The Impact of Muon Conversion Experiments on Linear Collider Studies

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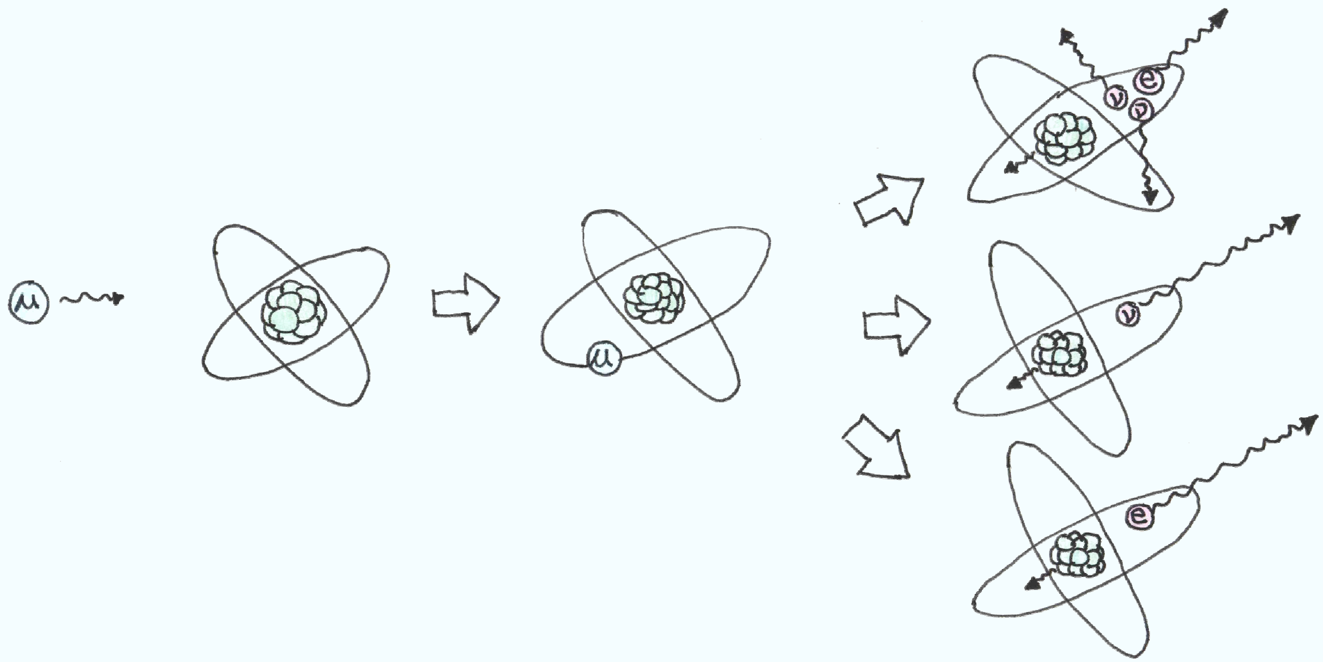
Based on
`Phys. Lett. B, 526, 157` with K. Tobe and J. Wells
`Phys. Rev. D, 65, 055003`
`eConf C010630` with J. Wells
Also work in progress with D. Maybury and C. Wagner

What is muon conversion?



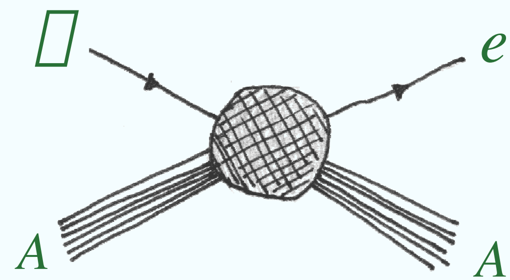
1.
Slow muons are captured

2.
A ground state muonic atom is formed.

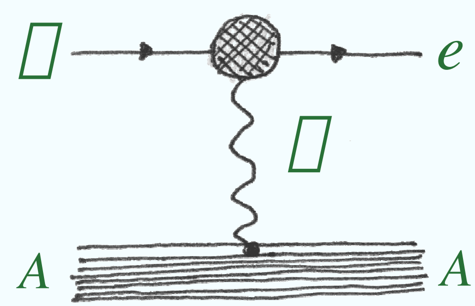
3.
The bound state decays.

What is muon conversion?

- In general:



- MSSM bias:



What is muon conversion?
- Description
- Muon decay
- Status
- $\mu \to e\gamma$
- Motivation

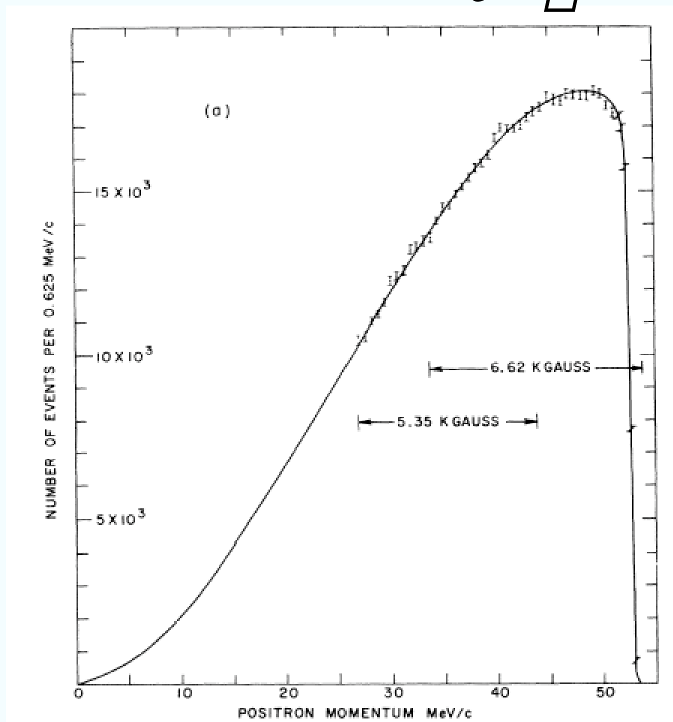
Case: *Z'*

Case: MSSM with a flavor *U*(1)

