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Diffractive parton densities in the semiclassical approach

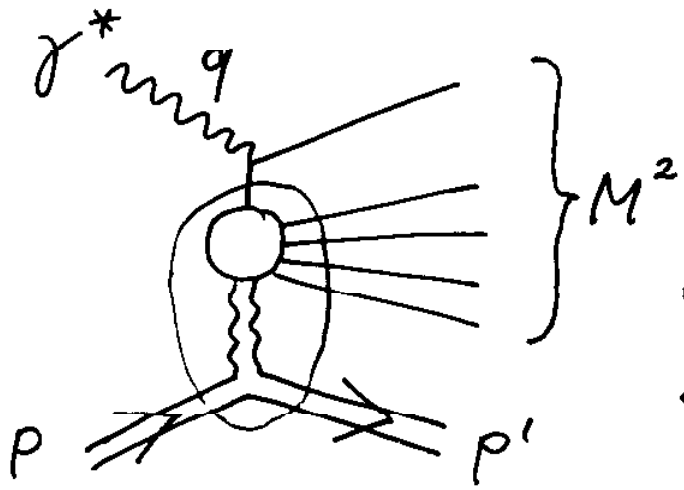
(hep-ph/970237

Outline

- Introduction
- Production of several high-$p_\perp$ particles in the semiclassical approach (scalar case)
- Identification of the correspondi diffractive parton density
- Generalization to the quark and gluon case
- Conclusions

Diffractive parton distributions

(Berera / Soper)



parameters:

DIS: x, Q^2

Diffraction: M^2

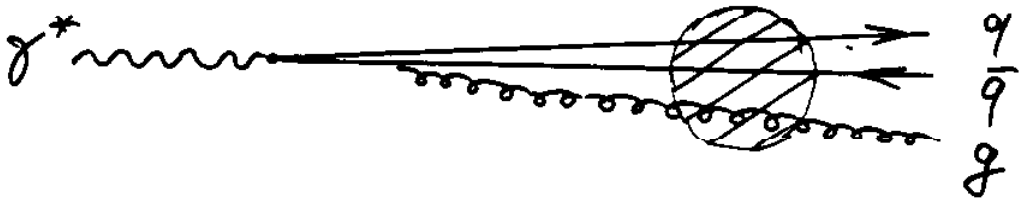
alternatively: $\underline{\underline{\xi}} = x \frac{M_+^2}{Q^2}$

basic quantity: $\frac{d\sigma_{L,T}(x, Q^2, \xi)}{d\xi}$

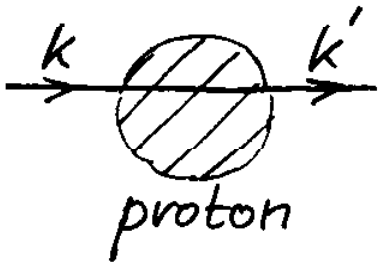
$$\frac{d\sigma(x, Q^2, \xi)}{d\xi} = \int dy\, \hat{\sigma}(y, x, Q^2) \frac{df^{diff}(\xi, y)}{d\xi}$$

The semiclassical approach

(in collab. with W. Buchmüller & M. McDermott



The basic ingredient is the effective vertex



$$= \quad 2\pi\,\delta(k_0'-k_0)\,2k_0\,\underline{\tilde{U}(k_\perp'-k_\perp}$$

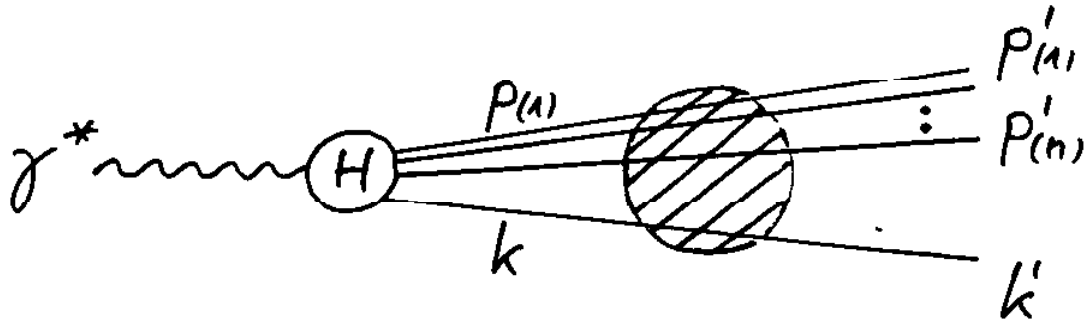
where

$$\underline{U(x_\perp)} = P\exp\left(-\frac{i}{2}\int A_- dx_+\right)$$

A <u>leading twist</u> contribution arises only if the photon wave function contains at least <u>one small-$k_\perp$ particl</u>

