Physics Beyond the Standard Model

Carlos E.M. Wagner

HEP Physics Division, Argonne National Laboratory

Enrico Fermi Institute, University of Chicago

Sixth Giambiagi Winter School, U.B.A., Buenos Aires, July 28-30, 2004.

Standard Model Particles

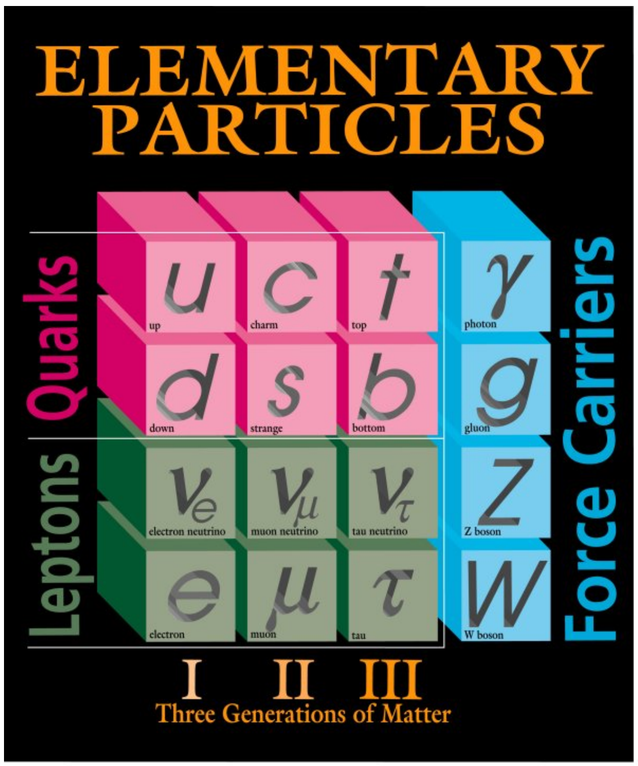
There are 12 fundamental gauge fields:

8 gluons, 3 W_μ's and B_μ

and 3 gauge couplings g_1, g_2, g_3

The matter fields:

3 families of quarks and leptons with same quantum numbers under gauge groups



Fermilab 95-759

But very different masses!

m_3/m_2 and $m_2/m_1 \simeq$ a few tens or hundreds

$m_e = 0.5\ 10^{-3}$ GeV, $\frac{m_\mu}{m_e} \simeq 200$, $\frac{m_\tau}{m_\mu} \simeq 20$

Largest hierarchies

$m_t \simeq 175$ GeV $\qquad m_t/m_e \propto 10^5$

neutrino masses smaller than as 10^{-9} GeV!

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2

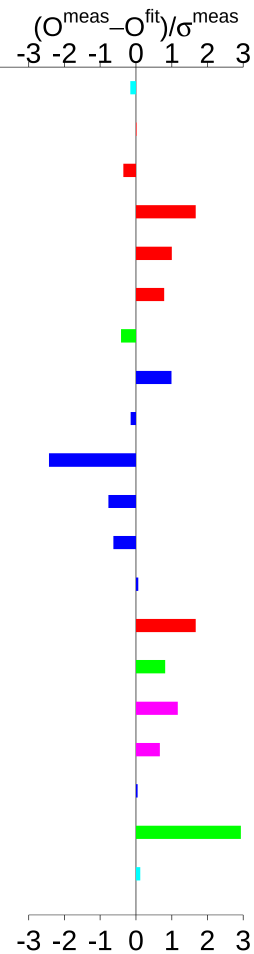
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Precision Tests of the SM

- The SM has been tested with very high precision (one part in a thousand) at experiments around the world: CERN, Fermilab, SLAC

Winter 2003

	Measurement	Pull
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	-0.16
m_Z [GeV]	91.1875 ± 0.0021	0.02
Γ_Z [GeV]	2.4952 ± 0.0023	-0.36
σ^0_{had} [nb]	41.540 ± 0.037	1.67
R_l	20.767 ± 0.025	1.01
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.79
$A_l(P_\tau)$	0.1465 ± 0.0032	-0.42
R_b	0.21644 ± 0.00065	0.99
R_c	0.1718 ± 0.0031	-0.15
$A^{0,b}_{fb}$	0.0995 ± 0.0017	-2.43
$A^{0,c}_{fb}$	0.0713 ± 0.0036	-0.78
A_b	0.922 ± 0.020	-0.64
A_c	0.670 ± 0.026	0.07
A_l(SLD)	0.1513 ± 0.0021	1.67
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.82
m_W [GeV]	80.426 ± 0.034	1.17
Γ_W [GeV]	2.139 ± 0.069	0.67
m_t [GeV]	174.3 ± 5.1	0.05
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	2.94
$Q_W(Cs)$	-72.83 ± 0.49	0.12



Spontaneous Symmetry Breakdown

Particle Masses arise through the Higgs mechanism: Spontaneous breakdown of gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{\mathrm{em}} \tag{1}$$

A scalar field, charged under the gauge group, acquires v.e.v.

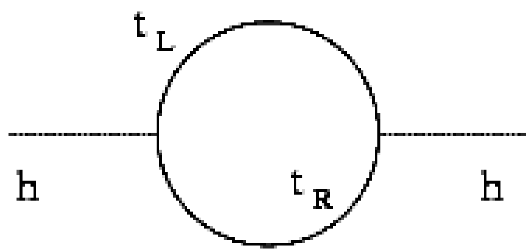
$$V(H) = m_H^2 H^\dagger H + \frac{\lambda}{2}\left(H^\dagger H\right)^2 \tag{2}$$

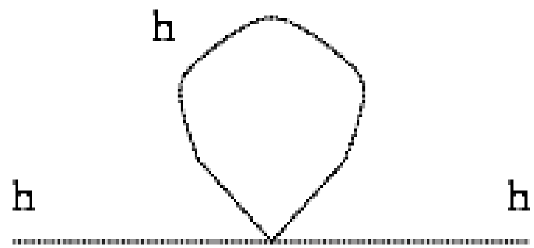
Therefore,

$$\langle H^\dagger H \rangle = -\frac{m_H^2}{\lambda} \tag{3}$$

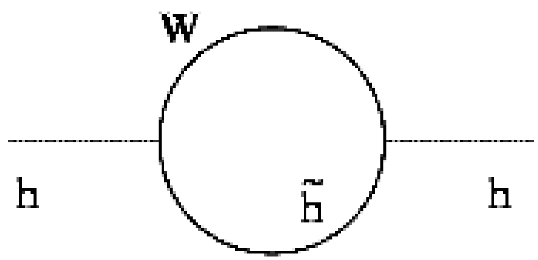
the v.e.v. of the Higgs field is fixed by the value of the negative mass parameter.
Problem: The mass parameter is unstable under quantum corrections.

