

# $B_s$ - $\bar{B}_s$ Mixing in $Z'$ Models with Flavor-Changing Neutral Currents

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## Abstract

In models with an extra  $U(1)'$  gauge boson, the family non-universal couplings to the weak eigenstates of the standard model fermions generally induce flavor-changing neutral currents. This phenomenon leads to interesting results in various  $B$  meson decays, for which recent data indicate hints of new physics involving significant contributions from  $b \rightarrow s$  transitions. We analyze the  $B_s$  system, emphasizing the effects of  $Z'$  on the mass difference and  $CP$  asymmetries.

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## I. INTRODUCTION

The study of  $B$  physics and the associated CP violating observables has been suggested as a good means to extract information of new physics at low energy scales [1–6]. Since  $B$ - $\bar{B}$  mixing is a loop-mediated process within the standard model (SM), it offers a good opportunity to see the footprints of physics beyond the SM. The currently observed  $\Delta M_{B_d} = 0.489 \pm 0.008 \text{ ps}^{-1}$  [7] and its mixing phase  $\sin 2\beta = 0.736 \pm 0.049$  extracted from the  $J/\psi K_S$  mode [8] agree well with constraints obtained from other experiments [9]. However, no such information other than a lower bound  $\Delta M_{B_s} > 14.4 \text{ ps}^{-1}$  [10] is available for the  $B_s$  meson yet.

Based upon the SM predictions,  $\Delta M_{B_s}$  is expected to be slightly larger than  $20 \text{ ps}^{-1}$  and its mixing phase  $\phi_s$  is only a couple of degrees. In contrast to the  $B_d$  system, its more than 25 times larger oscillation frequency and a factor of four less rate of hadronization from  $b$  quarks pose the primary challenges in the study of  $B_s$  oscillation and CP asymmetries. Since the  $B_s \rightarrow J/\psi\phi$  decay is dominated by a CKM favored tree-level process,  $b \rightarrow c\bar{c}s$ , that does not involve any new weak phase in the SM, its asymmetry provides the most reliable information about the mixing phase  $\phi_s$ . Although new physics contributions may not compete with the SM processes in most of the  $b \rightarrow c\bar{c}s$  decays, they can play an important role in the  $B_s$ - $\bar{B}_s$  mixing because its loop nature in the SM. In particular, such a mixing can be significantly modified in models where a tree-level bottom-strange quark coupling is allowed. It is thus seen that measuring the properties of  $B_s$  meson mixing is of high interest in future  $B$  physics studies and viewed as a means to reveal new physics [11, 12]. Since the current  $B$  factories do not run at the  $\Upsilon(5S)$  resonance to produce  $B_s$  mesons, we therefore consider it one of the primary objectives of hadronic colliders to study  $B_s$  oscillation and decay in the coming years [13, 14].

In  $E_6$  models, flavor changing neutral currents (FCNC) through an extra  $U(1)'$  gauge boson can arise when the  $Z'$  couplings to physical fermion eigenstates are non-diagonal [15–17]. This is achieved through the introduction of the exotic fermions with different  $U(1)'$  charges that mix with the SM fermions. However, to avoid inducing undesired FCNC mediated by the SM  $Z$  boson, one is restricted to schemes where only the right-handed fermions are mixed with the exotic fermions [18].

It is well-known that string models naturally give extra  $U(1)'$  groups, at least one of which

have family non-universal couplings with the SM fermions [19–22]. Generically, the physical and gauge eigenstates do not coincide. Therefore, unlike in the above-mentioned  $E_6$  setup, the off-diagonal couplings of fermions with the  $Z'$  boson without mixing with additional states can be obtained. In such models, both left-handed and right-handed fermions can have family non-diagonal couplings with the  $Z'$  while their couplings with the  $Z$  remain family diagonal. Moreover, by a suitable construction of intersecting branes, it is possible to have diminishing  $U(1)'$  charges for SM leptons [23]. In this paper, we will consider such leptophobic models with  $Z'$ -mediated FCNC in the quark sector.

Recently, we have studied the implications of a sizeable off-diagonal  $Z'$  coupling between the bottom and strange quark in the indirect CP asymmetry of  $B \rightarrow \phi K_S$  decay [24], which is seen to have a significant deviation from the SM prediction [5, 6, 25, 26]. Here we want to extend our analysis to the  $B_s$ - $\bar{B}_s$  mixing where the  $Z'$  contribution also enters at the tree-level.

