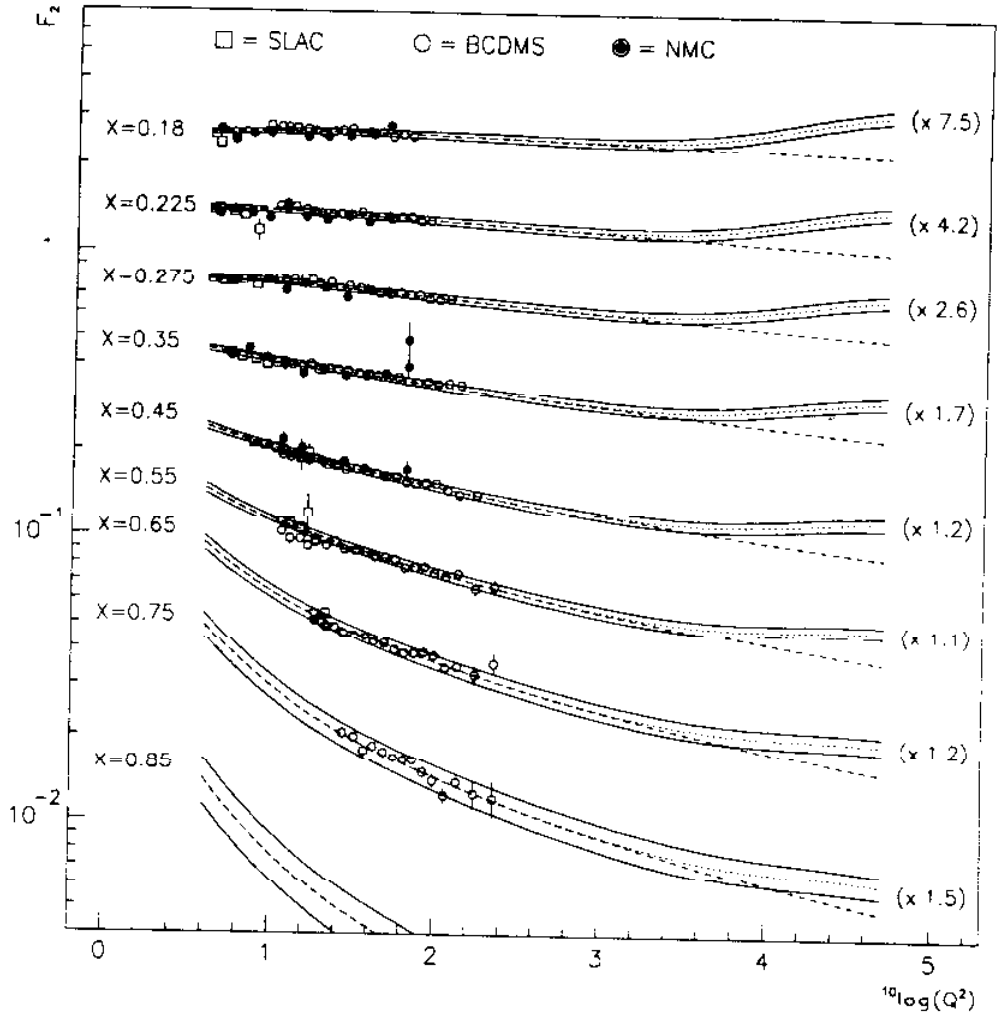
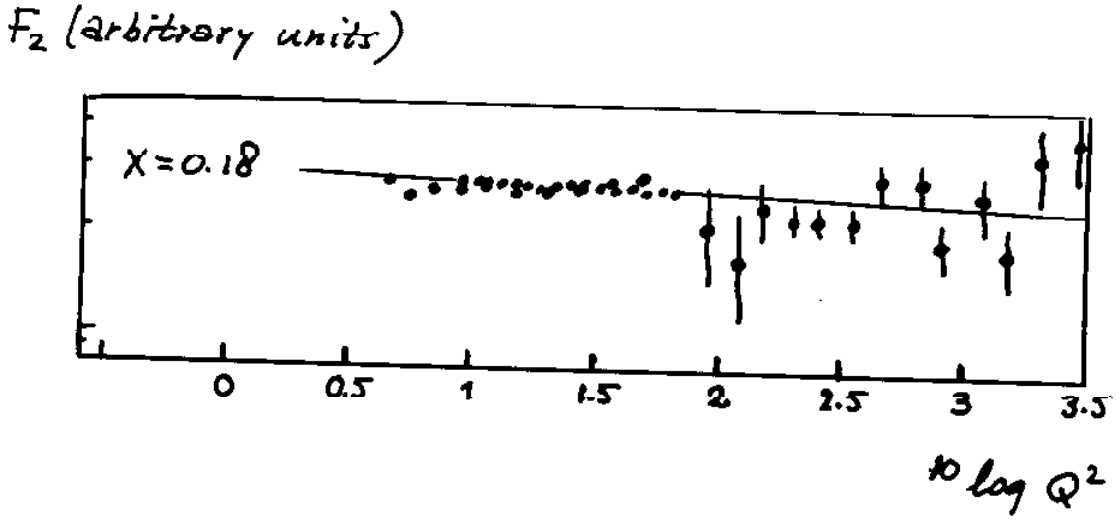


