Neutron-Antineutron Oscillations As Baryon Number Violating Processes In Grand Unified Theories

D.A. Fustin

June 7, 2006

Abstract

Experimental nonobservation of proton decay has led to doubt about a certain class of grand unified theories predicting this process, eliminating the simplest SU(5) theory of grand unification altogether. Because of this, theories invoking neutron-antineutron oscillations instead of proton decay have become increasingly popular. One such theory is the left-right symmetric $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$ model of partial unification. In order to explain this model, we draw heavily on Ref. [4]. Since experimental searches for neutron-antineutron oscillations have not excluded as much phase space as those searching for proton decay, neutron-antineutron oscillations remain a very viable option for baryon number nonconservation.

1 Introduction

Much has been made in recent years of attempts to classify all Standard Model physics into a larger group theoretical structure. The Standard Model (SM) as we know it is explained by the group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, where $\mathrm{SU}(3)_c$ describes the strong interactions, while $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ describes the electroweak interactions. This latter symmetry is spontaneously broken by the Higgs mechanism to $\mathrm{U}(1)_{em}$ giving mass to three of the gauge bosons (the W's and the Z mediating weak interactions) and leaving the gauge boson mediating the electromagnetic force (the photon) massless. The scale of this symmetry breaking is at the mass scale of the massive gauge bosons, that is $M_W \sim 100\,\mathrm{GeV}$. The SM group is rank 4, and therefore any larger structure in which we embed this group must be at least rank 4 as well. The only viable option of rank 4 is SU(5), making this group the first place to look to provide a Grand Unified Theory (GUT) explaining a larger gauge structure to the SM.

But, what does grand unification add to our understanding of physics? One aspect of a GUT's usefulness comes in helping to understand why there are three independent gauge couplings in the SM [1]. We have seen in class that the β functions describing the running of these couplings in the SM are negative for the weak and strong couplings (implying asymptotic freedom) and positive for the electromagnetic coupling. These couplings therefore could be expected to meet at some higher scale, called the GUT scale. In fact, in the SU(5) GUT with a coupling g_5, the SM couplings all meet at the scale of SU(5) symmetry breaking and have the values

$$g_5 = g_3 = g = \sqrt{\frac{5}{3}}g'. \tag{1}$$

Using the known particles from the SM to determine the β functions, we find that these couplings all do become relatively close at some scale between 10^{13} and 10^{17} GeV, however they do not exactly intersect. Adding supersymmetry to the SM changes the β functions (and in fact makes the weak β function become positive, thus removing asymptotic freedom) and allows these couplings to meet at a scale of 10^{16} GeV, interpreted as the scale of SU(5) symmetry breaking, referred to as the GUT scale. See Fig. 1.

With the larger gauge structure provided by SU(5) comes many additional gauge bosons. In fact, it doubles the number of bosons from 12 in the SM (eight gluons plus the four electroweak bosons) to 24. The spontaneous symmetry breaking of SU(5) at the GUT scale gives these 12 additional bosons mass, just as the spontaneous symmetry breaking of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ to $\mathrm{U}(1)_{em}$ gives mass to the W's and Z. These 12 extra gauge bosons mediate quark-lepton interactions which are absent in the SM and can lead to effective interactions such as

$$\Delta\mathcal{L} \quad \sim \quad \epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{kl}\left(\bar{q}^C_{i\alpha aL}q_{j\beta bL}\right)\left(\bar{q}^C_{k\gamma cL}l_{ldL}\right), \tag{2}$$

where α, β, and γ are SU(3) color indices, i, j, k, and l are SU(2) indices, and a, b, c, and d are generation indices [2]. All such possible interactions found in Ref. [2] conserve $B - L$ (baryon number minus lepton