The semiclassical approach

– more general framework –



amplitude:

$$i\,2\pi\delta(q_0-q_0')\,T = \int T_H \cdot$$

$$\cdot \prod_j \left(\frac{i}{p_{(j)}^2}\, 2\pi\delta(p'_{(j)0}-p_{(j)0})\, 2p_{(j)0}\, \underline{\tilde{U}(p'_\perp-p_\perp)}\, \frac{d^4p_{(j)}}{(2\pi)^4} \right.$$

$$\cdot \left(\frac{i}{k^2}\, 2\pi\delta(k'_0-k_0)\, 2k_0\, \underline{\tilde{U}(k'_\perp-k_\perp)} \right),$$

(for scalar partons)

use: $$\frac{1}{p_{(j)}^2} = \frac{1}{p_{(j)+}p_{(j)-} - p_{(j)\perp}^2 + i\varepsilon},$$

perform $p_{(j)+}$, $p_{(j)-}$ – integrations

result:

$$T = \int T_H \prod_j \left(\underline{\tilde{U}(p'_{(j)\perp} - p_{(j)\perp})} \frac{d^2 p_{(j)\perp}}{(2\pi)^2} \right) \frac{2k_0}{k^2} \underline{\tilde{U}(k'_\perp - k_\perp}$$

$p_\perp$ - integrations

- substitute $d^2 p_{(n)\perp} \longrightarrow d^2 k_\perp$
- make Fourier transformations explicit

$\downarrow$

exponentials:

$$\prod_{j=1}^{n-1} \exp\left[-i x_{(j)\perp} (p'_{(j)\perp} - \underline{p_{(j)\perp}})\right]$$

$$\cdot \exp\left[-i x_{(n)\perp} (p'_{(n)\perp} + k_\perp + \sum_1^{n-1} \underline{p_{(j)\perp}})\right]$$

$$\cdot \exp\left[-i y_\perp (k'_\perp - k_\perp)\right]$$

$p_\perp$-integrations $\Longrightarrow$ $\boxed{\prod_{j=1}^{n-1} \delta^2(x_{(n)\perp} - x_{(j)\perp})}$

(all high-$p_\perp$ parton are close together!

Colour structure

$$T_{colour} = T_H^{a_1 \dots a_n b} \Big(\prod_j U(x_\perp)^{a'_{(j)}}_{a_{(j)}} \Big) U(y_\perp)^{b'}_{b} \, P_{a'_1 \dots a'_n b'}$$

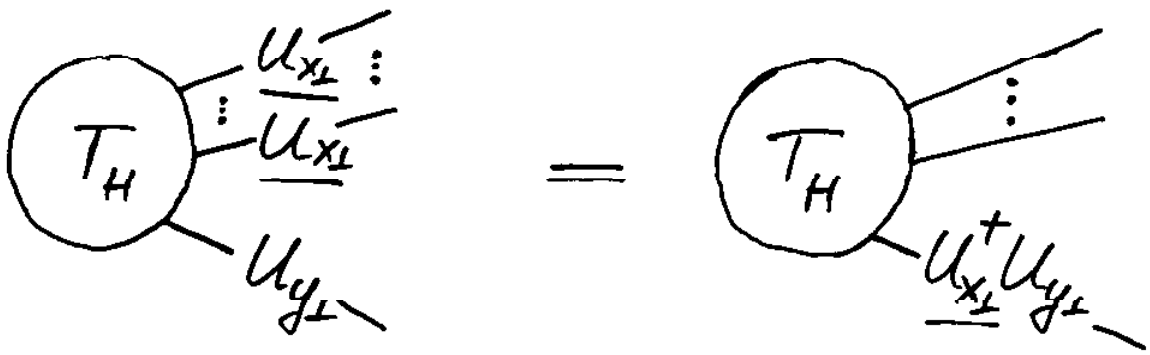
T_H is an invariant tensor in colour spac

e.g. for $n=1$

$$T_H^{ab} U_a^{a'} U_b^{b'} = T_H^{a'b}$$

$$T_H^{ab'} U_a^{a'} = T^{a'b} U_b^{+}$$

graphic representation:



result:

$$T_{colour} = T_H^{a'_1 \dots a'_n b} \left(U(x_\perp)^+ U(y_\perp) \right)^{b'}_{b} P_{a'_1 \dots a'_n b'}$$

with

$$T_H^{a'_1 \dots a'_n b} P_{a'_1 \dots a'_n b'} = \text{const.} \times \delta^{b}_{b'}$$