The paper is organized as follows. We review in Section II the basic formalism of  $B_s$ - $\bar{B}_s$  mixing. In Section III, we evaluate  $\Delta M$  in the SM. In Section IV, we include the  $Z'$  contributions, both left-handed and right-handed couplings, in the mixing. Our main results are summarized in Section VI.

## II. $B_s$ - $\bar{B}_s$ MIXING

If we write the heavy and light eigenstates as

$$\begin{aligned} |B_s\rangle_L &= p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \\ |B_s\rangle_H &= p|B_s^0\rangle - q|\bar{B}_s^0\rangle, \end{aligned} \tag{1}$$

then the mixing factor

$$\left(\frac{q}{p}\right)_{\text{SM}} \simeq \sqrt{\frac{M_{12}^{\text{SM}*}}{M_{12}^{\text{SM}}}}, \tag{2}$$

has a phase

$$\phi_s = 2 \arg(V_{tb}V_{ts}^*) = -2\lambda^2\eta = \mathcal{O}(-2^\circ), \tag{3}$$

where  $\Gamma_{12}^{\text{SM}} \ll M_{12}^{\text{SM}}$  is used. Here the off-diagonal element of the decay matrix,  $\Gamma_{12}^{\text{SM}}$ , is evaluated by considering decay channels that are common to both  $B_s$  and  $\bar{B}_s$  mesons, and  $M_{12}$  is the off-diagonal element of the mass matrix. It is dominated by the charm-quark

contributions over the up quarks in the box diagram due to the CKM enhancement. Unlike the kaon system,  $\Gamma_{12}^{\text{SM}}$  is much smaller than  $M_{12}^{\text{SM}}$  for  $B$  mesons. This is because the former is related to the  $B$  meson decays and thus set by the scale of its mass, whereas the latter is proportional to  $m_t^2$ . We can safely assume that  $\Gamma_{12}$  is not significantly modified by new physics, because  $\Gamma_{12}$  receives major contributions from CKM favored  $b \rightarrow c\bar{c}s$  decays in SM. Therefore, the relation  $\Gamma_{12} \ll M_{12}$  is unlikely to change.

The mass difference of the two physical states is

$$\Delta M \equiv M_H - M_L \simeq 2|M_{12}| . \quad (4)$$

The width difference is

$$\Delta\Gamma \equiv \Gamma_H - \Gamma_L = \frac{2\text{Re}(M_{12}^*\Gamma_{12})}{|M_{12}|} = 2|\Gamma_{12}| \cos\theta , \quad (5)$$

where the relative phase  $\theta = \arg(M_{12}/\Gamma_{12})$ . Since  $\Gamma_{12}$  is dominated by the contributions from CKM favored  $b \rightarrow c\bar{c}s$  decays, we have  $\theta = \arg((V_{tb}V_{ts}^*)/(V_{cb}V_{cs}^*)) \simeq \pi$ . Thus, in our convention,  $\Delta\Gamma = -2|\Gamma_{12}|$  is negative in the SM. Although  $\Gamma_{12}$  is unlikely to be affected by new physics, the width difference always reduces as long as the weak phase of  $M_{12}$  gets modified [27].

The observability of  $B_s$ - $\bar{B}_s$  oscillation is often measured by the parameter

$$x_s \equiv \frac{\Delta M}{\Gamma_{B_s}} , \quad (6)$$

where  $\Gamma_{B_s} = (4.51 \pm 0.18) \times 10^{-13}$  GeV, converted from the world average lifetime  $\tau_s = 1.461 \pm 0.057$  ps [7]. Too large a value of  $x_s$  will be a challenge for experimental searches. Currently, the result from all ALEPH [28], CDF [29], DELPHI [30], OPAL [31], and SLD [32] studies of  $\Delta M$  with a combined 95% CL sensitivity on  $\Delta M_s$  of  $17.8 \text{ ps}^{-1}$  gives [10]

$$\Delta M > 14.4 \text{ ps}^{-1} , \quad \text{and} \quad x_s > 20.6 . \quad (7)$$

It is also measured that  $\Delta\Gamma_s/\Gamma_s = 0.16_{-0.16}^{+0.15} (< 0.54)$  with the upper bound in the parentheses quoted at 95% CL [10].