Case: MSSM with seesaw

Free muon decay

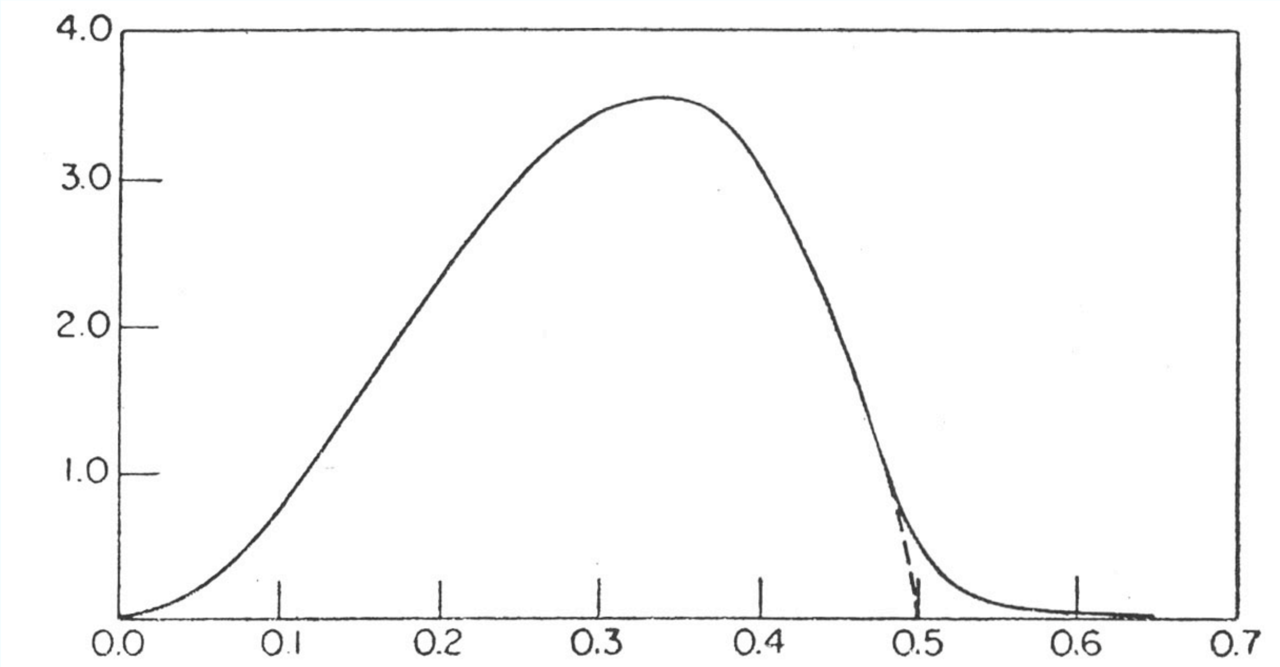
$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$



- The positron has a maximum energy of 56 MeV.
- Muon conversion will have an electron with a momentum of nearly all of the muon's rest mass, typically ~104 MeV.

Bardon et al, PRL 14, 449 (1965)

Muon decay in orbit



- Standard model decay of a muonic atom is the most prominent background.

Porter and Primakoff, Phys. Rev. 83, 849 (1951)

Experimental status

- Main observable:

$$\mathrm{BR}(\mu N \to eN) \equiv \frac{\Gamma(\mu N \to eN)}{\Gamma(\mu N \to \nu_\mu N')}$$

- Current limit:

$$\mathrm{BR}(\mu N \to eN) < 6.1 \times 10^{-13}$$
$$< 10^{-12.2}$$

(SINDRUM II)

- Future BR projections:

$$\mathrm{BR}(\mu N \to eN) \sim \text{ under } 10^{-16}$$ (MECO)

$$\mathrm{BR}(\mu N \to eN) \sim 10^{-18}$$ (PRIME)

- If muon conversion occurs at $\mathrm{BR} = 10^{-16}$, MECO will see 5 events with a background of 0.45 for 10^7 s (117 days).

MECO

MECO Sensitivity & Background

Expected Sensitivity

Contributions	Factor
Running time (s)	10^7
Proton flux (Hz)	4×10^{13}
μ reaching stopping target per incident proton	0.0043
μ stopping probability	0.58
μ capture probability	0.60
Fraction of μ capture in detection time window	0.49
Electron trigger efficiency	0.90
Fitting and selection criteria efficiency	0.19
Single Event Sensitivity	$R_{\mu e} = 2 \times 10^{-17}$

Expected Background

Source	Events
μ decay in orbit	0.25
Tracking errors	< 0.006
Radiative μ decay	< 0.005
Beam e^-	< 0.04
μ decay in flight	< 0.03
μ decay in flight	0.04
π decay in flight	< 0.001
Radiative π capture	0.07
Anti-proton induced	0.007
Cosmic ray induced	0.004
Total Background	**0.45**

Michael Hebert, UC Irvine Status and Perspectives of MECO, the Muon to Electron Conversion Exp. June 10, 2003 19

Relationship to $\mu \to e\gamma$

What is muon conversion?
- Description
- Muon decay
- Status
- $\mu \to e\gamma$
- Motivation

Case: *Z'*

Case: MSSM with a flavor *U*(1)

Case: MSSM with seesaw

- Experimental status:
  - Current: $\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ (MEGA)
  - Future: $\mathrm{BR}(\mu \to e\gamma) < 4.5 \times 10^{-14}$ (MEG 2005)

- Conventional wisdom:

$$\mathrm{BR}(\mu N \to eN) = \frac{\mathrm{BR}(\mu \to e\gamma)}{389} \quad \text{(for } ^{27}\mathrm{Al)}$$

- MEG may be equivalent to a MECO-like experiment with:

$$\mathrm{BR}(\mu N \to eN) < 1.2 \times 10^{-16} \quad \text{(for } ^{27}\mathrm{Al)}$$

Theoretical motivation

- Lepton flavor violation (LFV) is an accidental symmetry of the standard model.
- LFV may occur in:
  - Gauge hierarchy solutions:
    - MSSM
    - Dynamical symmetry breaking
    - Extra dimensions
    - Little Higgs
    - Non-minimal Higgs representations
  - Gauge unification
  - Neutrino physics
  - Flavor models
  - String models

Case: generation dependent *Z*'s

- String models:
  - Chouhuri, Chung, Lykken, Hockney
  - Cleaver, Faraggi, Nanopoulos, ter Veldhuis
- Dynamical symmetry breaking models:
  - Malkawi, Tait, Yuan
  - Chivukula, Simmons, Terning
- Typically, charge assignments are identical for 2 generations, but different for 1 generation.

Defining a Z' model

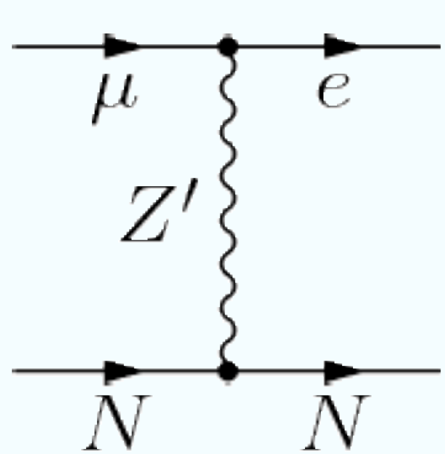
- Assume a conservative model:
  - No supersymmetry
  - Negligible Z-Z' mixing
  - Negligible phenomenology exotic fermions and scalars
  - Purely vectorial couplings
  - All Yukawa unitary transformations are $1_{3\text{x}3}$ except:
    - U^l parameterized
    - $U^{uL} = V_{\text{CKM}}$
  - Charge assignments -1, -1, +1 for leptons in the Z' gauge basis.