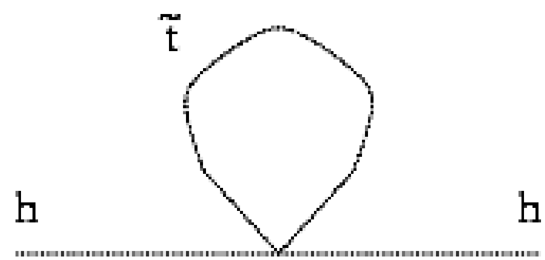
Quantum corrections induce quadratic divergent result





$$\delta\, m_H^2 \approx (-1)^{2\, S_i} \, \frac{n_i \, g_i^2}{16\pi^2} \, \Lambda^2$$

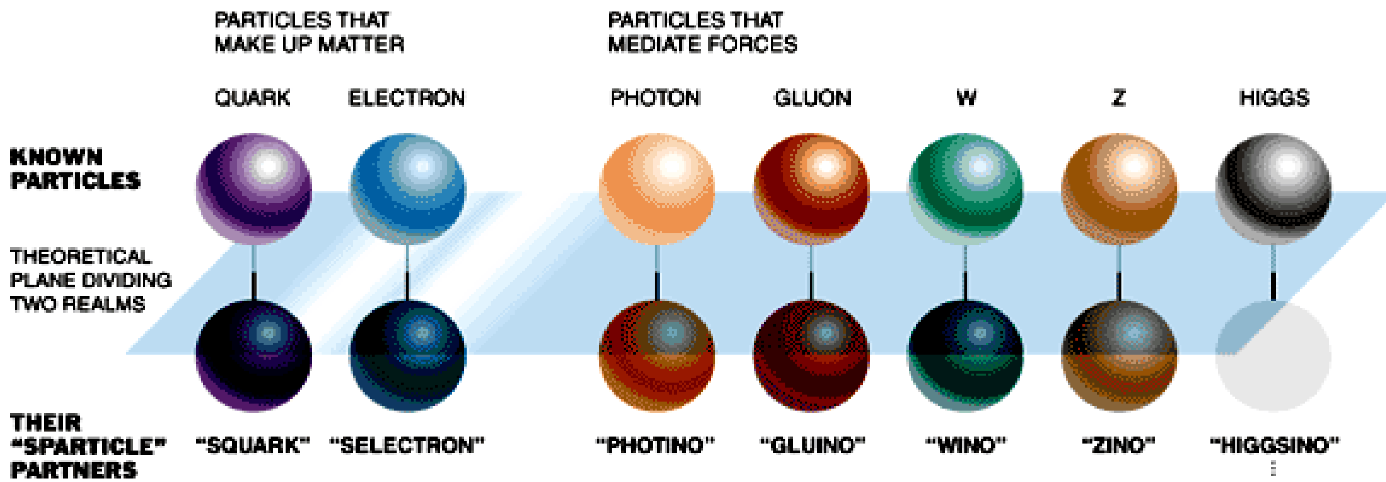




Cancelled if particles of different spin with same couplings are present. This happens naturally within a supersymmetric extension of the Standard Model

supersymmetry

fermions **bosons**



Photino, Zino and Neutral Higgsino: Neutralinos

Charged Wino, charged Higgsino: Charginos

No new dimensionless couplings. Couplings of supersymmetric particles equal to couplings of Standard Model ones.
Two Higgs doublets necessary. Ratio of vacuum expectation values denoted by $\tan\beta$

Why Supersymmetry ?

- Helps to stabilize the weak scale—Planck scale hierarchy

- Supersymmetry algebra contains the generator of space-time translations.
  Necessary ingredient of theory of quantum gravity.

- Minimal supersymmetric extension of the SM :
  Leads to Unification of gauge couplings

- Starting from positive masses at high energies, electroweak symmetry breaking is induced radiatively

- If discrete symmetry, $P = (-1)^{3B+L+2S}$ is imposed, lightest SUSY particle neutral and stable: Excellent candidate for cold Dark Matter.

Structure of Supersymmetric Gauge Theories

- The Standard Model is based on a Gauge Theory.
- A supersymmetric extension of the Standard Model has then to follow the rules of Supersymmetric Gauge Theories.
- These theories are based on two set of fields:
  - Chiral fields, that contain left handed components of the fermion fields and their superpartners.
  - Vector fields, containing the vector gauge bosons and their superpartners.
- Right-handed fermions are contained on chiral fields by means of their charge conjugate representation

$$(\psi_R)^C = \left(\psi^C\right)_L \tag{4}$$

- Higgs fields are described by chiral fields, with fermion superpartners

Generators of Supersymmetry

- Supersymmetry is a symmetry that relates boson to fermion degrees of freedom.
- The generators of supersymmetry are two component anticommuting spinors, Q_α, $\bar{Q}^{\dot{\alpha}}$, satisfying

$$\{Q_\alpha, Q_\beta\} = 0 \tag{5}$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \tag{6}$$

where $\sigma^\mu = (I, \vec{\sigma})$, $\bar{\sigma}^\mu = (I, -\vec{\sigma})$, and σ^i are the Pauli matrices.