Cross section:

$$d\sigma = \frac{1}{2q_0 N} \int \underbrace{|T_H|^2} \int_{x_\perp} \left| \int \frac{d^2k_\perp}{(2\pi)^2} \cdot \frac{\mathrm{tr}\, \tilde{W}_{x_\perp}(k_\perp' - k_\perp)}{k^2} \right|^2 \cdot$$

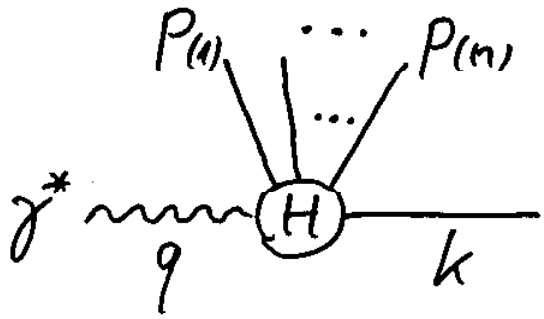
$$\cdot (2k_0)^2 (2\pi)^3 \underbrace{\delta^2\left(\sum p_{(j)\perp}\right) \delta(q_0' - q_0)} dX^{(n)}$$

where

$$W_{x_\perp}(y_\perp) = U(x_\perp) U^+(y_\perp) - \mathbb{1}$$

γ^*-parton scattering

compare our previous result with the cross section for γ^*-parton scattering



← now incoming partic

$$d\hat{\sigma} = \frac{1}{2(\hat{s}+Q^2)} |T_H|^2 (2\pi)^4 \delta^4(q-k-\Sigma p'_{(j)}) \, dX^{(n)}$$

introducing the variable y

$$\underline{-k_- = y P_-} \quad (P - \text{proton momentu}$$

this cross section can be understood as a function

$$d\hat{\sigma}(y)$$

(note: $-k_-$ is <u>large</u> in the Breit frame,

rewriting everything in terms of ξ and y the diffractive cross section reads

$$\frac{d\sigma}{d\xi} = \int_x^{\xi} dy \, d\hat{\sigma}(y) \left(\frac{df^{diff}(y,\xi)}{d\xi} \right)$$

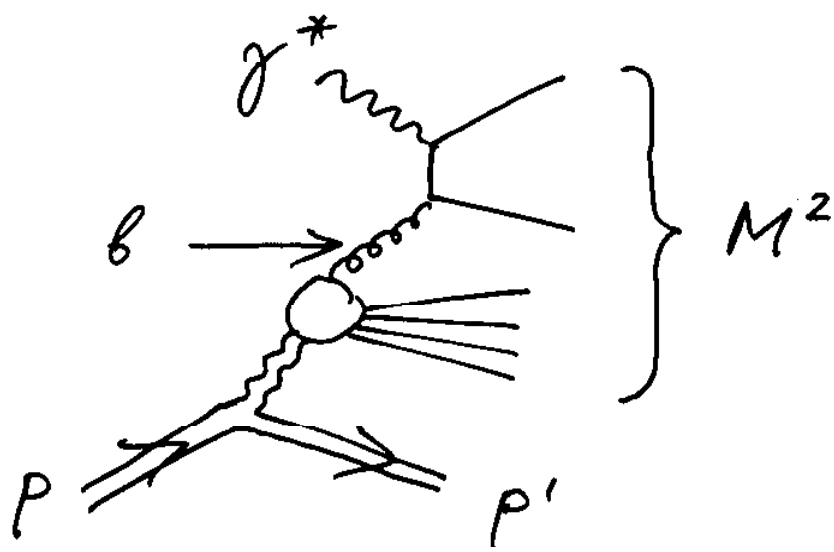
where, in the scalar case,

$$\frac{df^{diff}(y,\xi)}{d\xi} = \frac{1}{\xi^2} \cdot \frac{b}{1-b} \int \frac{d^2k_\perp' (k_\perp'^2)^2}{(2\pi)^4 N} \cdot$$