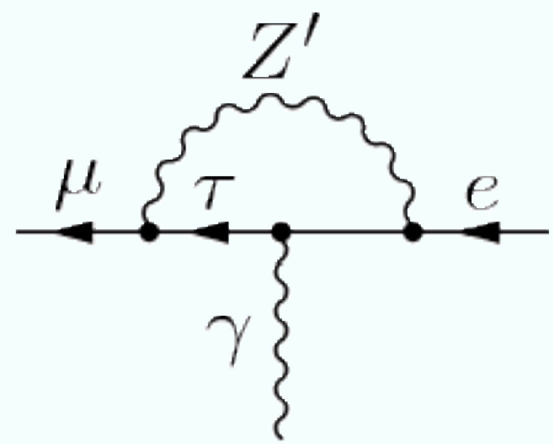
The origin of LFV

- After rotation to the mass basis:

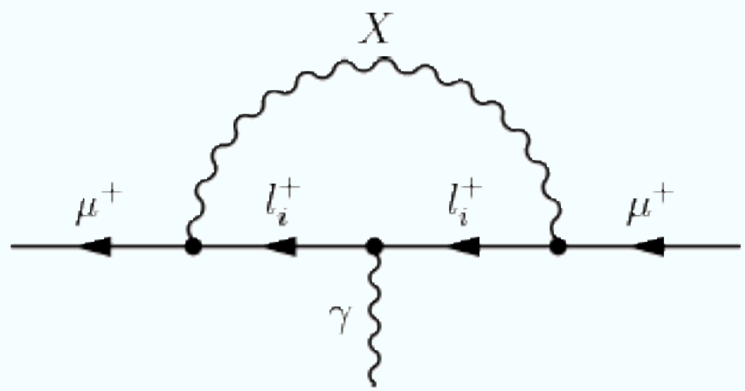
$$Q^l_{ij} = (q^l_{33} - q^l_{11}) U^{l\dagger}_{i3} U^l_{3j} \qquad (i \neq j)$$

- Muon conversion and $\mu \to e\gamma$:





Muon g-2 for reference



- The muon g-2 compatible parameter space lies in:

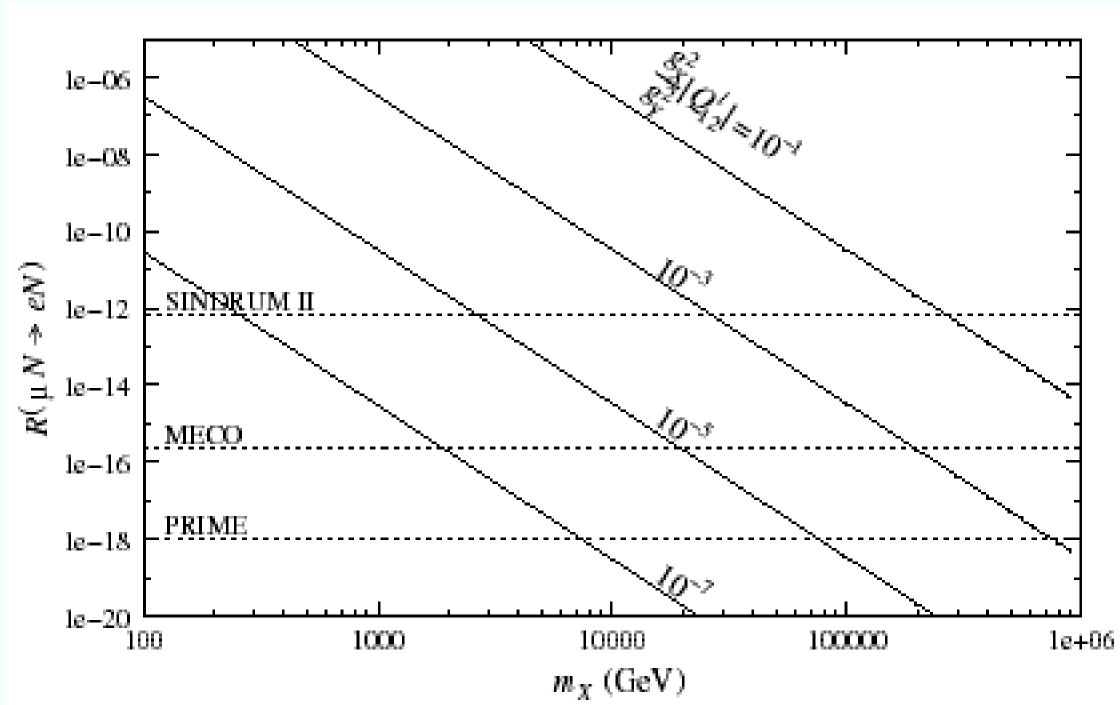
$$g_X = g_Y \quad \Rightarrow m_X \approx 1\,\text{TeV}$$

$$\frac{g_X}{\sin\theta_W} = \sqrt{4\pi} \quad \Rightarrow 2.8\,\text{TeV} \lesssim m_X \lesssim 4.8\,\text{TeV}$$

$$Q^l = \begin{pmatrix} -1 & \mathcal{O}(10^{-5}) & \mathcal{O}(10^{-5}) \\ & 0.1\text{ to }1 & 0.4\text{ to }1 \\ & & 0.1\text{ to }1 \end{pmatrix}$$

Branching ratio predictions

$$\mathrm{BR} = 3.2 \times 10^{-11} \left(\frac{g_X}{g_Y}\right)^4 \left(\frac{Q^l_{12}}{10^{-5}}\right)^2 \left(\frac{1\ \mathrm{TeV}}{m_X}\right)^4$$