- Two-spinors may be contracted to form Lorentz invariant quantities

$$\psi^\alpha \chi_\alpha = \psi^\alpha \epsilon_{\alpha\beta} \chi^\beta \tag{7}$$

Four-component vs. Two-component fermions

- A Dirac Spinor is a four component object whose components are

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_D^C = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{8}$$

- A Majorana Spinor is a four component object whose components are

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_M^C = \psi_M \tag{9}$$

- Gamma Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \tag{10}$$

- Observe that $\psi_{D,L} = \chi$; $\psi_{D,R} = \bar{\psi}$

- Usual Dirac contractions may be then expressed in terms of two component contractions.

$$\bar{\psi}_D = (\psi^\alpha \quad \bar{\chi}_{\dot{\alpha}}) \tag{11}$$

- For instance,

$$\bar{\psi}_D \, \psi_D = \psi\chi + h.c.; \tag{12}$$

$$\bar{\psi}_D \gamma^\mu \psi_D = \psi \bar{\sigma}^\mu \bar{\psi} + \bar{\chi} \sigma^\mu \chi \quad = \quad -\bar{\psi} \sigma^\mu \psi + \bar{\chi} \sigma^\mu \chi \tag{13}$$

- Other relations may be found in the literature.

Superspace

- In order to describe supersymmetric theories, it proves convenient to introduce the concept of superspace.
- Apart from the ordinary coordinates x^μ, one introduces new anticommuting spinor coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$; $[\theta] = [\bar{\theta}] =$ -1/2.
- This allows to represent fermion and boson fields by the same superfield.
- For instance, a generic chiral field, forming the base for an irreducible representation of SUSY, is given by

$$\Phi(x, \theta, \bar{\theta} = 0) = A(x) + \sqrt{2}\,\theta\,\psi(x) + \theta^2 F(x) \tag{14}$$

$$\Phi(x, \theta, \bar{\theta}) = \exp(-i\partial_\mu \theta \sigma^\mu \bar{\theta})\,\Phi(x, \theta, \bar{\theta} = 0) \tag{15}$$

- A, ψ and F are the scalar, fermion and auxiliary components.

Transformation of chiral field components

- Supersymmetry is a particular translation in superspace, characterized by a Grassman parameter ξ.
- Supersymmetry generators may be given as derivative operators

$$Q_\alpha = i\left[-\partial_\theta - i\sigma^\mu \bar{\theta}\partial_\mu\right] \tag{16}$$

- Under supersymmetric transformations, the components of chiral fields transform like

$$\begin{aligned} \delta A &= \sqrt{2}\xi\psi, \qquad\qquad \delta F = -i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu \partial_\mu \psi \\ \delta\psi &= -i\sqrt{2}\sigma^\mu \bar{\xi}\partial_\mu A + \sqrt{2}\xi F \end{aligned} \tag{17}$$

- Interestingly enough, the F component transforms like a total derivative and it is a good guidance to construct supersymmetric Lagrangians.

Properties of chiral superfields

- The product of two superfields is another superfield.
- For instance, the F-component of the product of two superfields Φ_1 and Φ_2 is obtained by collecting all the terms in θ^2, and is equal to

$$A_1 F_2 + A_2 F_1 + \psi_1 \psi_2 \tag{18}$$

- For a generic Polynomial function of several fields $P(\Phi_i)$, the result is

$$\left(\partial_{A_i} P(A)\right) F_i + \frac{1}{2}\left(\partial^2_{A_i,A_j} P(A)\right) \psi_i \psi_j \tag{19}$$

- Finally, a single chiral field has dimensionality $[A] = [\Phi] = 1$, $[\psi]=$ $3/2$ and $[F] = 2$. For $P(A)$, $[P(\Phi)]_F = [P(\Phi)] + 1$.

Vector Superfields

- Vector Superfields are generic hermitian fields. The minimal irreducible representations may be obtained by

$$V(x,\theta,\bar{\theta}) = -\left(\theta\sigma^{\mu}\bar{\theta}\right)V_{\mu} + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D \qquad (20)$$

- Vector Superfields contain a regular vector field V_{μ}, its fermionic supersymmetric partner λ and an auxiliary scalar field D.
- The D-component of a vector field transform like a total derivative.
- $[V]_D = [V] + 2$.

Superfield Strength and gauge transformations

- Similarly to $F_{\mu\nu}$ in the regular case, there is a field that contains the field strength. It is a chiral field, derived from V, and it is given by

$$W^{\alpha}(x,\theta,\bar{\theta}=0) = -i\lambda^{\alpha} + (\theta\sigma_{\mu\nu})^{\alpha} F^{\mu\nu} + \theta^{\alpha} D - \theta^2 \left(\bar{\sigma}^{\mu}\mathcal{D}_{\mu}\bar{\lambda}\right)^{\alpha} \quad (21)$$

- Under gauge transformations, superfields transform like

$$\begin{aligned} \Phi &\rightarrow \exp(-ig\Lambda)\Phi, \qquad W_{\alpha} \rightarrow \exp(-ig\Lambda)W_{\alpha}\exp(ig\Lambda) \\ \exp(gV) &\rightarrow \exp(-ig\bar{\Lambda})\exp(gV)\exp(ig\Lambda) \end{aligned} \quad (22)$$

where Λ is a chiral field of dimension 0.

Towards a Supersymmetric Lagrangian

- The aim is to construct a Lagrangian, invariant under supersymmetry and under gauge transformations.
- One should remember, for that purpose, that both the F-component of a chiral field, as well as the D-component of a vector field transform under SUSY as a total derivative.
- One should also remember that, if renormalizability is imposed, then the dimension of all interaction terms in the Lagrangian

$$[\mathcal{L}_{\rm int}] \leq 4 \tag{23}$$

- On the other hand,

$$[\Phi] = 1, \qquad [W_\alpha] = 3/2, \qquad [V] = 0. \tag{24}$$

and one should remember that $[V]_D = [V] + 2$; $[\Phi]_F = [\Phi] + 1$.