Figure 2: The structure function F_2^{ep} as function of Q^2 for several high-x bins, multiplied by the factors indicated. Not shown are data below the cuts $Q^2 = 4$ GeV^2 and $W^2 = 10$ GeV^2. The dotted curves correspond to the F_2 prediction including the contributions from γZ interference and Z exchange. The solid lines indicate the total error band. The dashed lines represent the γ exchange contribution $F_2^{\gamma m}$.

QCD extrapolation ⇔ F_2 (ZEUS, 1994)



- ZEUS
- BCDMS
- NMC

■ $xF_3^{\nu N}$ vs Q^2

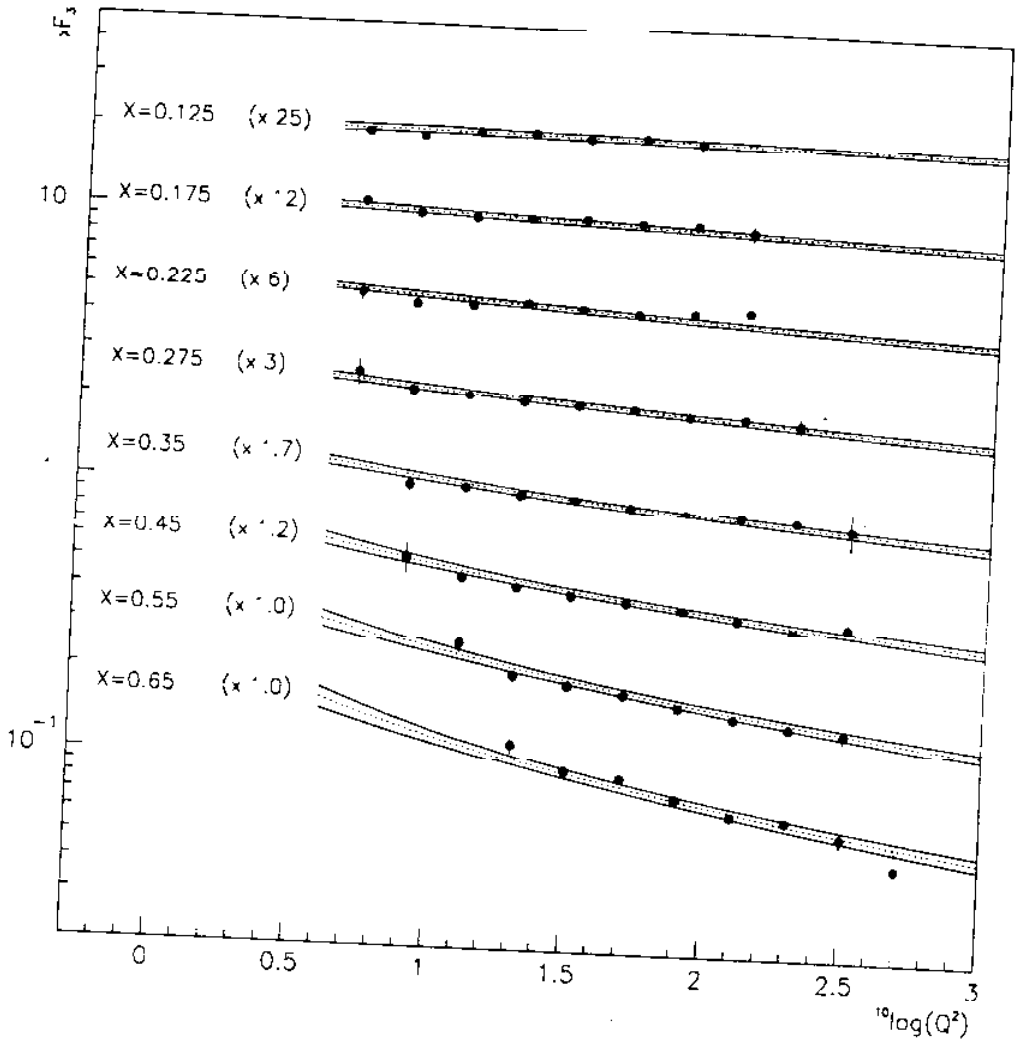


Figure 3: The structure function $xF_3^{\nu N}$ from CCFR as function of Q^2 for several high-x bins, multiplied by the