### III. $\Delta M$ IN SM

First, let us define the  $|\Delta B| = 2$  and  $|\Delta S| = 2$  operators relevant for later discussions:

$$O^{LL} = [\bar{s}\gamma_\mu(1 - \gamma_5)b][\bar{s}\gamma^\mu(1 - \gamma_5)b] ,$$

$$\begin{aligned}
O_1^{LR} &= [\bar{s}\gamma_\mu(1 - \gamma_5)b][\bar{s}\gamma^\mu(1 + \gamma_5)b] , \\
O_2^{LR} &= [\bar{s}(1 - \gamma_5)b][\bar{s}(1 + \gamma_5)b] , \\
O^{RR} &= [\bar{s}\gamma_\mu(1 + \gamma_5)b][\bar{s}\gamma^\mu(1 + \gamma_5)b] .
\end{aligned} \tag{8}$$

Because of the  $V - A$  structure in the SM weak interactions, only the operator  $O^{LL}$  contributes to  $B_s - \bar{B}_s$  mixing. The other three operators appear in the  $Z'$  models because of the right-handed couplings and operator mixing through renormalization, as we will see in the next section.

In the SM,

$$M_{12}^{\text{SM}} \simeq \frac{1}{2m_{B_s}} \langle B_s^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | \bar{B}_s^0 \rangle \tag{9}$$

is dominated by the top quark loop. The result accurate to the next-to-leading order (NLO) in QCD is given by [33]

$$M_{12}^{\text{SM}} = \frac{G_F^2}{12\pi^2} M_W^2 m_{B_s} f_{B_s}^2 (V_{tb} V_{ts}^*)^2 \eta_{2B} S_0(x_t) [\alpha_s(m_b)]^{-6/23} \left[ 1 + \frac{\alpha_s(m_b)}{4\pi} J_5 \right] B_{LL}(m_b) , \tag{10}$$

where  $x_t = (m_t(\mu_t)/M_W)^2$  and

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3} . \tag{11}$$

Using  $m_t(\mu_t) = \mu_t = 170 \pm 5 \text{ GeV}$  we find the numerical value of  $S_0$  at  $x_t$  is  $S_0(x_t) = 2.463$ . The NLO short-distance QCD corrections are encoded in  $\eta_{2B} \simeq 0.551$  and  $J_5 \simeq 1.627$  [33]. The bag parameter  $B_{LL}(\mu)$  is defined through the relation

$$\langle \bar{B}_s | O_{LL} | B_s \rangle \equiv \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B_{LL}(\mu) . \tag{12}$$

Recent lattice analyses give the hadronic parameters  $f_{B_s} = 230 \pm 30 \text{ MeV}$  and  $B_{LL}(m_b) = 0.872 \pm 0.005$  [34–36].

For numerical estimates throughout this paper, we will use  $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ ,  $m_{B_s} = 5369.6 \pm 2.4 \text{ MeV}$ , and  $M_W = 80.423 \pm 0.039 \text{ GeV}$ , and the Wolfenstein parameters [37] extracted various experiments are  $\lambda = 0.2240 \pm 0.0036$ ,  $A = 0.83 \pm 0.02$ ,  $\rho = 0.216 \pm 0.079$ , and  $\eta = 0.341 \pm 0.028$  [38]. We find

$$\begin{aligned}
\Delta M^{\text{SM}} &= (1.32 \pm 0.35) \times 10^{-11} \text{ GeV} \\
&= 20.0 \pm 5.4 \text{ ps}^{-1} , \\
x_s^{\text{SM}} &= 29 \pm 8 .
\end{aligned} \tag{13}$$

The main sources of errors are from  $m_t$  and  $f_{B_s}$  in  $\Delta M$ . For As noted before, the central value is slightly larger than the current sensitivity based upon the world average.

Latest studies show that with one year of data,  $\Delta M$  can be explored up to 30 ps<sup>-1</sup> (ATLAS), 26 ps<sup>-1</sup> (CMS), and 48 ps<sup>-1</sup> (LHCb) (corresponding to  $x_s$  up to 46, 42, and 75) using exclusive hadronic modes at the LHC [14]. Assuming a luminosity of 2 fb<sup>-1</sup> in a one-year run, the sensitivity of both BTeV and CDF on  $x_s$  can also reach up to 75 using the same modes [13].