Supersymmetric Lagrangian

- Once the above machinery is introduced, the total Lagrangian takes a particular simple form. The total Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\rm SUSY} &= \frac{1}{4g^2}\left(Tr[W^\alpha W_\alpha]_F + h.c.\right) + \sum_i \left(\bar{\Phi}\exp(gV)\Phi\right)_D \\ &+ \left([P(\Phi)]_F + h.c.\right) \end{aligned} \tag{25}$$

where $P(\Phi)$ is the most generic dimension-three, gauge invariant, polynomial function of the chiral fields Φ, and has the general expression

$$P(\Phi) = c_i\Phi_i + \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{\lambda_{ijk}}{3!}\Phi_i\Phi_k\Phi_k \tag{26}$$

- The D-terms of V^a and the F term of Φ_i do not receive any derivative contribution: Auxiliary fields that can be integrated out by equation of motion.

Lagrangian in terms of Component Fields

- The above Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$
\begin{aligned}
\mathcal{L}_{\rm SUSY} &= (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2}\bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \text{h.c.}\right) \\
&- \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left(\frac{i}{2}\bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \text{h.c.}\right) \\
&- \left(\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.\right) \\
&- V(F_i, F_i^*, D^a)
\end{aligned} \tag{27}
$$

- The last term is a potential term that depend only on the auxiliary fields

Scalar Potential

The scalar potential is given by

$$V(F_i, F_i^*, D^a) = \sum_i F_i^* F_i + \frac{1}{2} \sum_a (D^a)^2 \tag{28}$$

where the auxiliary fields may be obtained from their equation of motion, as a function of the scalar components of the chiral fields:

$$F_i^* = -\frac{\partial P(A)}{\partial A_i}, \qquad D^a = -g \sum_i (A_i^* T^a A_i) \tag{29}$$

Observe that the scalar potential is positive definite ! This is not a surprise. From the supersymmetry algebra, one obtains,

$$H = \frac{1}{4} \sum_{\alpha=1}^{2} \left(Q_\alpha^\dagger Q_\alpha + Q_\alpha Q_\alpha^\dagger \right) \tag{30}$$

- If for a physical state the energy is zero, this is the ground state.
- Supersymmetry is broken if the vacuum energy is non-zero !

Couplings

- The Yukawa couplings between scalar and fermion fields,

$$\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j + h.c. \tag{31}$$

are governed by the same couplings as the scalar interactions coming from

$$\left(\frac{\partial P(A)}{\partial A_i}\right)^2 \tag{32}$$

- Similarly, the gaugino-scalar-fermion interactions, coming from

$$-i\sqrt{2}gA_i^* T_a \psi_i \lambda^a + h.c. \tag{33}$$

are governed by the gauge couplings.

- No new couplings ! Same couplings are obtained by replacing particles by their superpartners and changing the spinorial structure.

Trilinear coupling

- A most useful example of the relation between couplings is provided by trilinear (Yukawa) couplings. To avoid complications, let's treat the abelian case:

$$P[\Phi] = h_t H U Q \tag{34}$$

where H is a Higgs superfield.

- Fermion Yukawa:

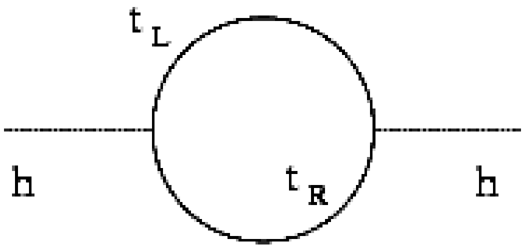
$$h_t H \psi_U \psi_Q + h.c. \qquad\qquad h_t \left(H \bar{\psi}_R \psi_L + h.c. \right) \tag{35}$$

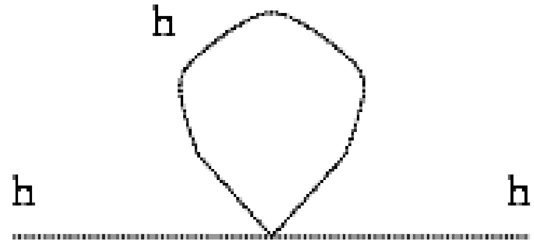
- Scalar Yukawas

$$|h_t|^2 |H|^2 \left(|Q|^2 + |U|^2 \right) \tag{36}$$

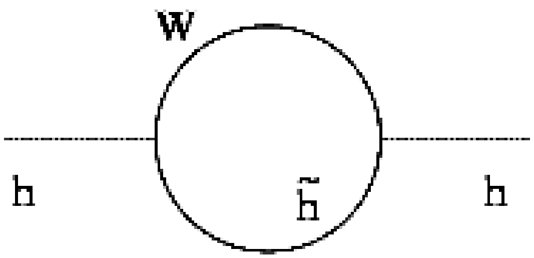
- As anticipated, same couplings of the Higgs field to fermions and to scalar fields.

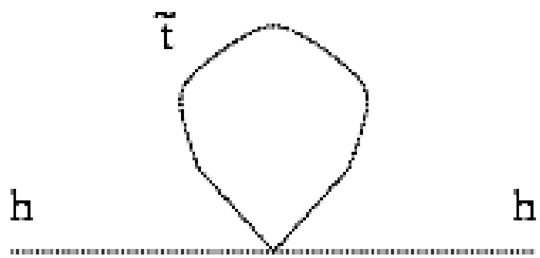
Quantum corrections induce quadratic divergent result





$$\delta m_H^2 \approx (-1)^{2 S_i} \frac{n_i \, g_i^2}{16\pi^2} \Lambda^2$$





Cancelled if particles of different spin with same couplings are present. This happens naturally within a supersymmetric extension of the Standard Model

Properties of supersymmetric theories

- To each complex scalar A_i (two degrees of freedom) there is a Weyl fermion ψ_i (sfermion, two degrees of freedom)
- To each gague boson V_μ^a, there is a gauge fermion (gaugino) λ^a.
- The mass eigenvalues of fermions and bosons are the same !
- Theory has only logarithmic divergences in the ultraviolet associated with wave-function and gauge-coupling constant renormalizations.
- Couplings in superpotential $P[\Phi]$ have no counterterms associated with them.
- The equality of fermion and boson couplings are essential for the cancellation of all quadratic divergences, at all oders in perturbation theory.