factors indicated. Not shown are data below the cuts $Q^2 = 4$ GeV^2 and $W^2 = 10$ GeV^2. The dotted curves correspond to the QCD fit described in the text; the solid curves indicate the total error band.

$$\tilde{\sigma}^- \equiv F_2 - \frac{y^2}{Y_+} F_L + x F_3$$

$$\Rightarrow x \sim 0.5 \quad Q^2 \sim 3.10^4 \quad \frac{\Delta\sigma^-}{\sigma^-} \sim 5\%$$

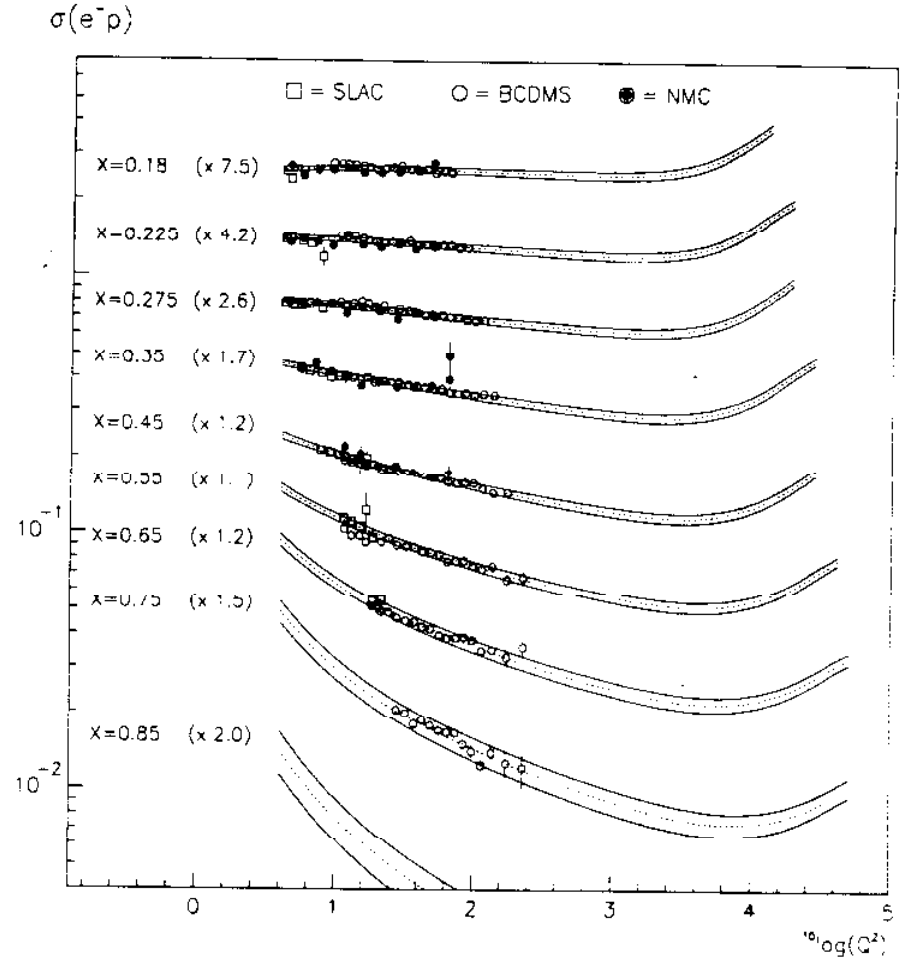


Figure 8: The reduced cross-section $\tilde{\sigma}$ (see text) for e^-p scattering at Hera energies ($E_e = 27.5$, $E_p = 820$ GeV). The bands represent the total error. The fixed target F_2 data are converted to $\tilde{\sigma}$ by applying a small correction for F_L.