#### IV. $Z'$ CONTRIBUTIONS

For simplicity, we assume that the  $U(1)'$  gauge group is orthogonal to the SM gauge group so that there is no mixing between the SM  $Z$  and the  $Z'$ . A purely left-handed off-diagonal  $Z'$  coupling to  $b$  and  $s$  quarks results in an effective  $|\Delta B| = 2$ ,  $|\Delta S| = 2$  Hamiltonian at  $M_W$  scale

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \left( \frac{g' M_Z}{g_1 M_{Z'}} B_{sb}^L \right)^2 O_{LL}(m_b) \equiv \frac{G_F}{\sqrt{2}} \rho_L^2 e^{2i\phi_L} O_{LL}(m_b), \quad (14)$$

where  $g'$  is the  $U(1)'$  gauge coupling,  $g_1 = e/\sin\theta_W$ ,  $M_{Z'}$  is the mass of the  $Z'$ , and  $B_{sb}^L$  is the FCNC  $Z'$  coupling between the bottom and strange quarks. The parameters  $\rho_L$  and the weak phase  $\phi_L$  in the  $Z'$  model are defined in the second part of the equation. Note that the  $Z'$  does not contribute to  $\Gamma_{12}$  at tree level because the intermediate  $Z'$  cannot be on shell. After evolving from the  $M_W$  scale to the  $m_b$  scale, the effective Hamiltonian becomes

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \left[ 1 + \frac{\alpha_s(m_b) - \alpha_s(m_w)}{4\pi} J_5 \right] R^{6/23} \rho_L^2 e^{2i\phi_L} O_{LL}(m_b), \quad (15)$$

where  $R = \alpha_s(M_W)/\alpha_s(m_b)$ .

One immediately notices that although the above effective Hamiltonian is largely suppressed by the ratio  $(g' M_Z)/(g_1 M_{Z'})$ , it has only one power of  $G_F$  in comparison with the corresponding quadratic dependence in the SM. This is because the  $Z'$ -mediated process occurs at tree level.

The full description of the running of the Wilson coefficient from the  $M_W$  scale to  $m_b$  scale can be found in [33]. We only repeat the directly relevant steps here.

The renormalization group equation for the Wilson coefficients  $\vec{C}$

$$\frac{d}{d \ln \mu} \vec{C} = \gamma^T(g) \vec{C}(\mu) \quad (16)$$

can be solved with the help of the  $U$  matrix

$$\vec{C}(\mu) = U(\mu, M_W)\vec{C}(M_W), \quad (17)$$

in which  $\gamma^T(g)$  is the transpose of the anomalous dimension matrix  $\gamma(g)$ . With the help of  $dg/d\ln\mu = \beta(g)$ ,  $U$  obeys the same equation as  $\vec{C}(\mu)$ . If we expand  $\gamma(g)$  to the first two terms in the perturbative expansion,

$$\gamma(\alpha_s) = \gamma^{(0)}\frac{\alpha_s}{4\pi} + \gamma^{(1)}\left(\frac{\alpha_s}{4\pi}\right)^2. \quad (18)$$

To this order the evolution matrix  $U(\mu, m)$  is given by

$$U(\mu, m) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}J\right)U^{(0)}(\mu, m)\left(1 - \frac{\alpha_s(m)}{4\pi}J\right) \quad (19)$$

$U^{(0)}$  is the evolution matrix in leading logarithmic approximation and the matrix  $J$  expresses the next-to-leading corrections to this evolution. We have

$$U^{(0)}(\mu, m) = V \left( \left[ \frac{\alpha_s(m)}{\alpha_s(\mu)} \right]^{\frac{\vec{\gamma}^{(0)}}{2\beta_0}} \right)_D V^{-1} \quad (20)$$

where  $V$  diagonalizes  $\gamma^{(0)T}$

$$\gamma_D^{(0)} = V^{-1}\gamma^{(0)T}V \quad (21)$$

and  $\vec{\gamma}^{(0)}$  is the vector containing the diagonal elements of the diagonal matrix  $\gamma_D^{(0)}$ .

If we define

$$G = V^{-1}\gamma^{(1)T}V \quad (22)$$

and a matrix  $H$  whose elements are

$$H_{ij} = \delta_{ij}\gamma_i^{(0)}\frac{\beta_1}{2\beta_0^2} - \frac{G_{ij}}{2\beta_0 + \gamma_i^{(0)} - \gamma_j^{(0)}} \quad (23)$$

the matrix  $J$  is given by

$$J = HVV^{-1} \quad (24)$$

The operators  $O_{LL}$  and  $O_{RR}$  do not mix with others under renormalization. Their Wilson coefficients follow exactly the same RGE, where the above-mentioned matrices are all simple numbers. The factor

$$\left[ 1 + \frac{\alpha_s(m_b) - \alpha_s(M_W)}{4\pi}J_5 \right] R^{6/23} \quad (25)$$

in Eqn. (15) reflects the RGE running. On the other hand,  $O_1^{LR}$  and  $O_2^{LR}$  form a sector mixed under RG running. Although the  $Z'$  boson only induces the operator  $O_1^{LR}$  at high energy scales,  $O_2^{LR}$  is generated after evolving down to low energy scales and, in particular, its Wilson coefficient  $C_2^{LR}$  is strongly enhanced by the RG effects [39].