Supersymmetric Extension of the Standard Model

- Apart from the superpotential $P[\Phi]$, all other properties are directly determined by the gauge interactions of the theory.
- To construct the superpotential, one should remember that chiral fields contain only left-handed fields, and right-handed fields should be represented by their charge conjugates.
- SM right-handed fields are singlet under $SU(2)$. Their complex conjugates have opposite hypercharge to the standard one.
- There is one chiral superfield for each chiral fermion of the Standard Model.
- In total, there are 15 chiral fields per generation, including the six left-handed quarks, the six right-handed quarks, the two left-handed leptons and the right-handed charged leptons.

Minimal Supersymmetric Standard Model

SM particle	SUSY partner	G_{SM}
(S = 1/2)	(S = 0)	
$Q = (t, b)_L$	$(\tilde{t}, \tilde{b})_L$	(3,2,1/6)
$L = (\nu, l)_L$	$(\tilde{\nu}, \tilde{l})_L$	(1,2,-1/2)
$U = \left(t^C\right)_L$	$\tilde{t}^*_R$	($\bar{3}$,1,-2/3)
$D = \left(b^C\right)_L$	$\tilde{b}^*_R$	($\bar{3}$,1,1/3)
$E = \left(l^C\right)_L$	$\tilde{l}^*_R$	(1,1,1)
(S = 1)	(S = 1/2)	
B_μ	$\tilde{B}$	(1,1,0)
W_μ	$\tilde{W}$	(1,3,0)
g_μ	$\tilde{g}$	(8,1,0)

The Higgs problem

- Problem: What to do with the Higgs field ?
- In the Standard Model masses for the up and down (and lepton) fields are obtained with Yukawa couplings involving H and $H^\dagger$ respectively.
- Impossible to recover this from the Yukawas derived from $P[\Phi]$, since no dependence on $\bar{\Phi}$ is admitted.
- Another problem: In the SM all anomalies cancel,

$$\begin{aligned} \sum_{quarks} Y_i &= 0; \qquad \sum_{left} Y_i = 0; \\ \sum_i Y_i^3 &= 0; \qquad \sum_i Y_i = 0 \end{aligned} \tag{37}$$

- In all these sums, whenever a right-handed field appear, its charge conjugate is considered.
- A Higgsino doublet spoils anomaly cancellation !

Solution to the problem

- Solution: Add a second doublet with opposite hypercharge.
- Anomalies cancel automatically, since the fermions of the second Higgs superfield act as the vector mirrors of the ones of the first one.
- Use the second Higgs doublet to construct masses for the down quarks and leptons.

$$P[\Phi] = h_u Q U H_2 + h_d Q D H_1 + h_l L E H_1 \tag{38}$$

- Once these two Higgs doublets are introduced, a mass term may be written

$$\delta P[\Phi] = \mu H_1 H_2 \tag{39}$$

- μ is only renormalized by wave functions of H_1 and H_2.

Higgs Fields

- Two Higgs fields with opposite hypercharge.

(S = 0)	(S = 1/2)	
H_1	$\tilde{H}_1$	(1,2,-1/2)
H_2	$\tilde{H}_2$	(1,2,1/2)

- It is important to observe that the quantum numbers of H_1 are exactly the same as the ones of the lepton superfield L.

- This means that one can extend the superpotential $P[\Phi]$ to contain terms that replace H_1 by L.

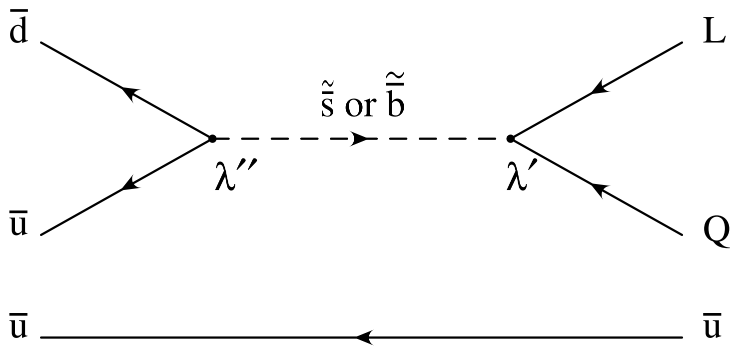
Baryon and Lepton Number Violation

- General superpotential contains, apart from the Yukawa couplings of the Higgs to lepton and quark fields, new couplings:

$$P[\Phi]_{\rm new} = \lambda' \, LQD + \lambda \, LLE + \lambda'' \, UDD \tag{40}$$

- Assigning every lepton chiral (antichiral) superfield lepton number 1 (-1) and every quark chiral (antichiral) superfield baryon number 1/3 (-1/3) one obtains :
  - Interactions in $P[\Phi]$ conserve baryon and lepton number.
  - Interactions in $P[\Phi]_{\rm new}$ violate either baryon or lepton number.
- One of the most dangerous consequences of these new interaction is to induce proton decay, unless couplings are very small and/or sfermions are very heavy.

Proton Decay



- Both lepton and baryon number violating couplings involved.
- Proton: Lightest baryon. Lighter fermions: Leptons

R-Parity

- A solution to the proton decay problem is to introduce a discrete symmetry, called R-Parity. In the language of component fields,

$$R_P = (-1)^{3B+2S+L} \tag{41}$$

- All Standard Model particles have $R_P = 1$.
- All supersymmetric partners have $R_P = -1$.
- All interactions with odd number of supersymmetric particles, like the Yukawa couplings induced by $P[\Phi]_{\rm new}$ are forbidden.
- Supersymmetric particles should be produced in pairs.
- The lightest supersymmetric particle is stable.
- Good dark matter candidate. Missing energy at colliders.

Supersymmetry Breaking

- No supersymmetric particle have been seen: Supersymmetry is broken in nature
- Unless a specific mechanism of supersymmetry breaking is known, no information on the spectrum can be obtained.
- Cancellation of quadratic divergences:
  - Relies on equality of couplings and not on equality of the masses of particle and superpartners.
- Soft Supersymmetry Breaking: Give different masses to SM particles and their superpartners but preserves the structure of couplings of the theory.

Supersymmetry Breaking Parameters

Standard Model quark, lepton and gauge boson masses are protected by chiral and gauge symmetries.
Supersymmetric partners are not protected.
Explanation of absence of supersymmetric particles in ordinary experience/ high-energy physics colliders: Supersymmetric particles can acquire gauge invariant masses, as the one of the SM-Higgs.