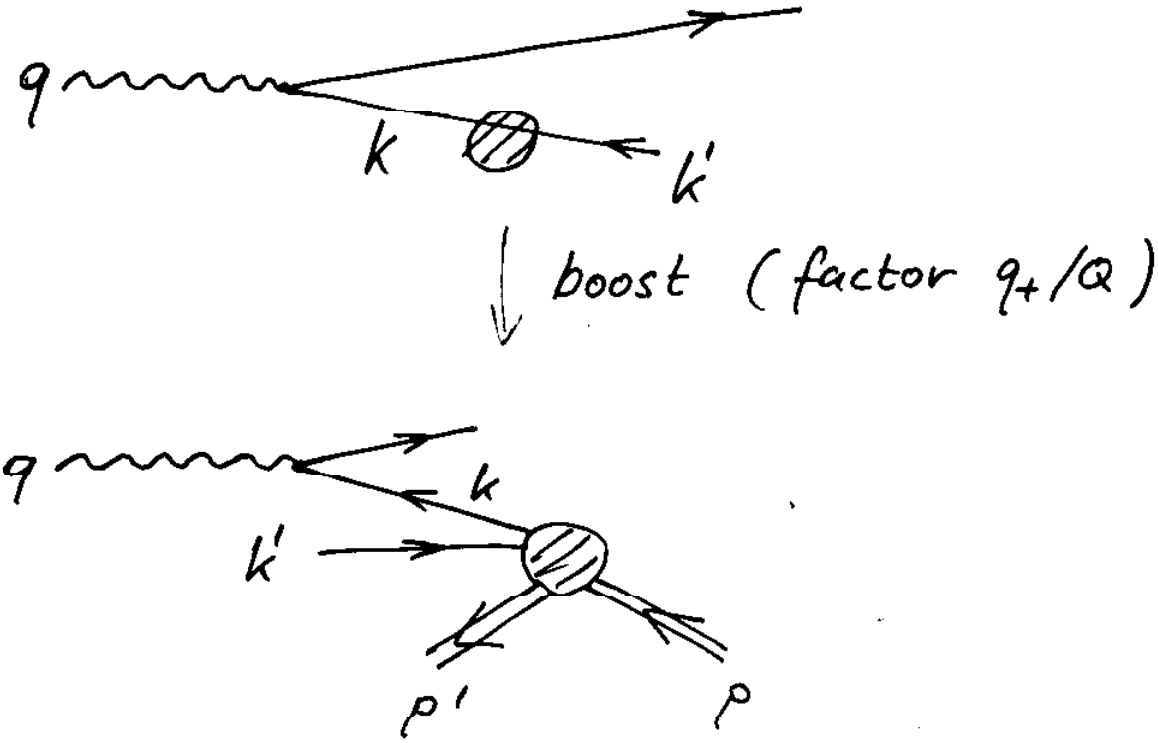
$$\cdot \int_{x_\perp} \left| \int \frac{d^2k_\perp}{(2\pi)^2} \cdot \frac{\mathrm{tr}\, \boxed{\tilde{W}_{x_\perp}(k_\perp' - k_\perp)}}{k_\perp'^2 b + k_\perp^2 (1-b)} \right|^2$$

and $\quad b = y/\xi$

<u>Note:</u> $\quad b \neq \beta = x/\xi$ in general



Another viewpoint: the Breit-frame



$$q = (q_+, q_-, q_\perp) = (Q, Q, 0_\perp)$$

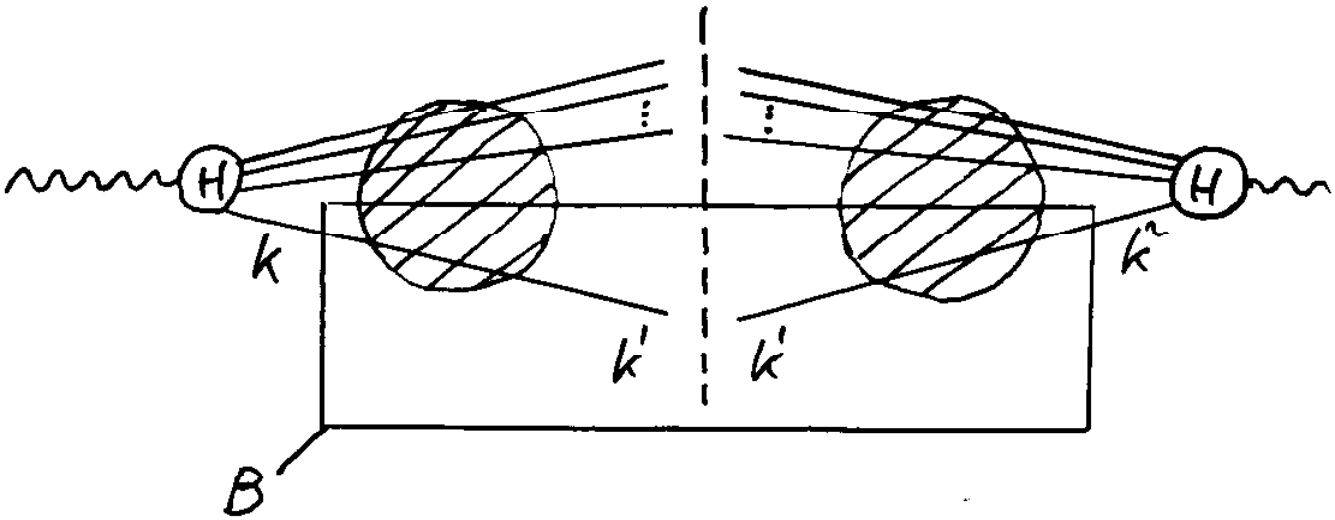
$$k = (k_+, \underline{\underline{k_-}}, k_\perp) = (\alpha Q, \underline{\underline{-Q}}, k_\perp)$$