With both contributions from the SM and the  $Z'$  boson with only left-handed FCNC couplings included, the mass difference

$$\begin{aligned}\Delta M &= \Delta M^{SM} \left( 1 + \frac{\Delta M^{Z'}}{\Delta M^{SM}} \right) \\ &= 20.0 |1 + 3.566 \times 10^5 \rho_L^2 e^{2i\phi_L}| \text{ ps}^{-1},\end{aligned}\tag{26}$$

where  $\Delta M^{SM}$  is the contribution in the SM and  $\Delta M^{Z'}$  is the contribution from  $Z'$ . Similarly, the oscillation parameter

$$x_s(\rho_L, \phi_L) = 29.2 |1 + 3.566 \times 10^5 \rho_L^2 e^{2i\phi_L}|.\tag{27}$$

It is noticed that with couplings of only one chirality are considered, the physical observables  $\Delta M$ ,  $x_s$ , and  $\sin 2\phi_s$  to be considered below are periodic functions of the new weak phase with a period of  $180^\circ$ .

The effect of including a  $Z'$  with left-handed coupling is shown in Fig. 1 (a). It is noted that if  $\rho$  is too small,  $x_s$  is dominated by the SM contribution and has a value  $\sim 29$ . For  $\phi_L$  around  $90^\circ$  and small enough  $\rho_L$ , the  $Z'$  contribution tends to cancel that of the SM and reduces  $x_s$  to be smaller than 29.2, the SM value. We show the contour plot of  $x_s$  in Fig. 1 (b) in the parameter space of the  $Z'$  model. We only show the range of  $0^\circ \leq \phi_L \leq 180^\circ$  because  $x_s$  has a period of  $\pi$  in  $\phi_L$ . The region inside of the  $x_s = 20.6$  contour is ruled out by the currently measured lower bound on  $x_s$ . If the experimental value turns out to be significantly larger than that, one is led to a new physics explanation. From Fig. 1 we see that the  $Z'$  contribution dominates if  $\rho \gtrsim 0.002$ , independent of the actual value of  $\phi_L$ . The planned resolution of Fermilab Run II and LHCb are both about 75 [13, 14]. From another point of view, if  $x_s$  is measured to fall within a range, one can read from the plot what is the allowed region for the  $Z'$ -model parameters.

In Fig. 2 (a), we show the sine of twice the mixing phase  $\phi_s$  as a function of  $\rho_L$  and  $\phi_L$ . The contours of fixed  $\sin 2\phi_s$  with values of  $-0.5$ ,  $0.0$  and  $0.5$  are shown as solid, dashed and dotted lines in Fig. 2 (b). Once  $x_s$  and  $\sin 2\phi_s$  are extracted  $B_s$  decays, one can combine