Different kind of parameters:

Squark and slepton masses $m^2_{\tilde{q}}$, $m^2_{\tilde{l}}$
Gaugino (Majorana) masses M_i, $i = 1\text{-}3$
Trilinear scalar masses $(\tilde{f}^*_L \tilde{f}_R H_i)$ A_f, $-\mu^*$
Higgsino Mass μ Higgs Mass Parameters $|\mu|^2 + m^2_{H_i}$

Gaugino/Higgsino Mixing

- Just like the gauge boson mixes with the Goldstone modes of the theory after spontaneous breakdown of the gauge symmetry, gauginos mix with the Higgsinos.
- Mixing comes from the interaction $\sqrt{2} g A_i^* T_a \psi_i \lambda^a$, when one takes $A_i \equiv H_i$, and $\lambda^a \equiv \tilde{W}^a$, $\tilde{B}$.
- Charged Winos, $\tilde{W}_1 \pm i\tilde{W}_2$, mix with the charged components of the Higgsinos $\tilde{H}_{1,2}$. The mass eigenstates are called charginos $\tilde{\chi}^{\pm}$.
- Neutral Winos and Binos, $\tilde{B}$, $\tilde{W}_3$ mix with the neutral components of the Higgsinos. The mass eigenstates are called neutralinos, $\tilde{\chi}^0$.
- Charginos form two Dirac massive fields. Neutralinos give four massive Majorana states.

Unification of Couplings

- The value of gauge couplings evolve with scale according to the corresponding RG equations:

$$\frac{1}{\alpha_i(Q)} = \frac{b_i}{2\pi} \ln\left(\frac{Q}{M_Z}\right) + \frac{1}{\alpha_i(M_Z)} \tag{42}$$

- Unification of gauge couplings would occur if there is a given scale at which couplings converge.

$$\frac{1}{\alpha_3(M_Z)} = \frac{b_3 - b_1}{b_2 - b_1}\frac{1}{\alpha_1(M_Z)} - \frac{b_3 - b_2}{b_2 - b_1}\frac{1}{\alpha_2(M_Z)} \tag{43}$$

- This leads to a relation between $\alpha_3(M_Z)$ and $\sin^2\theta_W(M_Z) = \alpha_1^{SM} / \left(\alpha_1^{SM} + \alpha_2^{SM}\right)$.

Rules to compute the beta-functions

- The one-loop beta-functions for the $U(1)$ and $SU(N)$ gauge couplings are given by,

$$\begin{aligned} \frac{5}{3} b_1 &= -\frac{1}{6}\sum_f y_f^2 - \frac{1}{12}\sum_s y_s^2 \\ b_N &= \frac{11\,N}{3} - \frac{n_f}{3} - \frac{n_S}{6} - \frac{2N}{3} n_A \end{aligned} \tag{44}$$

- In the above, $y_{f,s}$ are the hypercharges of the charged chiral fermion and scalar fields, $n_{f,s}$ are the number of fermions and scalars in the fundamental representation of $SU(N)$, n_A are fermions in the adjoint representation and the factor 5/3 is just a normalization factor, so that over one generation

$$Tr\left[T^3 T^3\right] = \frac{3}{5} Tr\left[\left(\frac{y_f}{2}\right)^2\right] \tag{45}$$

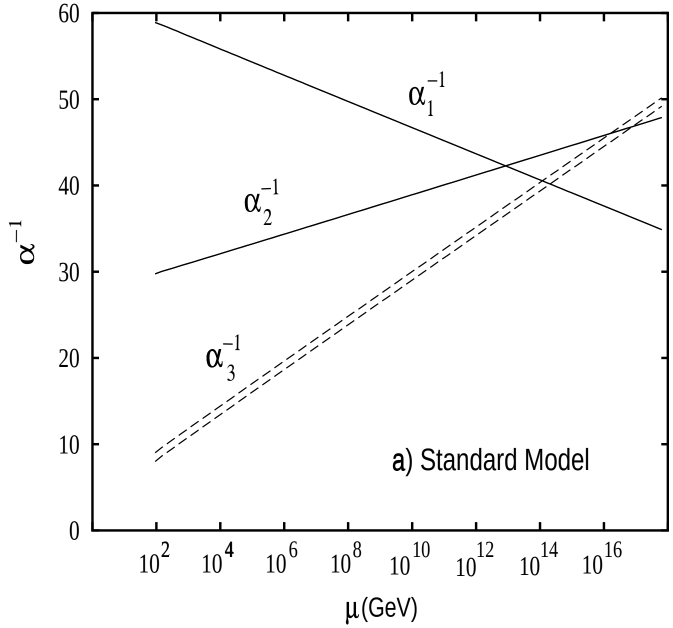
- The coupling g_1 is not asymptotically free, but becomes strong at scales far above M_{Pl}, where the effective theory description breaks down anyway.
- A full generation contributes to the same amount to all β-functions. It does not affect the unification conditions. ($\beta_{\rm gen}^{SM} = 4/3$; $\beta_{\rm gen}^{\rm SUSY} = 2$.)
- Only incomplete $SU(5)$ representations affect one-loop unification.

$$\begin{aligned} b_1^{SM} &= -41/10, \quad b_2^{SM} = 19/6, \quad b_3^{SM} = 7. \\ b_1^{\rm SUSY} &= -33/5, \quad b_2^{\rm SUSY} = -1, \quad b_3^{\rm SUSY} = 3. \end{aligned} \tag{46}$$

$$M_G = M_Z \exp\left[\left(\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)}\right)\frac{2\pi}{b_2 - b_1}\right] \tag{47}$$

