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Minimal Supersymmetric Standard Model

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Abstract

The Standard Model is extremely succesful, but is viewed as a low-energy effective theory rather than a fundamental theory. One problem in the Standard Model is the so-called hierarchy problem, which involves loop corrections to the Higgs self-energy. To avoid the precise fine tuning that such corrections require, supersymmetry is introduced.

In this paper we draw an analogy to the chiral symmetry between the positron and electron introduced to explain fine-tuning of the electron self-energy. In the Minimal Supersymmetric Standard Model (MSSM), the Standard Model particles are assembled into chiral and gauge supermultiplets with their superpartners, along with an extra Higgs doublet. We use the properties of the generators of the supersymmetry algebra to deduce properties of particle states in the same supermultiplet. Since Standard Model and MSSM particles are not found with degenerate mass in nature, supersymmetry must be broken at some scale. We discuss the supersymmetric and soft super-symmetry breaking terms in the Lagrangian.

1 Introduction

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group of the Standard Model provides a very successful phenomenological description of presently known high energy phenomena.[1] The distance scale of the Standard Model is given by the vacuum expectation value of the Higgs boson condensate. However, it is likely only a low-energy effective theory of a more fundamental theory.

1.1 Motivation for Supersymmetry

1.1.1 Positron Analogue

In classical electrodynamics, an electron in vacuum has a Coulomb field around it. The Coulomb field has the energy:

$$\Delta E_{Coulomb} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e}, \tag{1}$$

where r_e is the "size" of the electron introduced to cut off the divergent Coulomb self-energy. The electron mass receives an additional contribution from the Coulomb

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self-energy:

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{Coulomb} \tag{2}$$

From experiment, the electron is localized within $r_e < \; 10^{-17}$ cm, so the self-energy $\Delta E_{Coulomb}$ is at least 10 GeV. Consequently the "bare" electron mass must be negative to obtain the observed electron mass:

$$0.511\text{MeV} = -999.489\text{MeV} + 10000.000\text{MeV} \tag{3}$$

Ignoring the conceptual issue of what a negative electron mass means, there is an unnaturally finely-tuned cancellation between the "bare" mass of the electron and the Coulomb energy.

This problem was resolved through the introduction of the positron, which doubled the degrees of freedom in the theory. The Coulomb self-energy is represented by a diagram in which the electron emits and reabsorbs a virtual photon. Now one can consider vacuum fluctuations that produce an electron-positron pair from the vacuum along with a photon. The electron in the vacuum can also annihilate with the positron and the photon in the vacuum fluctuation. Then the electron remains as a real electron.

The contribution of this latter process to the electron self-energy is negative and perfectly cancels the linearly divergent piece of $1/r_e$ of the Coulomb self-energy [2]:

$$\Delta E_{pair} = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e} \tag{4}$$

Then in the limit $r_e \to 0$, the leading contribution is:

$$\Delta E = \Delta E_{Coulomb} + \Delta E_{pair} = \frac{3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e}. \tag{5}$$

The correction ΔE is proportional to the electron mass, so the total mass is proportional to the bare electron mass:

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} \left(\frac{1 + 3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e} \right). \tag{6}$$

The correction is only a fraction of the bare mass, and the correction depends only logarithmically on the "size" of the electron. Even if this size is taken to be the Planck length $r_e = 1/M_{Pl} = 1.6 \times 10^{-33}$cm, the correction is only a 9% increase in the electron mass.

The correction is proportional to the mass because of the introduction of chiral symmetry. In the limit of exact chiral symmetry, the electron is massless and is prevented from acquiring mass from self-energy corrections. The non-zero electron mass explicitly breaks the chiral symmetry, so the correction must be proportional to the electron mass. [3]

1.1.2 Higgs Self-Energy

In the Standard Model, the Higgs potential is:

$$V = \mu^2 |H|^2 + \lambda |H|^4, \tag{7}$$

where $v^2 =< H >^2 = -\mu^2/2\lambda = (176\text{GeV})^2$ is the vacuum expectation value of the Higgs. $\lambda \lesssim 1$ by perturbative unitarity, so $-\mu^2 \sim (100\text{GeV})^2$. But μ^2 receives a quadratically divergent contribution from self-energy corrections. For example, the Higgs doublet can split into a pair of top quarks and recombine to the Higgs boson gives the correction:

$$\Delta\mu^2_{top} = -6 \frac{h_t^2}{4\pi^2} \frac{1}{r_H^2}, \tag{8}$$

where r_H is the "size" of the Higgs boson and $h_t \approx 1$ is the top quark Yukawa coupling.