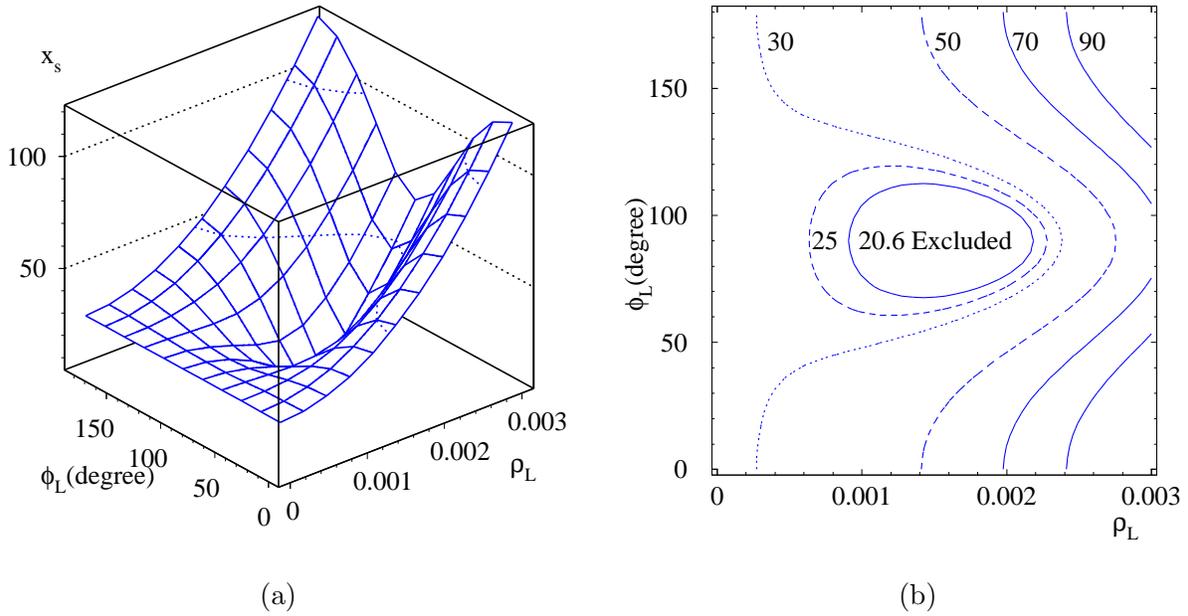


FIG. 1: Plot (a): Three-dimensional plot of  $x_s$  in the presence of  $Z'$ -mediated FCNC for left-handed  $b$  and  $s$  quarks as a function of  $\rho_L$  and  $\phi_L$ , defined in Eq. (14). Plot (b): A contour plot of  $x_s$  in the presence of a  $Z'$ -mediated FCNC for left-handed  $b$  and  $s$  quarks.

Figs. 1 (b) and 2 (b) to determine  $\rho_L$  up to a two-fold ambiguity and  $\phi_L$  up to a four-fold ambiguity in general, except for the special case when  $\sin 2\phi_s \simeq 0$ .

Once the right-handed  $Z'$  couplings are introduced, we immediately have the new  $|\Delta B| = 2$  operators  $O_1^{LR}$ ,  $O_2^{LR}$ , and  $O^{RR}$  defined in Eq. (8) in the effective Hamiltonian that contribute to  $B_s$ - $\bar{B}_s$  mixing. The matrix element of  $O^{RR}$  is the same as that of  $O^{LL}$ , while those of  $O_1^{LR}$  and  $O_2^{LR}$ , according to [35], are

$$\langle \bar{B}_s | O_1^{LR} | B_s \rangle = -\frac{4}{3} \left( \frac{m_{B_s}}{m_b(m_b) + m_s(m_b)} \right)^2 m_{B_s}^2 f_{B_s}^2 B_1^{LR}(m_b) \quad (28)$$

$$\langle \bar{B}_s | O_2^{LR} | B_s \rangle = 2 \left( \frac{m_{B_s}}{m_b(m_b) + m_s(m_b)} \right)^2 m_{B_s}^2 f_{B_s}^2 B_2^{LR}(m_b) \quad (29)$$

For the  $Z'$  coupling to right handed currents, we define new parameters  $\rho_R$  and weak phase  $\phi_R$

$$\rho_R e^{i\phi_R} \equiv \frac{g' M_Z}{g_1 M_{Z'}} B_{sb}^R. \quad (30)$$

At the  $M_W$  scale, due to the right handed currents, we have addition contribution to the effective Hamiltonian that is similar to Eqn. (14). The effective Hamiltonian due to the