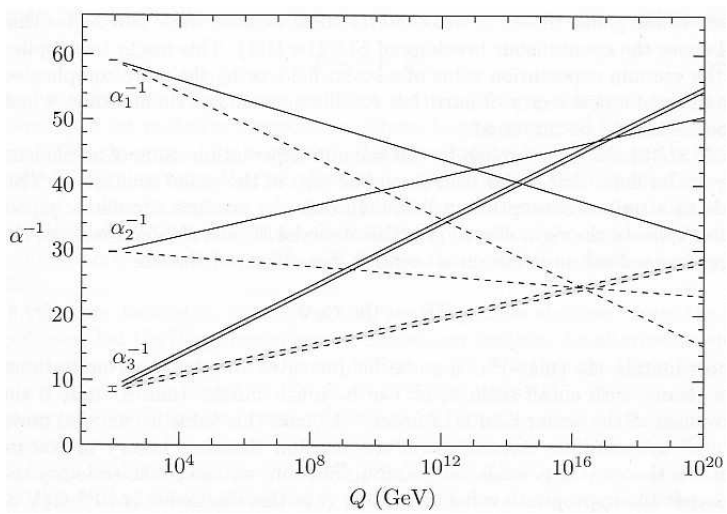

Figure 1: Plot showing the running of the SM couplings at energies above $M_Z \sim 100$ GeV. The solid lines correspond to the SM while the dashed lines correspond to having supersymmetry added to the theory (taken from Ref. [1]).

number) and can lead to a coupling between a u quark and a positron along with a coupling between a d quark and an $\bar{u}$ quark, resulting in the decay of the proton, $p \to e^+\pi^0$.

However, experimental nonobservation of proton decay setting a lower limit on the lifetime of the proton of about 10^{31} to 10^{33} years [3] has led to the exclusion of standard SU(5) as a possible GUT. There are many possibilities to get around this problem, including modifying our gauge structure to allow for longer proton lifetimes. Another possibility focuses on theories that predict $B - L$ nonconservation, particularly $B - L$ symmetry breaking by two units. These theories predict the B-violating process of neutron-antineutron (n-$\bar{n}$) oscillations in favor of proton decay. n-$\bar{n}$ oscillations have not been searched for as rigorously as proton decay and therefore GUTs employing $B - L$ violation are not as restricted by experiment as those predicting the latter process.

2 An Example of a Unification Group Violating $B - L$

Since SU(5) is the only viable unification group of rank 4, we instead look at higher rank groups. In particular, we can look at the left-right symmetric group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_c$ embedded in the partial unification group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$, both of which have rank 5, proposed in Ref. [4]. Now, this theory starts with the assumption that the weak interactions have parity restored above a scale of $M_{W_R} \gg M_{W_L}$ where the group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{L+R}$ spontaneously breaks to the standard electroweak symmetry of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. We build on this symmetry because the vector U(1) generator in this model is associated with $B - L$, instead of the hypercharge Y like in the SM case [4]. Because of this, the electric charge is given by

$$Q = I_{3L} + I_{3R} + \frac{1}{2}(B - L), \tag{3}$$

analogous to the familiar SM relation

$$Q = I_{3L} + \frac{1}{2}Y. \tag{4}$$

What this implies is that in the low energy region described by the SM, Eq. 3 gives

$$\Delta I_{3R} = -\frac{1}{2}\Delta(B - L), \tag{5}$$

thus opening the possibility of $B - L$ violation modulo two.

To provide unification by adding quark-lepton interactions, we take this symmetry and embed it into a higher gauge structure $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$ first proposed by Pati and Salam with $\mathrm{SU}(4)_c$ having L as the fourth color [5]. In our case, however, we will make the fourth color $B-L$. This symmetry breaks at a unification scale of M_X to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_c$ as above, breaking again at a scale of M_{W_R} to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_c$, which is the SM. Given that $M_X > M_{W_R} \gg M_{W_L}$, our unification group gives the same predictions as the SM at all energies currently probed. To better understand this symmetry-breaking and exactly how n-$\bar{n}$ oscillations are allowed in this model, we must first understand its structure a bit more.