$$\tilde{\sigma}^{+} \equiv F_2 - \frac{y^2}{Y_+} F_L - x F_3$$

(24)

$$\Rightarrow x \sim 0.5 \quad Q^2 \sim 3.10^4 \quad \frac{\Delta\sigma^+}{\sigma^+} \sim 9\%$$

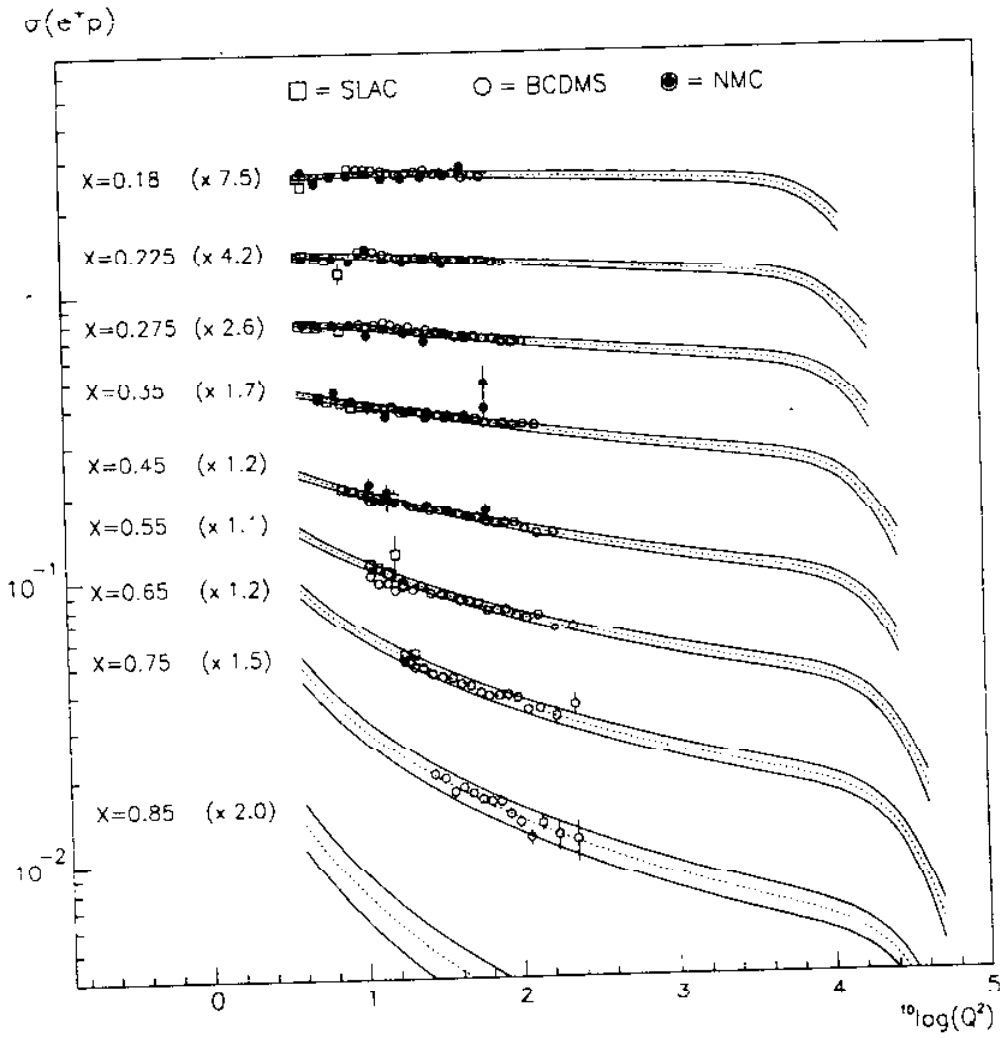


Figure 9: The reduced cross-section $\tilde{\sigma}$ (see text) for e^+p scattering at Hera energies ($E_e = 27.5$, $E_p = 820$ GeV). The bands represent the total error. The fixed target F_2 data are converted to $\tilde{\sigma}$ by applying a small correction for F_L.

$$\tilde{\sigma}^{+} \equiv F_2 - \frac{y^2}{Y_+} F_L - x F_3$$

o Comparison with MRSA and CTEQ3M

⇒ Comparing pdf sets does not give a reliable estimate of the uncertainty.