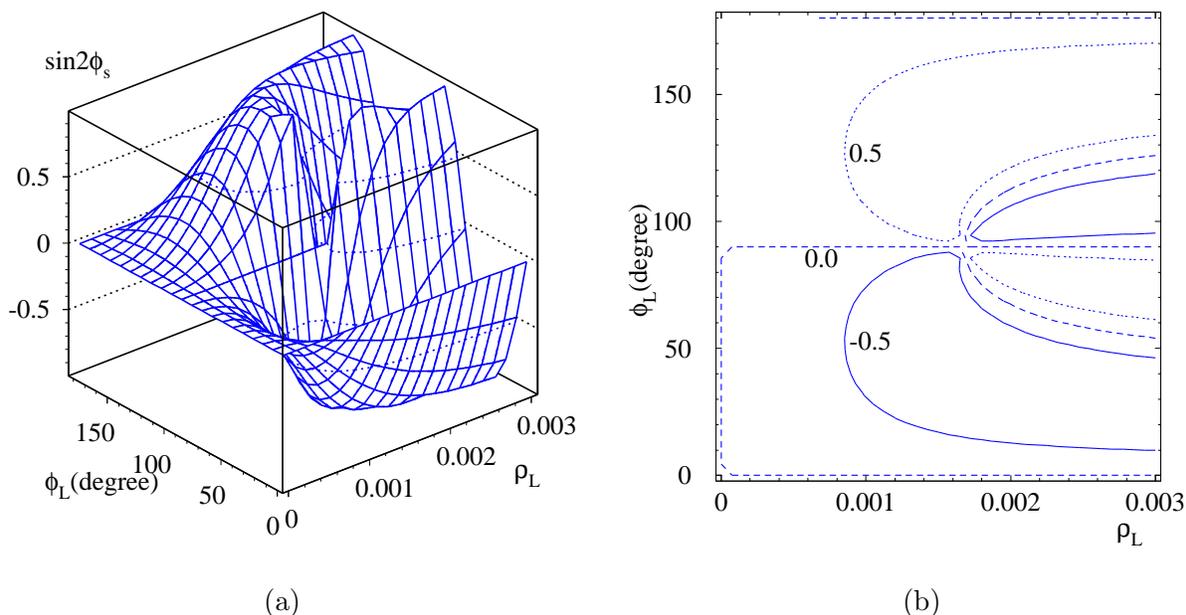


FIG. 2: Plot (a): Three-dimensional plot of  $\sin 2\phi_s$  in the presence of  $Z'$ -mediated FCNC for left-handed  $b$  and  $s$  quarks as a function of  $\rho_L$  and  $\phi_L$ , defined in Eq. (14). Plot (b): A contour plot of  $\sin 2\phi_s$  in the presence of a  $Z'$ -mediated FCNC for left-handed  $b$  and  $s$  quarks.

lift-right mixing is

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \rho_L \rho_R e^{-i(\phi_L - \phi_R)} (O_1^{LR}, O_2^{LR}) \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (31)$$

In the RGE running, the Wilson coefficient for  $O_1^{LR}$  will mix with that of  $O_2^{LR}$  and the relevant anomalous dimension matrices are [39]

$$\gamma^{(0)} = \begin{pmatrix} \frac{6}{N_c} & 12 \\ 0 & -6N_c + \frac{6}{N_c} \end{pmatrix}, \quad \text{and} \quad (32)$$

$$\gamma^{(1)} = \begin{pmatrix} \frac{137}{6} + \frac{15}{2N_c^2} - \frac{22}{3N_c} f & \frac{200}{3} N_c - \frac{6}{N_c} - \frac{44}{3} f \\ \frac{71}{4} + \frac{9}{N_c} - 2f & -\frac{203}{6} N_c^2 + \frac{479}{6} + \frac{15}{2N_c^2} + \frac{10}{3} N_c f - \frac{22}{3N_c} f \end{pmatrix}, \quad (33)$$

where  $N_c$  is the number of colors and  $f$  is the number of active quarks. At the scale of the  $B$  meson, we have  $f = 5$ .

We will use  $m_b(m_b) = 4.4$  GeV,  $\Lambda_{\overline{MS}}^{(5)} = 225$  MeV and  $m_b(m_b) + m_s(m_b) = 4.6$  GeV. Following Eqns. (20-24), we find the effective Hamiltonian at  $m_b$  for the operator  $O_{1,2}^{LR}$  to be

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \rho_R \rho_L e^{-i(\phi_L - \phi_R)} (O_1^{LR}, O_2^{LR}) \begin{pmatrix} 0.930 \\ -0.711 \end{pmatrix}. \quad (34)$$