If we simplify our model to just one family, the fermions in $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$ are in the following representations:

$$\Psi_{L,i} = \left(\frac{1}{2}, 0, 4\right) \tag{6}$$

$$\Psi_{R,i} = \left(0, \frac{1}{2}, 4\right), \tag{7}$$

with i denoting color such that

$$\Psi = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}. \tag{8}$$

We call our couplings to the SU(2)'s $g = g_L = g_R$, and the coupling to the SU(4) g_c. According to Ref. [4], the breakdown of $\mathrm{SU}(4)_c \to \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_c$ is accomplished by giving mass to the appropriate gauge bosons via a Higgs multiplet Σ in the representation $(1, 1, \bar{15})$ with a vacuum expectation value (vev) of $\langle \Sigma \rangle = \mathrm{diag}(1, 1, 1, -3) M_X / g_c$.

Employing other Higgs multiplets $\Phi = \left(\frac{1}{2}, \frac{1}{2}, 1\right)$, $\Delta_L = (1, 0, \bar{10})$, and $\Delta_R = (0, 1, \bar{10})$, we can further break the symmetry from $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_c$ to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_c$ and finally to $\mathrm{U}(1)_{em} \times \mathrm{SU}(3)_c$. We can write the Δ's by expressing them explicitly with their indices, giving $\Delta^a_{L,ij}$ and $\Delta^a_{R,ij}$ with i and j (with i, j symmetry) the color indices of $\mathrm{SU}(4)_c$ and a the flavor index. Our spontaneous symmetry breaking is caused by assigning the following vev's:

$$\left\langle \Delta^{1+i2}_{R,44} \right\rangle = v \tag{9}$$

$$\langle \Delta_{L,ij} \rangle = 0 \tag{10}$$

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}. \tag{11}$$

The Φ vev causes our typical SM Higgs mechanism, so that $M_{W_L} = g\kappa \sim 100\,\mathrm{GeV}$. The higher scale at which $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_c \to \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_c$ is at $M_{W_R} = gv$.

Now, according to Ref. [4], the possible allowed gauge-invariant Yukawa couplings are

$$\mathcal{L}_Y = ih\left(\Psi^{\mathrm{T}}_{L,i}\tau_2\tau_a C^{-1}\Psi_{L,j}\Delta^a_{L,ij} + \Psi^{\mathrm{T}}_{R,i}\tau_2\tau_a C^{-1}\Psi_{R,j}\Delta^a_{R,ij}\right) + h_1\bar{\Psi}_L\phi\Psi_R + h_1'\bar{\Psi}_L\tilde{\phi}\Psi_R + \mathrm{h.c.} \tag{12}$$

As an aside, Eq. 12 along with our Higgs vev's lead immediately to a mass matrix for neutrinos giving $M_{\nu_R} = hv = gM_{W_R}$ and $m_{\nu_L} = {m_e}^2/gM_{W_R}$. This implies that our SM neutrinos are Majorana particles and therefore $\Delta L = 2$, as expected [4]. Focusing instead on the hadronic sector, we first note that there is a self-coupling of the Δ's such that

$$\mathcal{L}_S = \lambda\left[\epsilon^{ikmp}\epsilon^{jlnq}\Delta^a_{L,ij}\Delta^a_{L,kl}\Delta^b_{L,mn}\Delta^b_{L,pq} + (L \leftrightarrow R) + \mathrm{h.c.}\right]. \tag{13}$$

Since only the Δ_R has a non-zero vev, we note that using the R components of Eq. 13 and Eq. 12 we can develop a six fermion interaction given by

$$\Delta\mathcal{L} \sim h_{\mathrm{eff}} d^{\mathrm{T}}_{R,\alpha} C^{-1} d_{R,\beta} d^{\mathrm{T}}_{R,\gamma} C^{-1} d_{R,\lambda} u^{\mathrm{T}}_{R,\rho} C^{-1} u_{R,\sigma} \epsilon^{\alpha\lambda\rho}\epsilon^{\beta\lambda\sigma} + \text{similar terms}, \tag{14}$$

where $h_{\mathrm{eff}} = \lambda h^3 \langle \Delta_{R,44} \rangle / {M_{\Delta_R}}^6$. It is this interaction that allows for the $\Delta B = 2$ process of n-$\bar{n}$ oscillations, as shown in Fig. 2.