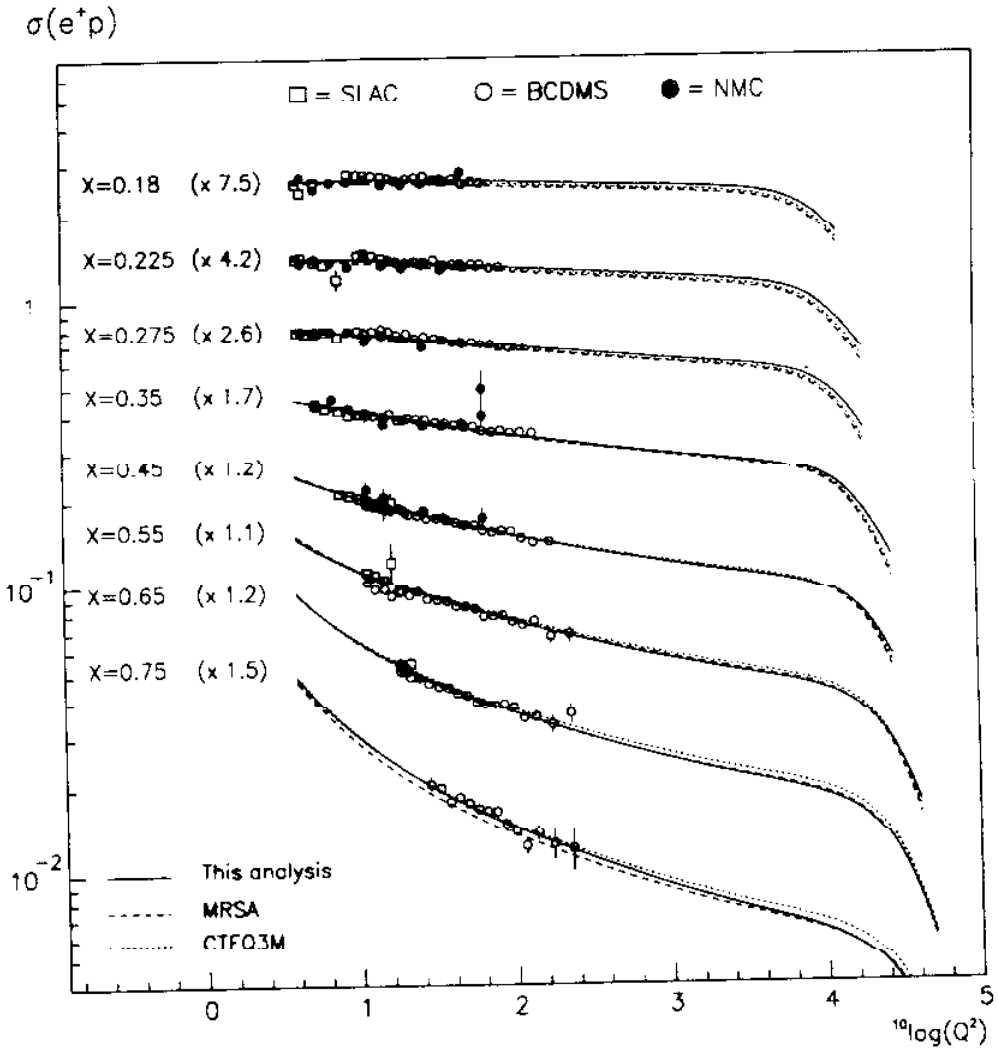


Figure 12: The reduced cross-section $\tilde{\sigma}$ (see text) for e^+p scattering at Hera energies from the present analysis (solid curves) compared to predictions based on the parton distributions from MRSA (dashed curves) and CTEQ3M (dotted curves).

Summary

- In the kinematic range investigated, NLO QCD evolution gives a good description of the F_2 data.

- The gluon distribution is extracted down to $x \sim 5.10^{-5}$ with an accuracy of $\sim 20\%$

- The main additional source of uncertainty ($\sim 10\%$) is due to $\Delta \alpha_s(M_Z^2) \simeq 0.005$.

- The QCD prediction of F_2^c is consistent with the ZEUS data.

- $x \sim 0.5 \quad Q^2 \sim 3.10^4 \qquad \frac{\Delta\sigma}{\sigma}(e^-p)^{NC} \sim 5\%$

  $\frac{\Delta\sigma}{\sigma}(e^+p)^{NC} \sim 9\%$

  from QCD extrapolation of $x > 0.1$ fixed tgt data + propagation of stat & syst. errors