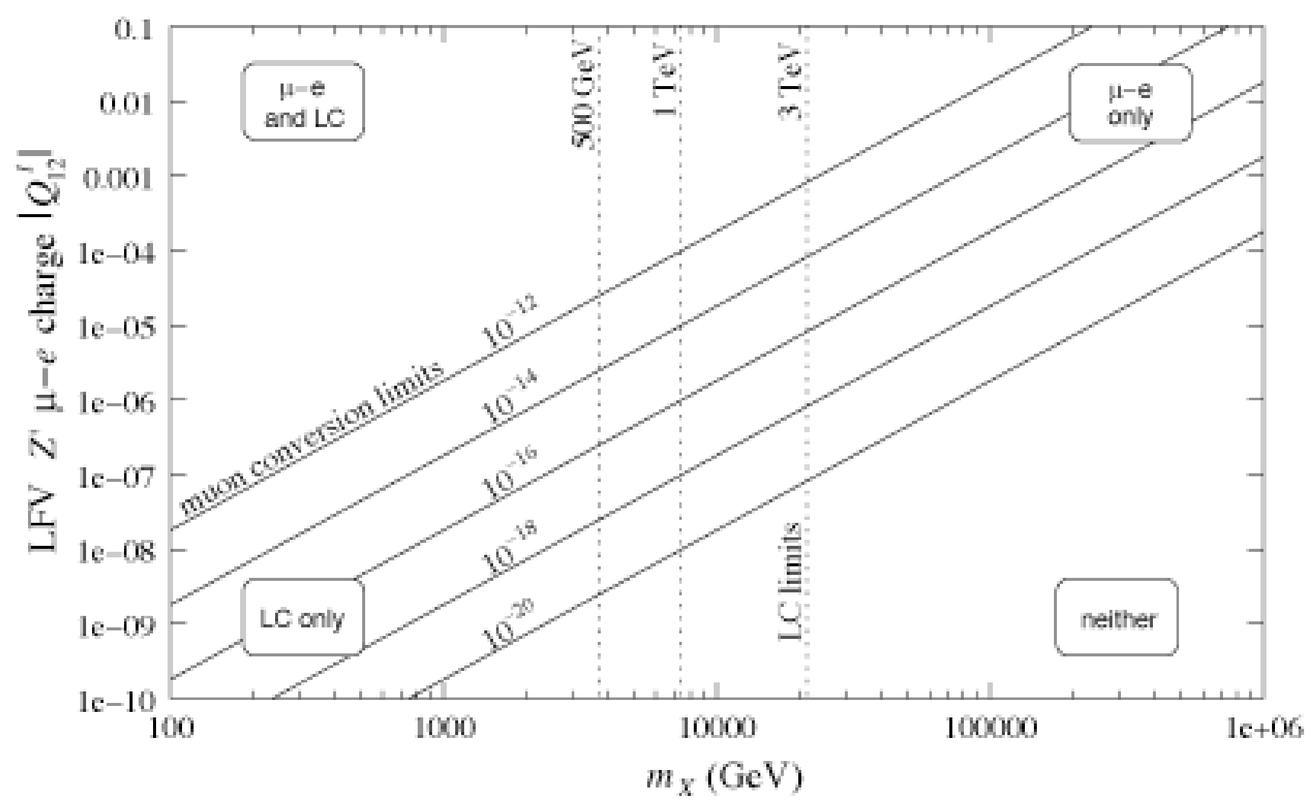


- Large LFV *Z'* masses and small charges are accessible to MECO and PRIME.
- Using the muon g-2 compatible parameter space ($Q^l_{12} = 10^{-5}$), muon conversion has probed to 2 TeV.

Complementarity with an NLC

What is muon conversion?
Case: *Z'*
- Motivation
- Our model
- Origin of LFV
- Muon *g*-2
- Branching ratio predictions
- **Complementarity with an NLC**

Case: MSSM with a flavor *U*(1)
Case: MSSM with seesaw



- The linear collider observable is $\sigma(e^+e^- \to \mu^+\tau^-)$.

Case: MSSM with a flavor $U(1)$

Focus on the case of 1 heavy $U(1)$ with lepton-flavor violation only

Ingredients for an example model:

⋆ The supermultiplets are:

X	vector	flavor $U(1)_X$
S_+, S_-	chiral	charged under X
ϕ_i	chiral	MSSM

⋆ Include a superpotential to give S_+ and S_- vevs.

$$W \supset \frac{\lambda}{2m_p} S_+^2 S_-^2$$

The scalar potential

⋆ The scalar potential:

F-terms: $F^i = -\partial W/\partial \phi_i$

$$V \supset \frac{\lambda^2}{m_P^2}(|S_+|^2|S_-|^4 + |S_+|^4|S_-|^2)$$

D-terms: $D^a = -g(\phi^\dagger T^a \phi)$

$$V \supset \frac{g_X^2}{2}(|S_+|^2 - |S_-|^2 + Q_{ij}\phi_i^*\phi_j)^2$$

Soft terms:

$$V \supset {m_+}^2|S_+|^2 + {m_-}^2|S_-|^2 + \frac{a}{2m_P}S_+^2 S_-^2$$

Scale of the Z'

⋆ 2nd derivatives of V:

$$\Rightarrow m_+^2 + m_-^2 < 0.$$

A large Yukawa coupling to some chiral multiplet can do this.

⋆ The D-terms are almost flat:

$$\Rightarrow \langle |S_+|^2 \rangle \approx \langle |S_-|^2 \rangle = \frac{aM_P}{6\lambda^2}\left[1 + \sqrt{1 - 6\lambda^2(m_+^2 + m_-^2)/a^2}\right]$$

No unnatural parameters required.

The LFV D-term

⋆ Minimize the potential:

$$\Rightarrow \langle |S_+|^2 \rangle - \langle |S_-|^2 \rangle = -\frac{\langle D_X \rangle}{g_X} \approx -\frac{1}{2g_X^2}(m_+^2 - m_-^2)$$

$\langle D_X \rangle$ and the scalar vevs are related.

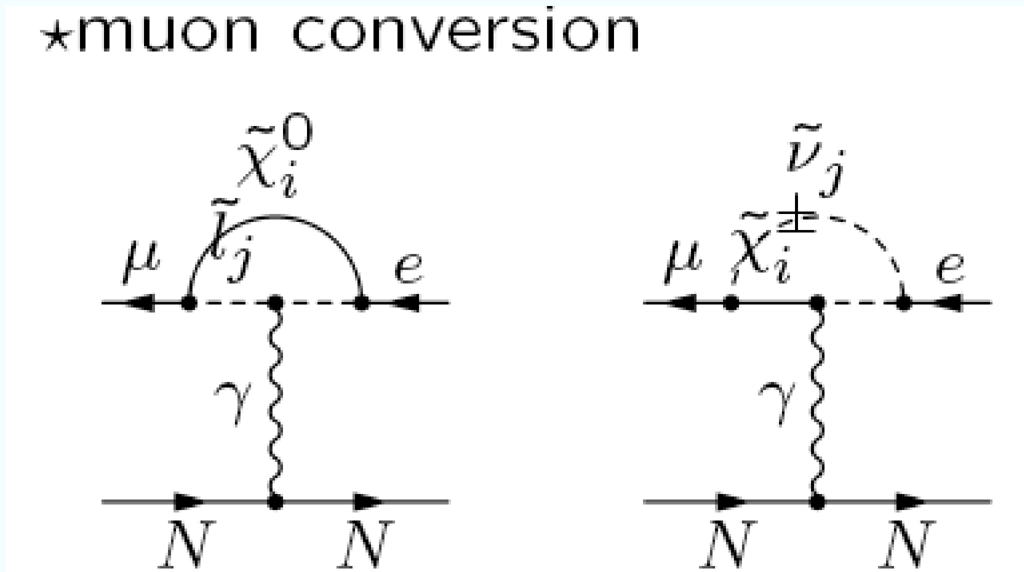
⋆To see flavor violation, replace the scalars by their vevs.

$$\mathcal{L} \supset -\frac{g_X^2}{2}(\langle |S_+|^2 \rangle - \langle |S_-|^2 \rangle + Q_{ij}\phi_i^* \phi_j)^2$$