As with the positron, supersymmetry doubles the degrees of freedom by introducing an explicitly broken new symmetry. In particular, the top quark has a superpartner called the "stop"' (scalar top), whose loop diagram cancels the leading piece in $1/r_H$ of the gives a contribution to the self-energy of the Higgs boson self-energy:

$$\Delta\mu^2_{stop} = +6 \frac{h_t^2}{4\pi^2} \frac{1}{r_H^2}, \tag{9}$$

This leaves the correction:

$$\Delta\mu^2_{top} + \Delta\mu^2_{stop} = -6 \frac{h_t^2}{4\pi^2} (m_{\tilde{t}}^2 - m_t^2) \log \frac{1}{r_H^2 m_{\tilde{t}}^2}. \tag{10}$$

For $\Delta\mu^2$ to be the same order of magnitude as the tree-level value $\mu^2 = -2\lambda v^2$, $m_{\tilde{t}}^2$ must be near the electroweak scale. This naturalness constraint on the superparticle masses also applies to the other superpartners that couple directly to the Higgs doublet.

Besides supersymmetry, there exist other solutions to fine-tuning of the Higgs boson mass-squared term. For example, technicolor replaces the Higgs doublet with a composite techni-quark condensate, so that $r_H \sim 1$ TeV is the true physical size of the Higgs doublet and no fine-tuning is needed. Alternatively, large extra spatial dimensions have been proposed to lower the Planck scale to the TeV scale. Fortunately, these ideas all make predictions that will be tested at future colliders such as the LHC.

2 Supermultiplets

The generator of supersymmetry Q must be an anticommutating spinor such that:

$$Q|\text{Boson} >= |\text{Fermion} > \tag{11}$$
$$Q|\text{Fermion} >= |\text{Boson} > \tag{12}$$

Because Q and $Q^\dagger$ are fermionic operators, each carries spin angular momentum 1/2. They must satisfy the (anti)commutation relations: $\{Q, Q^\dagger\} = P^\mu$, $\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$, and $[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$, where P^μ generates spacetime translations [4].

In supersymmetry the original Standard Model multiplets are extended to supermultiplets, which are irreducible representations of the supersymmetry algebra. A supermultiplet contains both fermions and bosons, which are superpartners of each other.

The $(\mathrm{mass})^2$ operator $-P^2$ commutes with Q, $Q^\dagger$, and with the spacetime rotation and translation operators. Hence all particles in the same supermultiplet have the same eigenvalues of $-P^2$, i.e. they all have the same mass.

In addition, Q and $Q^\dagger$ commute with the generators of gauge transformations. Hence all particles in the same multiplet transform in the same representation of the gauge group. This means that they all have the same electric charge, weak isospin, and color degrees of freedom.

Lastly, it can be shown that there are equal number of fermionic and bosonic degrees of freedom in each supermultiplet. Consider the operator $(-1)^{2s}$, where s is the spin angular momentum. This operator has eigenvalue +1 for a bosonic state and -1 for a fermionic state. Now $(-1)^{2s}$ anticommutes with Q and $Q^\dagger$. Let $|\mathrm{i}>$ index the states in a supermultiplet with the same eigenvalue p^μ of the operator P^μ. Using the anticommutation relations, as well as the completeness relation $\sum_i |i><i| = 1$, one can show that:

$$\sum_i < i|(-1)^{2s} P^\mu |i>= 0. \tag{13}$$

But this is equal to $p^\mu \mathrm{Tr}[(-1)^{2s}]$. This is in turn proportional to $n_B - n_F$, where n_B and n_F are the number of bosonic and fermionic degrees of freedom. Thus $n_B = n_F$ for $p^\mu \neq 0$ in each supermultiplet.

The simplest way to build a supermultiplet that satisfies the above requirements is to combine a Weyl fermion (which has two helicity states, so $n_F = 2$) with a complex scalar field. This is called a "chiral supermultiplet".

The next simplest way is to combine a massless spin-1 vector boson (two helicity states, so $n_B = 2$) with a massless spin-1/2 Weyl fermion (two helicity states, so $n_F = 2$). Note that the boson must be massless and the fermion must be massless and spin-1/2 (as opposed to spin-3/2) in order to preserve renormalizibility of the theory. This is called a "gauge supermultiplet".

Lastly, we have to introduce an additional Higgs doublet, so there are now two such doublets with hypercharge $Y = \pm\frac{1}{2}$. This is necessary to prevent triangle gauge anomalies, which manifest themselves in terms proportional to $\mathrm{Tr}[Y^3]$ and $\mathrm{Tr}[Y]$.