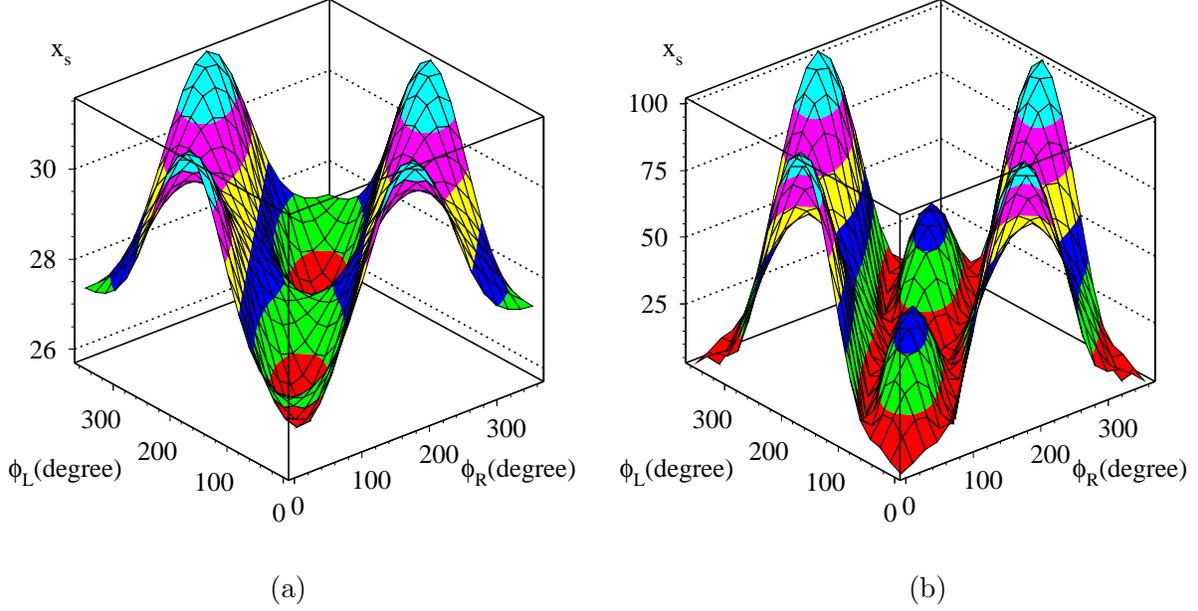


FIG. 3: Plot (a):  $x_s$  as a function of  $\phi_L$  and  $\phi_R$  for  $\rho_L = \rho_R = 0.0002$ . Plot (b):  $x_s$  as a function of  $\phi_L$  and  $\phi_R$  for  $\rho_L = \rho_R = 0.001$ .

Note that at  $M_W$  scale, the operator  $O_2^{LR}$  does not contribute. Through the operator mixing in RGE running, its effect becomes important at the  $m_b$  scale.

The bag parameters from Ref. [35] are  $B_1^{LR}(m_b) = 1.753 \pm 0.021$  and  $B_2^{LR}(m_b) = 1.162 \pm 0.007$  and the decay constant  $f_{B_s}$  is the same as before. The mass difference with all  $Z'$  contributions included is

$$\Delta M = 20.01 + 3.566 \times 10^5 \rho_L^2 e^{2i\phi_L} + 3.566 \times 10^5 \rho_R^2 e^{2i\phi_R} - 1.851 \times 10^6 \rho_L \rho_R \cos(\phi_L - \phi_R) | \text{ ps}^{-1}. \quad (35)$$

The overall contribution for the SM and  $Z'$  to  $x_s$  is,

$$x_s = 29.2 |1 + 3.566 \times 10^5 \rho_L^2 e^{2i\phi_L} + 3.566 \times 10^5 \rho_R^2 e^{2i\phi_R} - 1.851 \times 10^6 \rho_L \rho_R \cos(\phi_L - \phi_R)|. \quad (36)$$

To see the interference among different contributions, we set  $\rho_L = \rho_R = 0.0002$  or  $0.001$  and plot  $x_s$  versus the weak phases  $\phi_L$  and  $\phi_R$  in Fig. 3.

First we note that after the RGE running, the operators  $O_1^{LR}$  and  $O_2^{LR}$  interfere constructively. After adding the corresponding contributions from the  $RL$  counterparts, they

become the most dominant term. As one of the phase approaching  $180^\circ$  and the other phase  $0^\circ$ , the three terms from  $Z'$  all contribute positively. When the phases are close to be equal, the most dominant contribution approaches 0.

## V. MISC

The inclusive branching ratio  $b \rightarrow s\gamma$  is computed to the NLO in the SM to be  $(3.79_{-0.53}^{+0.39}) \times 10^{-4}$  [41], where uncertainties of CKM parameters, charm mass and scale dependence are taken into account. Thus, the SM predicts the ratio

$$\left( \frac{BR(B_d \rightarrow X_s \gamma)}{\Delta M} \right)_{\text{SM}} = (2.94 \pm 1.03) \times 10^7 \text{ GeV}^{-1} . \quad (37)$$

### Possible things to discuss

1. **Include**  $Br(b \rightarrow s\gamma)^{SM} / \Delta M_{B_s}^{SM}$  **and**  $Br(b \rightarrow s\mu^+\mu^-)^{SM} / \Delta M_{B_s}^{SM}$ .
2. **Discuss**  $\Delta\Gamma$ .

## VI. CONCLUSION

### Conclusions here.

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