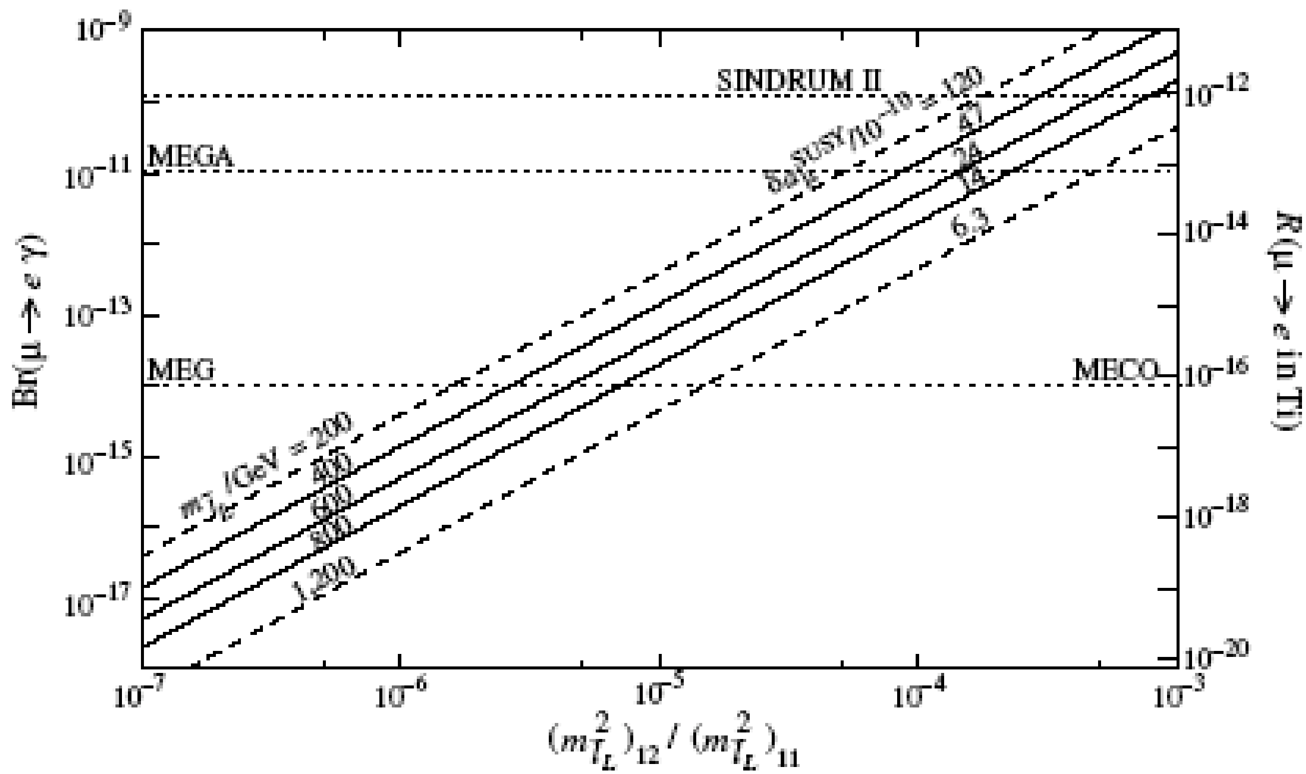
$$\Rightarrow \Delta m_{ij}^2 = \frac{1}{2} Q_{ij}(m_+^2 - m_-^2)$$

No boson mass! No gauge coupling!
Apply to slepton sector.

The muon conversion amplitudes



Branching ratio predictions



- $\tan\beta = 30$
- $M_2(m_Z) = 125$ GeV
- $\mu > 0$
- $A_0 = 0$.

Case: MSSM with seesaw

- The origin of LFV in the MSSM with seesaw:

$$W \ni \bar{e}\mathbf{y}_e L H_d + \nu_R \mathbf{y}_\nu L H_u + \frac{1}{2}\nu_R \mathbf{m}_R \nu_R$$

1. At some scale, only 2 of the 3 matrices can be simultaneously diagonalized.
2. Suppose $\mathbf{y}_e$ and $\mathbf{m}_R$ are diagonalized at the Majorana scale.
3. Off-diagonal slepton masses: $\frac{d}{dt}m_{\tilde{L}}^2 \propto \mathbf{y}_\nu^+ \mathbf{y}_\nu$
4. RG flow down to the low scale induces LFV.

Seesaw parameters

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

- If we measure all
  - Neutrino masses (6)
  - Mixings and phases (6)
  - …then there are still 6 unknown parameters.
- Formulate the unknowns into a complex orthogonal matrix **R** that relates all input parameters. i.e.,

$$\mathbf{y}_\nu \left\langle H_u^0 \right\rangle = \sqrt{\mathbf{m}_R}\,\mathbf{R}\sqrt{\mathbf{m}_L}\,\mathbf{U}_{\mathrm{MNS}}^{+}$$

Reducing parameters

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

- Motivated guesses:
  - Light neutrinos: hierarchical, inverted, or nearly degenerate?
  - Heavy neutrinos: hierarchical, inverted, or degenerate?
  - R taken to be real.
  - mSUGRA input parameters.

Subcase: all neutrino masses hierarchical

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

$$\mathbf{m}_L = \mathrm{diag}\left(0,\ \sqrt{\Delta m^2_{\mathrm{sol}}},\ \sqrt{m^2_{\mathrm{atm}}}\right)$$

$$\mathbf{m}_R = \mathrm{diag}(0,\ 0,\ m_{R3})$$

- Only elements R_{32} and R_{33} show up in $\mathrm{BR}(\mu \to e\gamma)$.
- Inspired by $SO(10)$, the largest neutrino Yukawa is equal to $y_t(m_{\mathrm{GUT}})$.
- This leaves only one parameter $R_{32}/R_{33} = \tan\theta_1$ in:

$$R = \begin{pmatrix} c_2c_3 & -c_1s_3 - s_1s_2c_3 & s_1s_3 - c_1s_2c_3 \\ c_2s_3 & c_1c_3 - s_1s_2s_3 & -s_1c_3 - c_1s_2s_3 \\ s_2 & s_1c_2 & c_1c_2 \end{pmatrix}$$