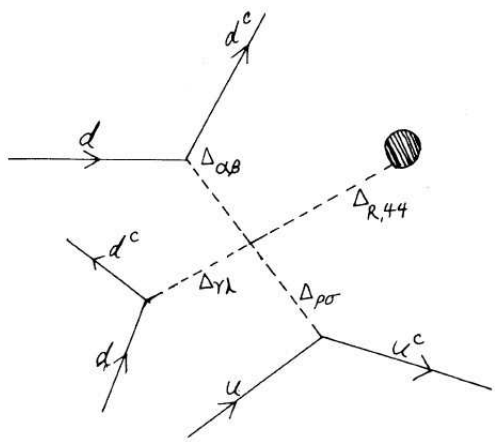

Figure 2: Diagram of the six fermion interaction of Eq. 14 leading to n-$\bar{n}$ oscillations (taken from Ref. [4]).

It should be noted that there are other theories predicting n-$\bar{n}$ oscillations as well. For instance, Ref. [6] develops a means of using an SU(5) invariant theory to violate $B - L$ conservation. Further, just as typical GUTs are improved by adding supersymmetry (allowing the couplings to meet at the GUT scale), the left-right symmetric model discussed can also be augmented by supersymmetry [7]. On top of improving the running of the SM couplings, the supersymmetric left-right symmetric (SUSYLR) model has the added benefit of improving upon the hierarchy of neutrino masses. In the non-supersymmetric case, using the seesaw mechanism to provide neutrino masses requires a ν_R mass scale of M_{W_R}, giving a neutrino mass hierarchy in the eV-keV-MeV range. While this is quite unappealing, adding SUSY to the picture allows a lower ν_R mass scale giving neutrino masses in the meV to eV range [7]. In short, there are many interesting GUTs employing n-$\bar{n}$ oscillations rather than proton decay that lead to exciting physical possibilities in explaining things such as neutrino masses at the same time as predicting B-nonconservation.

3 Detecting n-$\bar{n}$ Oscillations

The phenomenon of n-$\bar{n}$ oscillations is considerably altered in the presence of external fields. For instance, unbound neutrons in terrestrial experiments are still affected by the earth's magnetic field, decreasing the probability of oscillation. In the same way, neutrons bound within nuclei experience a similar increase in their oscillation time. In fact, the free n-$\bar{n}$ oscillation time ($\tau_{n\bar{n}}$) is related to the nuclear lifetime associated with n-$\bar{n}$ oscillations in nuclei (T) by

$$T = {\tau_{n\bar{n}}}^2 T_R, \tag{15}$$

where T_R is a nuclear suppression factor which varies for different nuclei (see, for example, Ref. [8]).

As seen in Ref. [9], for free neutrons which are part of a double state $\psi = \begin{pmatrix} n \\ \bar{n} \end{pmatrix}$, the mass Lagrangian is given by

$$\mathcal{L}_{\text{mass}} = \bar{\psi} M \psi \tag{16}$$

where M is the mass matrix given by

$$M = \begin{pmatrix} A & \delta m \\ \delta m & A \end{pmatrix}. \tag{17}$$

A non-zero value of δm in Eq. 17 results from n-$\bar{n}$ oscillations. The mass eigenstates are

$$n_{1,2} = \frac{1}{\sqrt{2}} (n \pm \bar{n}) \tag{18}$$

with masses

$$m_{1,2} = A \pm \delta m. \tag{19}$$

Calculating the probability of n-$\bar{n}$ oscillation is analogous to neutrino oscillation probability calculation. Neutrons and antineutrons are in an initial state

$$|n(\bar{n})\rangle = \frac{1}{\sqrt{2}}\left(|n_1\rangle \pm |n_2\rangle\right). \tag{20}$$