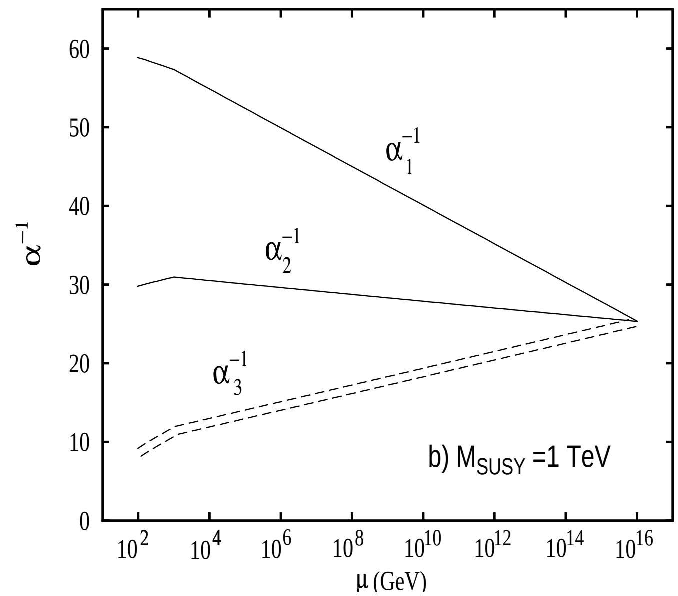
SM:

Couplings tend to converge at high energies, but unification is quantitatively ruled out.



MSSM:

Unification at $\alpha_{GUT} \simeq 0.04$ and $M_{GUT} \simeq 10^{16}$ GeV.



Experimentally, $\alpha_3(M_Z) \simeq 0.118 \pm 0.004$

Bardeen, Carena, Pokorski & C.W.

in the MSSM: $\alpha_3(M_Z) = 0.127 - 4(\sin^2\theta_W - 0.2315) \pm 0.008$

Remarkable agreement between Theory and Experiment!!

Minimal Supergravity Model

All scalars acquire a common mass m_0^2 at the Grand Unification scale
All gauginos acquire a common mass $M_{1/2}$ at the GUT scale
Masses evolve differently under R.G.E. At low energies,

Squark Masses: $m_0^2 + 6\ M_{1/2}^2$
Left-Slepton Masses $m_0^2 + 0.5\ M_{1/2}^2$
Right-Slepton Masses $m_0^2 + 0.15\ M_{1/2}^2$

Wino Mass $M_2 = 0.8\ M_{1/2}$.
Gluino Mass $M_3 = \frac{\alpha_3}{\alpha_2} M_2$
Bino Mass $M_1 = \frac{\alpha_1}{\alpha_2} M_2$

Lightest SUSY particle tends to be a Bino.

If SUSY exists, many of its most important motivations demand some SUSY particles at the TeV range or below

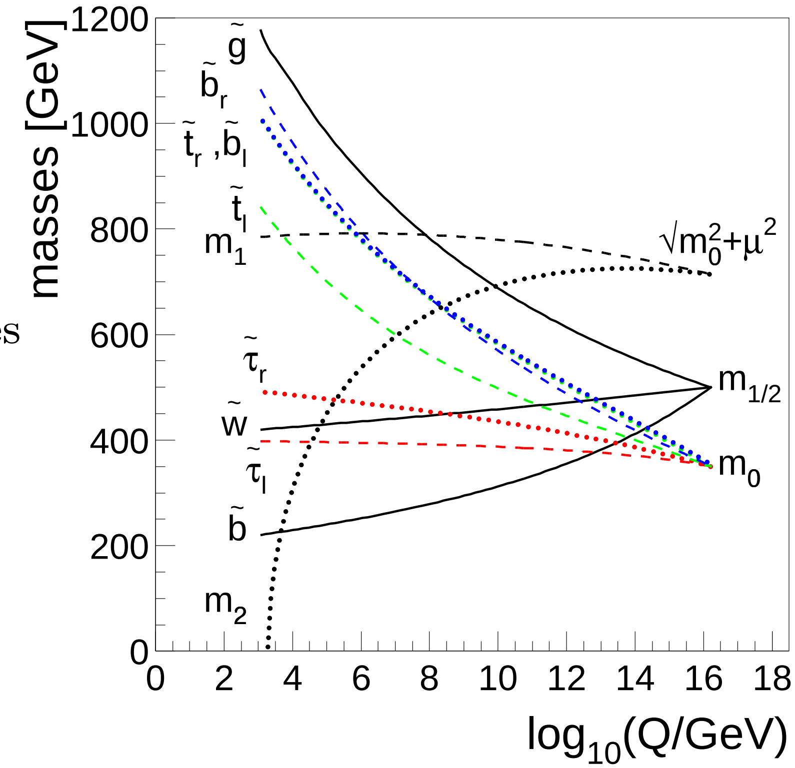
$\star$ Solve hierarchy/naturalness problem by having $\Delta m^2 \simeq \mathcal{O}(v^2)$

$\boxed{\text{SUSY breaking scale must be at or below 1 TeV if SUSY is associated with EWSB scale !}}$

$\star$ EWSB is radiatively generated

In the evolution of masses from high energy scales
$\longrightarrow$ a negative Higgs mass parameter is induced via radiative corrections

$\Longrightarrow$ *important top quark effects!*



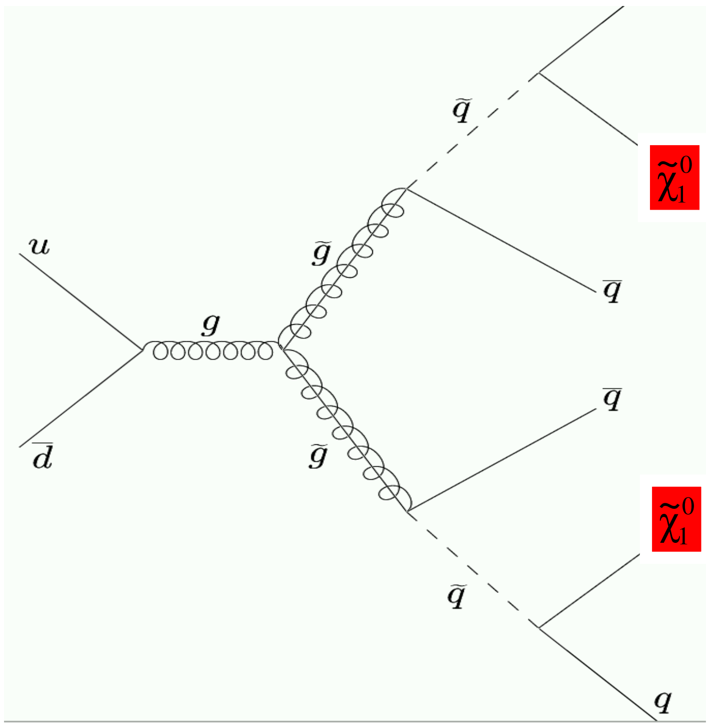
Supersymmetry at colliders

Gluino production and decay: Missing Energy Signature

Supersymmetric Particles tend to be heavier if they carry color charges.