The $\mu \to e\gamma$ branching ratio

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

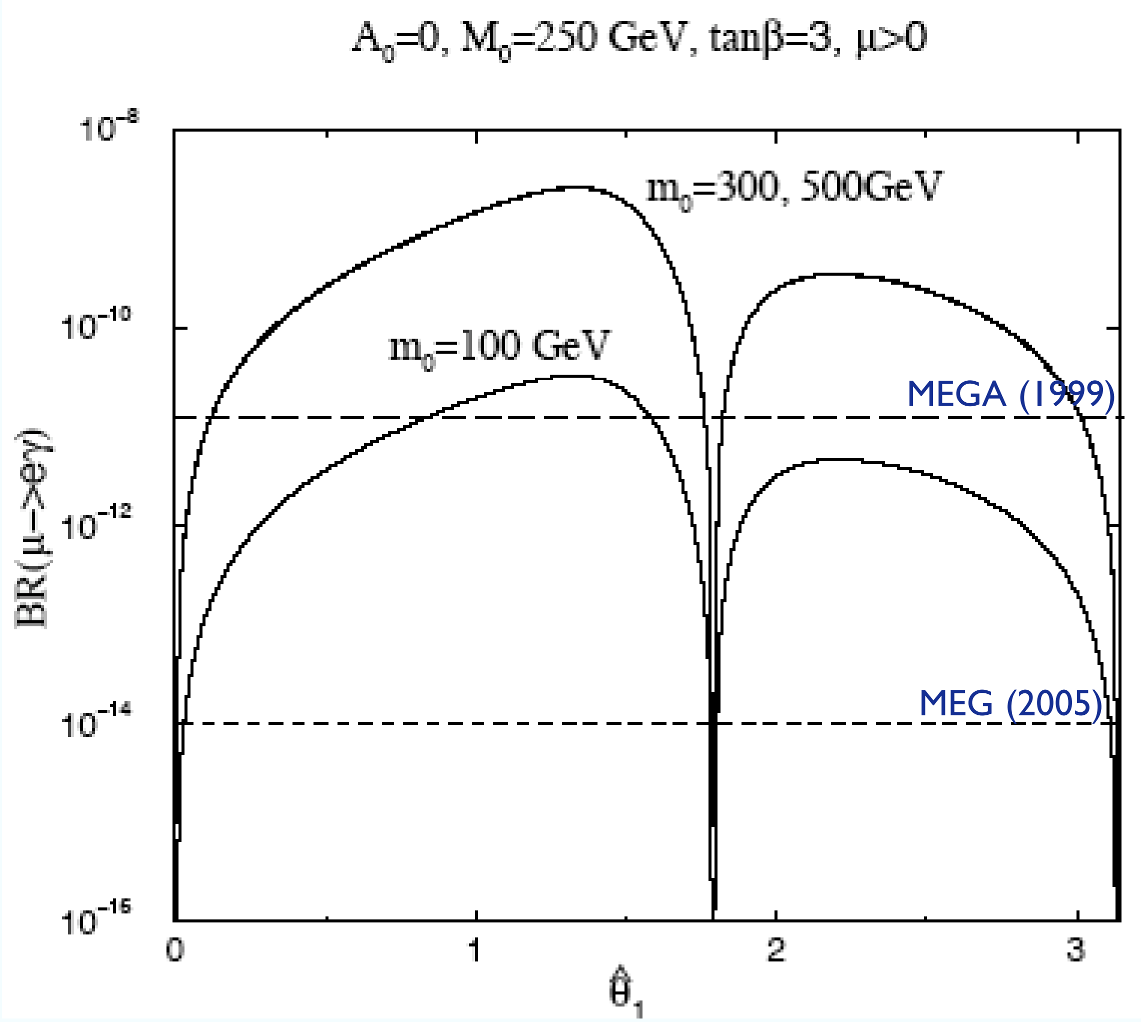
- Leading log approximation:

$$\mathbf{BR}(\mu \to e\gamma) \approx \frac{\alpha^3}{64\pi^4 m_{\tilde{l}}^8} \left| \left(3m_0^2 + A_0^2\right) \ln \frac{m_{\mathrm{GUT}}}{m_R} \right|^2 \left| \left(\mathbf{y}_\nu^+ \mathbf{y}_\nu\right)_{21} \right|^2 \tan^2 \beta$$

- Note the dependences.

Branching ratio predictions

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)



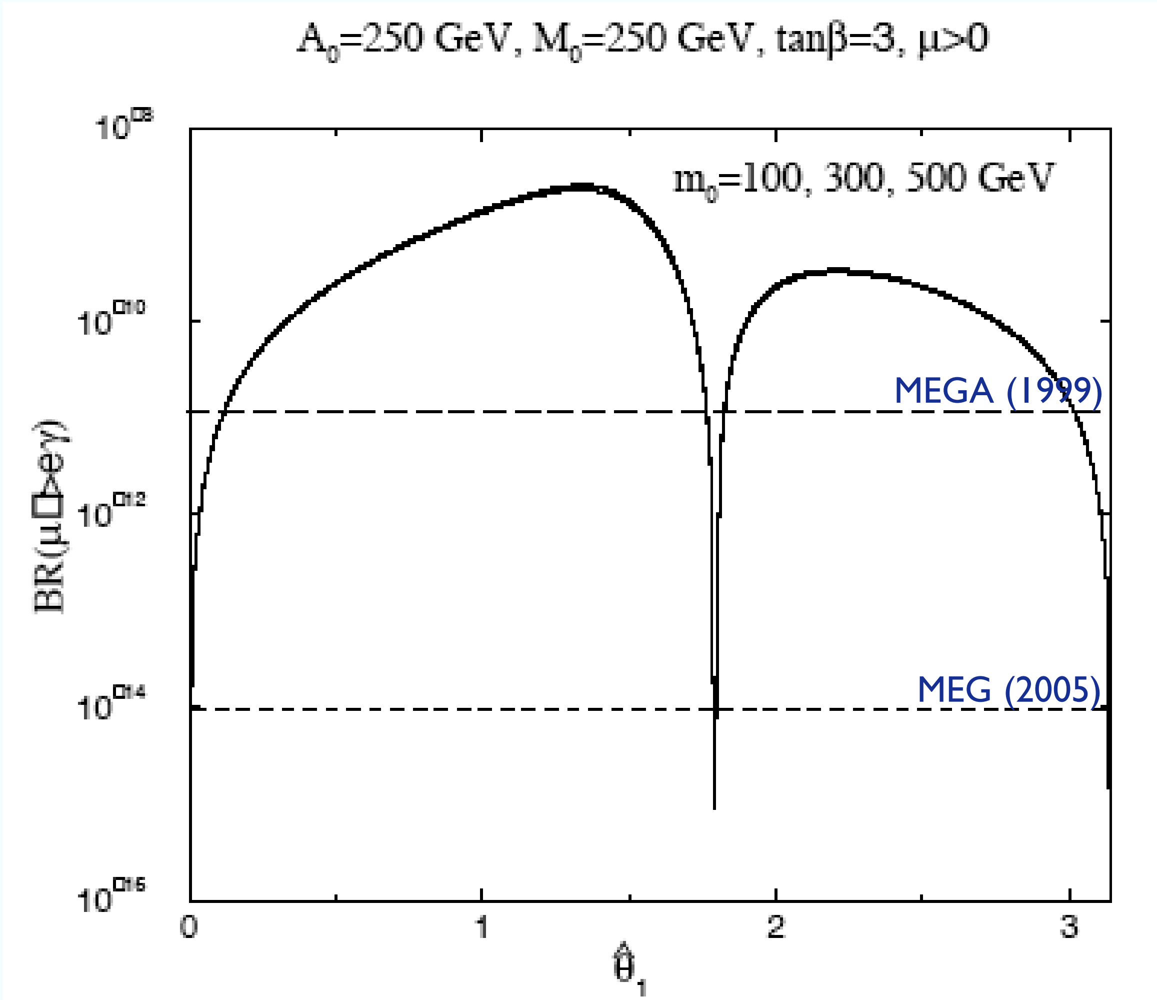
Implications for Yukawa textures

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

- Cancellations arise in $(\mathbf{y}_{\square}^{+}\mathbf{y}_{\square})_{21}$.
- Possibly natural, $\square_1 \sim 0$.
- This results in:

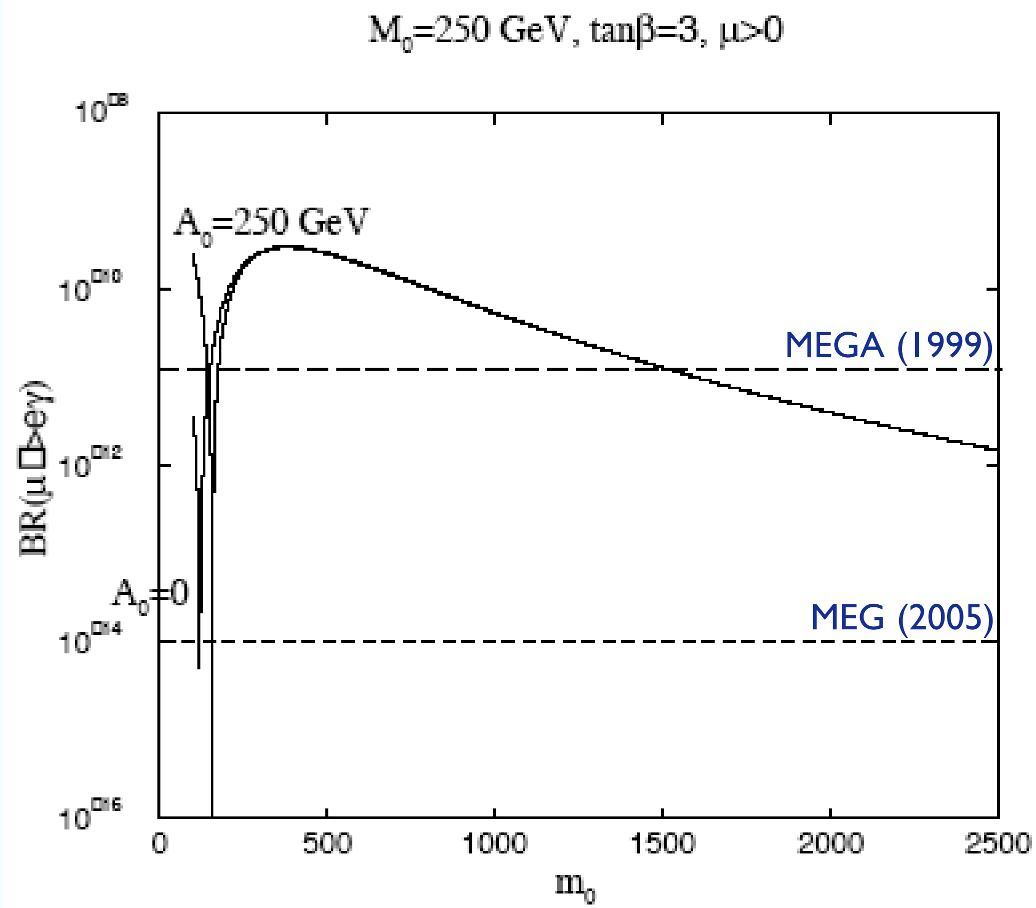
$$\mathbf{y}_{\square} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Branching ratio predictions



Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

Branching ratio predictions

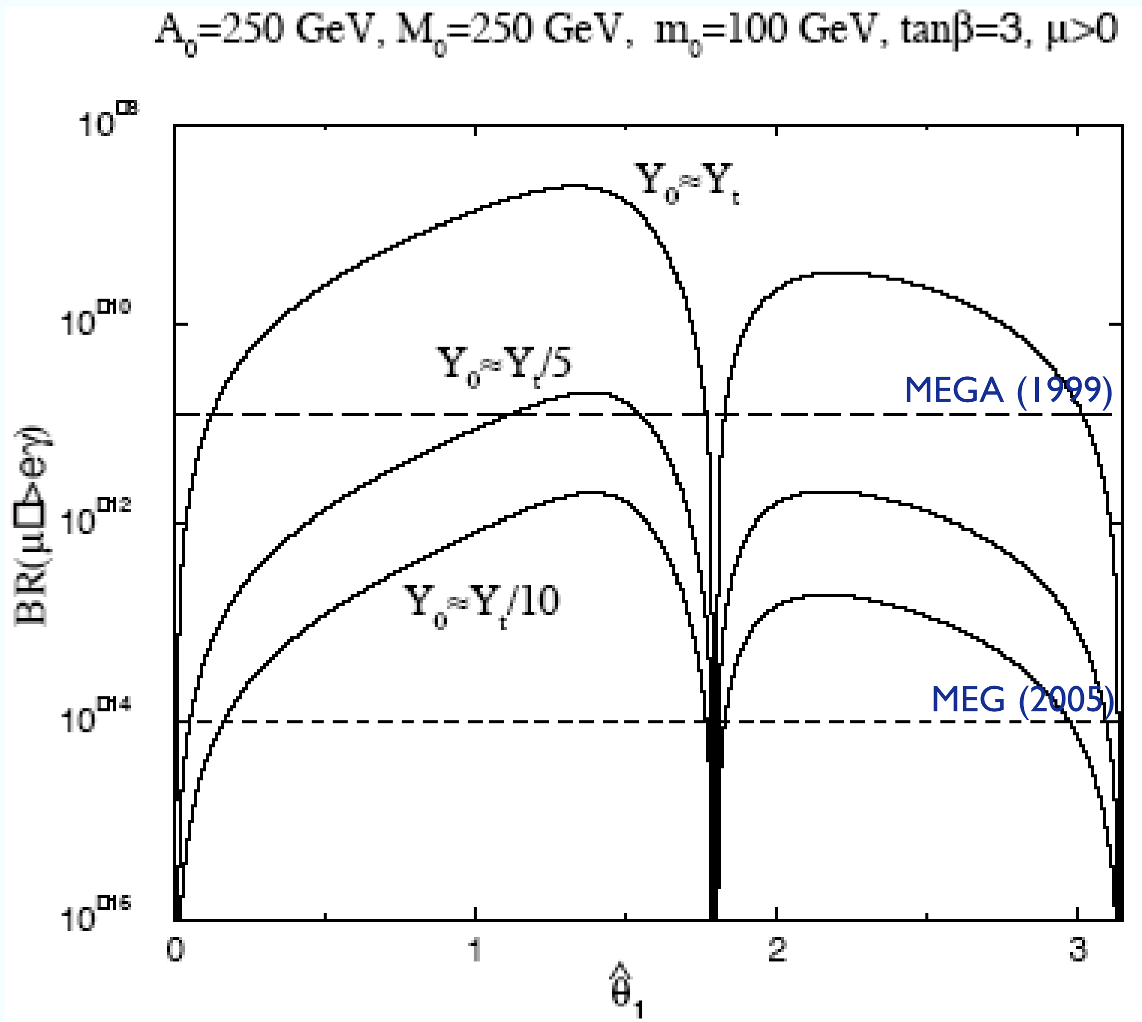


Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

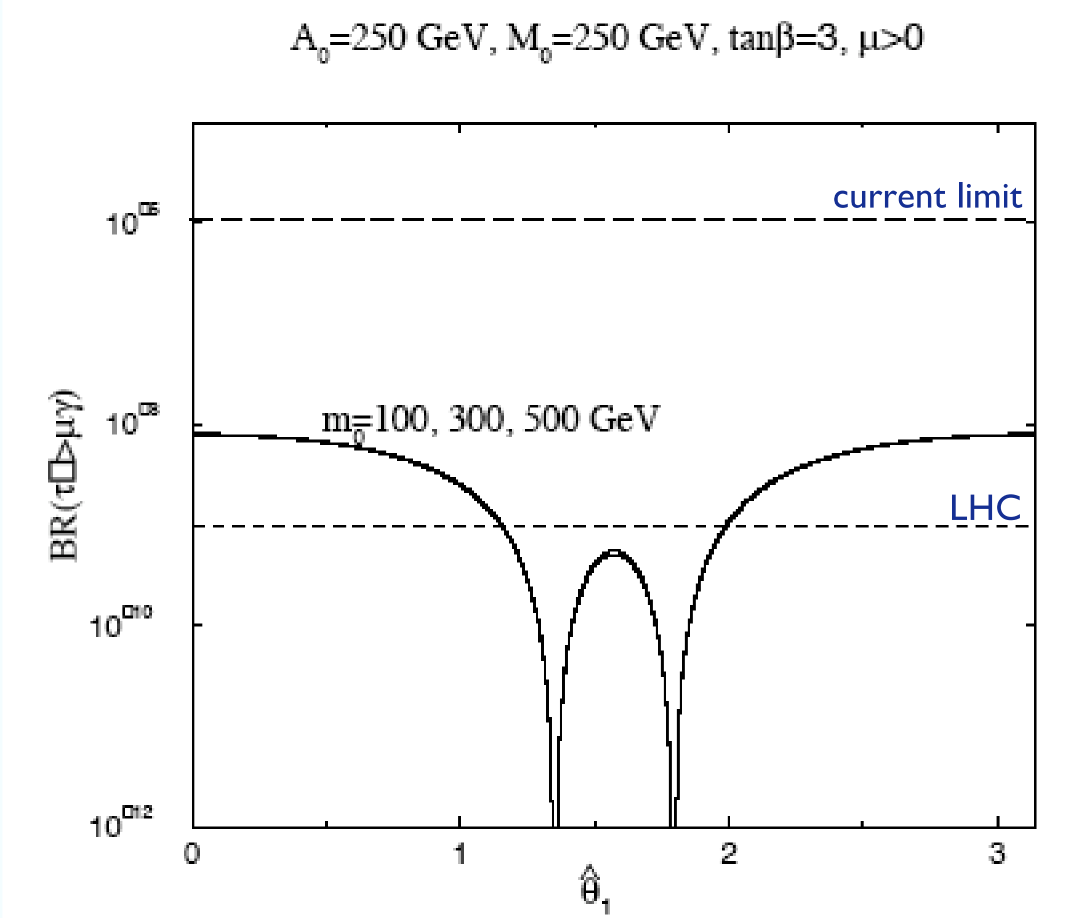
- Scalars must be very light or very heavy.

Branching ratio predictions

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)



$\tau \to \mu\gamma$ at the LHC

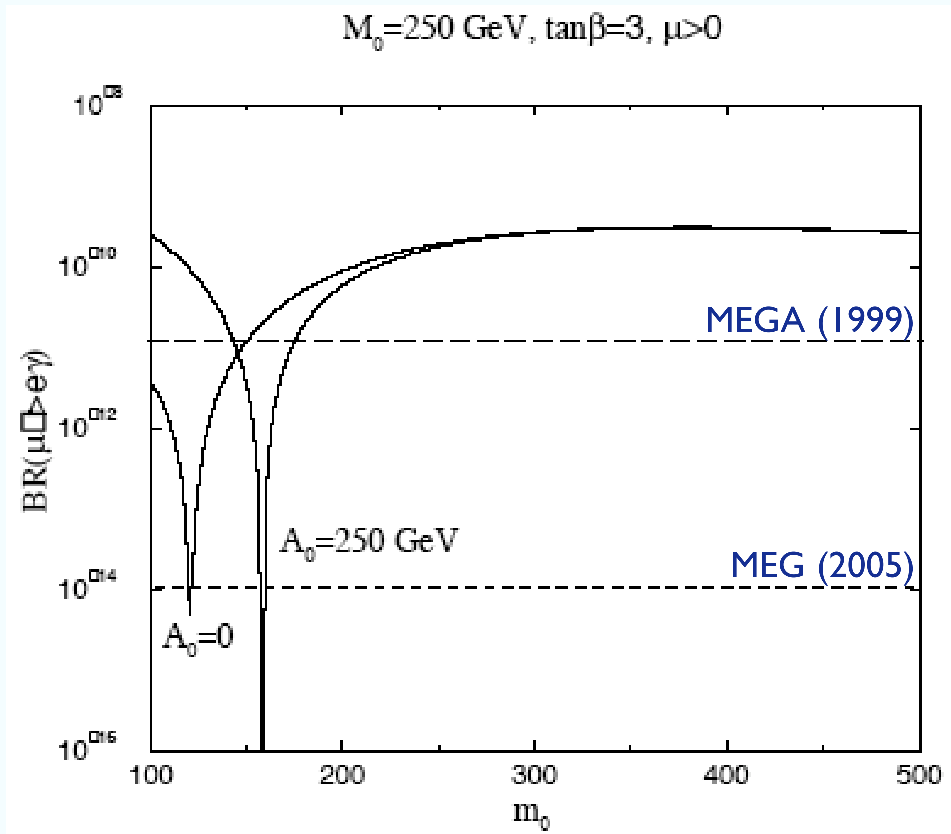


Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

Subcase: degenerate Majorana masses

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

- Recall: $\mathbf{y}_{\Box}\left\langle H_u^0\right\rangle = \sqrt{\mathbf{m}_R}\,\mathbf{R}\sqrt{\mathbf{m}_L}\,\mathbf{U}^{+}_{\mathrm{MNS}}$
- Here, real R drops out of $\mathbf{y}_{\Box}^{+}\mathbf{y}_{\Box}$.



Subcase: light and heavy masses degenerate

Casas and Ibarra, Nucl. Phys. B, 618, 171 (2001)

- Recall: $\mathbf{y}_\nu \left\langle H_u^0 \right\rangle = \sqrt{\mathbf{m}_R}\,\mathbf{R}\sqrt{\mathbf{m}_L}\,\mathbf{U}^+_{\mathrm{MNS}}$
- Perfect degeneracy: $\mathbf{y}_\nu^+ \mathbf{y}_\nu \sim 1$
- Small differences: $\left(\mathbf{y}_\nu^+ \mathbf{y}_\nu\right)_{21} \sim \frac{\Delta m^2_{\mathrm{sol}}}{m_L^2}$
- Same results as previous subcase, but rates reduced by a factor of 10^{-8}.

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Summary

- Observable LFV is likely in motivated extensions of the standard model.
- MEG (2005), MECO (2008?), and PRIME (2008?) have unprecedented potential to observe muon LFV.
- For *Z'*s:
  - Muon conversion will probe high boson masses and small couplings.
  - Linear colliders can probe the diagonal charges of light *Z'*s.
- For supersymmetric high scale flavor *U*(1)'s:
  - Observable LFV persists through *D*-terms.
  - No decoupling solution possible if supersymmetry dominates muon *g*-2.
- For mSUGRA with seesaw:
  - Muon conversion and $\mu \to e\gamma$ can help reconstruct the high scale neutrino parameters.
  - In the case of hierarchical neutrino masses, if MEG does not see $\mu \to e\gamma$ then the LHC should see [illegible]
  - MEG, MECO, and PRIME should probe most of the interesting parameter space of this scenario.