$\Rightarrow$ 'incoming' quark with momentum $\underline{\underline{-k}}$
(usual parton picture of DIS)

$\Rightarrow$ this is a <u>non-perturbative</u> model for correlated, colour-neutral $q\bar{q}$-pairs in the proton.

see also: Nikolaev / Zakharov (1994)
Abramowicz / Frankfurt / Strikman (19
Bartels / Wüsthoff (1996)
Brodsky / Hoyer / Magnea (1996)
...

Diffractive quark & gluon densities



Only the 'box' B depends on the spin of the 'soft' parton.

The differences come from

- <u>the propagator</u> $\left(\frac{1}{k^2}, \frac{\not k}{k^2}, \frac{g^{\mu\nu}}{k^2} \right)$
- <u>the effective vertex</u> $V(k', k)$

One has to calculate and compare

$$B_{\underline{scalar}}, \quad B_{\underline{spinor}}, \quad B_{\underline{vector}}$$

Final formulae

scalar

$$\frac{df}{d\xi} = \frac{1}{\xi^2}\cdot\frac{b}{1-b}\int\frac{d^2k_\perp'(k_\perp'^2)^2}{(2\pi)^4\,\underline{\underline{N}}}\int_{x_\perp}\left|\int\frac{d^2k_\perp}{(2\pi)^2}\,\frac{\mathrm{tr}\,\tilde{W}_{x_\perp}(k_\perp'-k_\perp)}{k_\perp'^2 b + k_\perp^2(1-b)}\right|^2$$

spinor

$$\frac{df}{d\xi} = \frac{1}{\xi^2}\cdot 2\int\frac{d^2k_\perp'(k_\perp'^2)}{(2\pi)^4\,\underline{\underline{N}}}\int_{x_\perp}\left|\int\frac{d^2k_\perp}{(2\pi)^2}\cdot\frac{k_\perp\cdot\mathrm{tr}\,\tilde{W}_{x_\perp}(k_\perp'-k_\perp)}{k_\perp'^2 b + k_\perp^2(1-b)}\right|^2$$

vector

$$\frac{df}{d\xi} = \frac{1}{\xi^2}\cdot\frac{b}{1-b}\int\frac{d^2k_\perp'(k_\perp'^2)^2}{(2\pi)^4\,\underline{\underline{(N^2-1)}}}\int_{x_\perp}\left|\int\frac{d^2k_\perp}{(2\pi)^2}\cdot\frac{t^{ij}\,\mathrm{tr}\,\tilde{W}_{x_\perp}(k_\perp'-k_\perp)}{k_\perp'^2 b + k_\perp^2(1-b)}\right|^2$$

$$t^{ij} = \delta^{ij} + \frac{2k_\perp^i k_\perp^j}{k_\perp'^2}\cdot\frac{1-b}{b}$$

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general structure:

$$\frac{df(y,\xi)}{d\xi} = \frac{1}{\xi^2}\cdot g(b)$$

$$b = y/\xi$$

Conclusions

- the semiclassical (or eikonal) approac is consistent (at leading order) with th concept of diffractive parton densities
- previous results are reproduced in a unified, covariant way
- a model for the non-perturbative inpu (at low scale) is provided

Outlook

- confront models for the proton colour field with HERA data
- consider α_s-corrections
- introduce non-trivial (soft) ξ-dependence

- does the data really demand a hard (perturbative?) ξ-dependence