If we start with a decaying (with decay width γ) neutron and propagate each state in time using e^{-imt}, we find that the state at a time t is

$$\begin{aligned} |n(t)\rangle &= \frac{e^{-\gamma t/2}}{\sqrt{2}}\left(|n_1\rangle e^{-im_1 t} + |n_2\rangle e^{-im_2 t}\right) \\ &= \frac{e^{-\gamma t/2}}{\sqrt{2}} e^{-im_1 t}\left(|n_1\rangle + |n_2\rangle e^{2i\delta m t}\right). \end{aligned} \tag{21}$$

The amplitude of oscillation in an $\bar{n}$ state is given by

$$\begin{aligned} \langle\bar{n}|n(t)\rangle &= \frac{e^{-\gamma t/2}}{2} e^{-im_1 t}\left(\langle n_1| - \langle n_2|\right)\left(|n_1\rangle + |n_2\rangle e^{2i\delta m t}\right) \\ &= \frac{e^{-\gamma t/2}}{2} e^{-im_1 t}\left(1 - e^{2i\delta m t}\right), \end{aligned} \tag{22}$$

and therefore the oscillation probability is

$$\begin{aligned} P_{n\bar{n}} &= |\langle\bar{n}|n(t)\rangle|^2 \\ &= \frac{e^{-\gamma t}}{2}\left(1 - \cos 2\delta m t\right). \end{aligned} \tag{23}$$

Thus determining the probability of n-$\bar{n}$ oscillations experimentally will also give the mass splitting between the neutron and antineutron.

There are many way in which one can search for n-$\bar{n}$ oscillations in experiment, both by observing "free" neutrons in a beam (see, for example, Ref. [10]) and by observing neutrons bound within nuclei (see, for example, Ref. [11]). As noted in Eq. 15, the time-scale for the latter process is considerably longer than that of the former, however it is quite easy to collect a large quantity of nuclei and wait for an annihilation signature. In fact, nonobservation of n-$\bar{n}$ oscillations in both types of experiments has led to similar limits on $\tau_{n\bar{n}}$: $\tau_{n\bar{n}} > 0.86 \times 10^8$ s for free neutrons [10] and $\tau_{n\bar{n}} > 1.3 \times 10^8$ s for bound neutrons [11]. It has also been noted that with the excess of neutrons due to the deuterium in the heavy water used in the Sudbury Neutrino Observatory (SNO), a similar search can be performed to that in Ref. [11]. By conservative estimates, an experimental search for n-$\bar{n}$ oscillations in SNO, if it were carried out, would increase this limit to at least $\tau_{n\bar{n}} > 2 \times 10^8$ s [12].

SNO is a heavy water Čerenkov detector employing an array of approximately 10000 photomultiplier tubes (PMTs) to detect Čerenkov light from particles associated primarily with neutrino interactions [13]. However, these PMTs can also be employed to detect Čerenkov light generated from multiple pions resulting from an antineutron-nucleon annihilation after an n-$\bar{n}$ oscillation occurs in a nucleus. The experimental signature of such an event would be a multiple ring pattern in the PMTs. The systematics of performing this search are worked out in Ref. [12].

4 Conclusion

While many attempts at grand unification predict B-violation (but a conservation of $B - L$) resulting in proton decay, still others predict $B - L$ breaking by two units resulting in n-$\bar{n}$ oscillations. The latter have become more appealing as of late due to the experimental nonobservation of proton decay up to a time scale of 10^{31} to 10^{33} years. In comparison, the lower limit on the n-$\bar{n}$ oscillation time in free space is only approximately 10^8 seconds. Further, models predicting n-$\bar{n}$ oscillations such as the left-right symmetric $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$ model of partial unification provide other compelling explanations of physical phenomena like the generation of neutrino masses. Better experimental searches for n-$\bar{n}$ oscillations such as the one proposed in Ref. [12] should be encouraged along with improvements to limits set on the proton lifetime in limiting the possible theories providing a unified understanding of particle physics.

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