3 Minimal Supersymmetric Standard Model

The extension of the Standard Model with minimal particle content is called the Minimal Supersymmetric Standard Model. The superpartners of the gauge bosons are called "gauginos", and the superpartners of the leptons are called "sleptons". The prependage of "s" stands for "scalar". Similarly, superpartners of quarks are called "squarks".

The total particle content can be organized into the following two tables:

Chiral Supermultiplets		Spin 0	Spin 1/2	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Squarks, quarks ($\times$3 generations)	Q	$(\tilde{u}_L\ \tilde{d}_L)$	$(u_L\ d_L)$	$(3,\ 2,\ \frac{1}{6})$
	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{3},\ 1,\ -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{3},\ 1,\ \frac{1}{3})$
Sleptons, leptons ($\times$3 generations)	L	$(\tilde{\nu}\ \tilde{e}_L)$	$(\nu\ e_L)$	$(1,\ 2,\ -\frac{1}{2}\ )$
	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(1,\ 1,\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ H_u^0)$	$(\tilde{H}_u^+\ \tilde{H}_u^0)$	$(1{,}2,\ +\frac{1}{2})$
	H_d	$(H_d^0\ H_d^-)$	$(\tilde{H}_d^0\ \tilde{H}_d^-)$	$(1{,}2,\ -\frac{1}{2})$

Gauge Supermultiplets	Spin 1/2	Spin 1	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Gluino, gluon	$\tilde{g}$	g	(8, 1, 0)
Winos, W bosons	$\tilde{W}^\pm\ \tilde{W}^0$	$W^\pm\ W^0$	(1, 3, 0)
Bino, B boson	$\tilde{B}^0$	B^0	(1, 1, 0)

4 Supersymmetric Lagrangian

Let the chiral supermultiplet ϕ denote three fields: a complex scalar field A, a Weyl fermion $\frac{1-\gamma_5}{2}\psi$, and an auxiliary complex field F. The Kähler potential contains the kinetic terms for ϕ in the Lagrangian[3]:

$$\int d^4x\phi_i^*\,\phi_i = \partial_\mu A_i^*\partial^\mu A_i + \bar{\psi}_i i\gamma^\mu\partial_\mu\psi_i + F_i^*F_i \tag{14}$$

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Since F does not contain derivatives, it can be solved for explicitly and eliminated. In particular, the superpotential is a homomorphic function $W(\phi)$ of the chiral supermultiplets ϕ. It contributes to the Lagrangian:

$$-\int d^2x\,W(\phi) = -\frac{1}{2}\left.\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\right|_{\phi_i=A_i}\psi^i\psi^j + \left.\frac{\partial W}{\partial\phi_i}\right|_{\phi_i=A_i}F_i \tag{15}$$

If we solve for F then:

$$F_i^* = -\left.\frac{\partial W}{\partial \phi_i}\right|_{\phi_i = A_i}. \tag{16}$$

Eliminating F in the Lagrangian yields:

$$-V_F = -\left|\frac{\partial W}{\partial \phi_i}\right|^2_{\phi_i = A_i}. \tag{17}$$

Let W_α be a gauge multiplet, the generalization of the gauge bosons. It consists of three components in the adjoint representation of the gauge group, indexed by a. These are a Weyl fermion λ (gaugino), a vector gauge field A_μ, and an auxiliary real scalar field D. Their kinetic terms are:

$$\int d^2x W_\alpha^a W^{\alpha a} = -\frac{1}{4}F_{\mu\nu} + \bar{\lambda}^a i \not{D} \lambda^a + \frac{1}{2} D^a D^a. \tag{18}$$

As with F, the field D does not contain derivatives, so it can be eliminated from the Lagrangian as follows.

To preserve gauge invariance of the Lagrangian, chiral supermutiplets that transform non-trivially under the gauge group should also couple to the gauge multiplets. Then the Kähler potential is modified to:

$$\int d^4x \phi_i^\dagger e^{2gV} \phi_i = D_\mu A_i^\dagger D^\mu A_i + \bar{\psi}_i i \gamma^\mu D_\mu \psi_i + F_i^\dagger F_i - \sqrt{2} g (A^\dagger T^a \lambda^a \psi) - g A^\dagger T^a D^a A \tag{19}$$

Eliminating D in the Lagrangian yields:

$$-V_D = -\frac{g^2}{2}(A^\dagger T^a A)^2 \tag{20}$$

Now the potential terms V_F and V_D that we have solved for determine the supersymmetric Lagrangian. By the non-renormalization theorem of the superpotential, no superpotential mass terms can be generated by renormalizations.

As a simple example, suppose there are two chiral supermultiplets ϕ_1 and ϕ_2 with a superpotential

$$W = m\phi_1\phi_2. \tag{21}$$

Then the Dirac fermion components have mass terms in the Lagrangian:

$$-\frac{1}{2}\frac{\partial^2 W}{\partial \phi_i \partial \phi_j}\psi^i \psi^j = -m\psi_1\psi_2. \tag{22}$$

The two complex scalars have a mass term:

$$-\left.\frac{\partial W}{\partial \phi_i}\right|^2_{\phi_i = A_i} = -m^2|A_1|^2 - m^2|A_2|^2. \tag{23}$$

Thus we see that the fermionic and bosonic components are degenerate with the same mass m, as we showed previously.

5 Soft Supersymmetry-Breaking Lagrangian

Superpartners with the same mass as the Standard Model particles have not been observed, so supersymmetry must be a broken symmetry.[4] There are various techniques to spontaneously break supersymmetry. However, for a low-energy effective theory, it suffices to simply add terms to the supersymmetric Lagrangian that explicitly break supersymmetry.

These terms must keep the Higgs mass-squared only logarithmically divergent, since that is the original motivation for supersymmetry. They have all been classified for a renormalizable superpotential:

$$W = \frac{1}{2}\mu_{ij}\phi_i\phi_j + \frac{1}{6}\lambda_{ijk}\phi_i\phi_j\phi_k. \tag{24}$$

The soft supersymmetry breaking terms can have the forms: $m^2_{ij}A^*_iA_j$, $M\lambda\lambda$, $\frac{1}{2}b_{ij}\mu_{ij}A_iA_j$, and $\frac{1}{6}a_{ijk}\lambda_{ijk}A_iA_jA_k$.

The first term gives mass to the scalar components in the chiral supermultiplets, removing degeneracy between the scalars and spinors. The second term gives mass to gauginos so that they are no longer degenerate with gauge bosons. The final two terms, which have parameters b_{ij} and a_{ijk} of mass dimension one, are the bilinear and trilinear soft breaking terms.

As a calculational example, consider the coupling of the Higgs chiral supermultiplet H to left-handed Q and right-handed T chiral supermultiplets, given by superpotential:

$$W = h_tQTH_u. \tag{25}$$

This contributes to the Lagrangian:

$$h_tQTH_u - h_t^2|\tilde{Q}|^2|H_u|^2 - h_t^2|\tilde{T}|^2|H_u|^2 - m_Q^2|\tilde{Q}|^2 - m_T^2|\tilde{T}|^2 - h_tA_t\tilde{Q}\tilde{T}H_u. \tag{26}$$

where m_Q^2, m_T^2, and A_t are soft parameters, the fields Q and T are spinor components of the chiral supermultiplets, and $\tilde{Q}$, $\tilde{T}$, and H_u are the scalar components.

The field H_u can be shifted around the vacuum expectation value, generating mass terms for the top quark and scalars. Its one-loop self-energy diagram can be calculated. The diagram with a top quark loop from the first term contributed to the Lagrangian is negative and quadratically divergent. The contractions with $\tilde{Q}$ or $\tilde{T}$ in the following two terms provide positive contributions. If we had $m_Q^2 = m_T^2 = 0$, then these two contributions would cancel each other exactly. They are not zero, but for simplicity suppose $m_Q^2 = m_T^2 = \tilde{m}^2$. Then the correction is:

$$\delta m_H^2 = -\frac{6h_t^2}{16\pi^2}\tilde{m}^2 \log\frac{\lambda^2}{\tilde{m}^2}, \tag{27}$$

where Λ is the UV cutoff for one-loop. This is only logarithmically divergent. Similarly, the diagrams with two h_tA_t couplings with a scalar top loop are only logarithmically divergent as well. Thus we see that the mass-squared parameters are in fact "soft" because they don't have power divergences.

6 Conclusions

The Minimal Supersymmetric Standard Model provides a viable solution to the Hierarchy Problem. The corrections to the Higgs self-energy from internal loops of Standard Model particles are cancelled those of their superpartners, resolving the unnatural fine-tuning that otherwise appears.

However, there are alternative solutions to the Hierarchy Problem: technicolor and large extra dimensions. Future high energy experiments at the LHC and beyond will determine which of these theories, if any, is correct.

References

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