Particles with large Yukawas tend to be lighter.

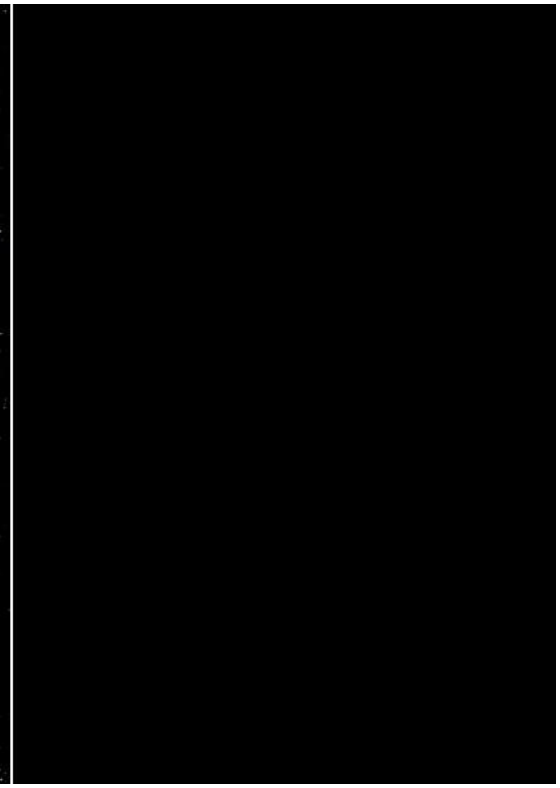
Charge-less particles tend to be the lightest ones.



- **Lightest supersymmetric particle = Excellent**
Cold dark matter candidate.

What is the Dark Matter ?

Luminous Matter

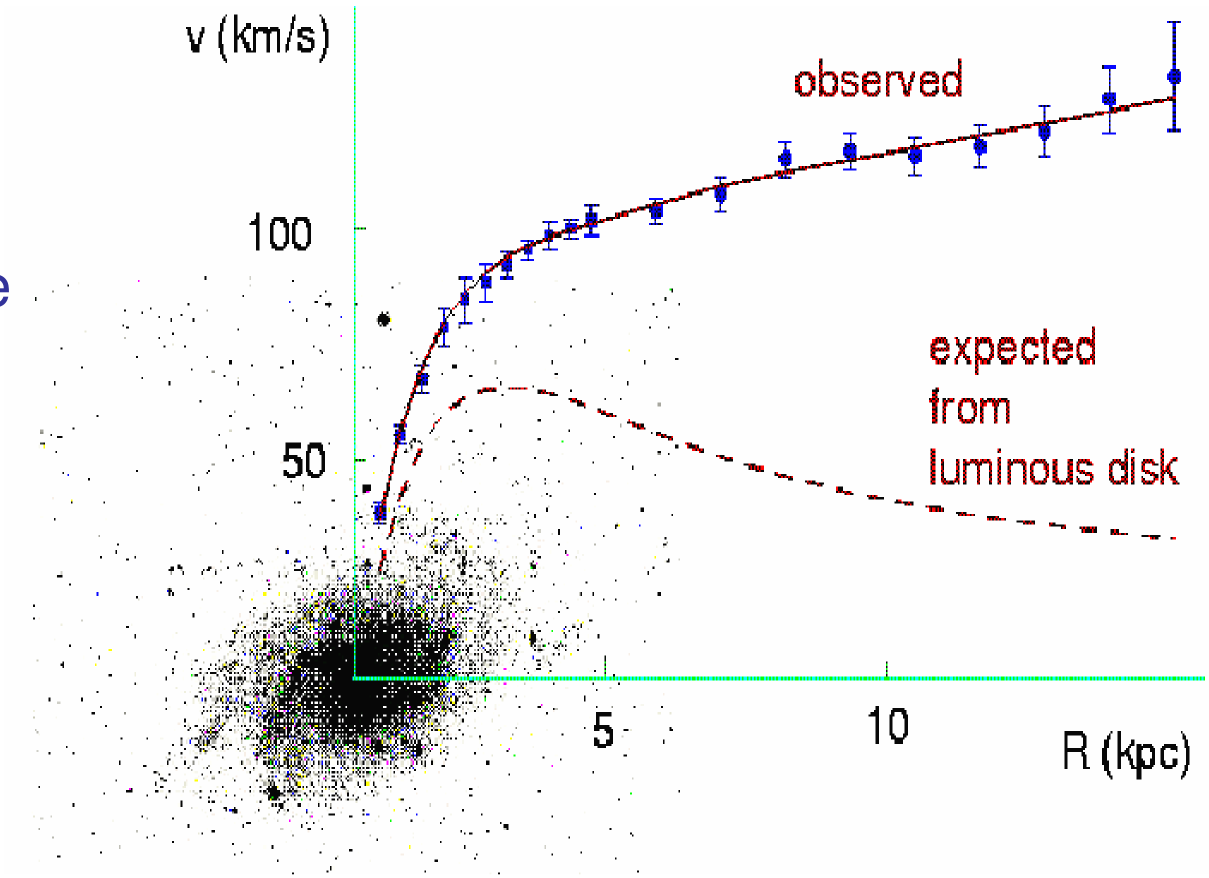
Luminous Matter Dark Matter

Evidence for Dark Matter:

Rotation velocity of stars far from galactic center . Gravity prediction:

$$\frac{v^2}{r} = G_N \frac{M(r)}{r^2} \quad \Rightarrow \quad v^2 \propto \frac{1}{r}$$

Strong evidence for additional, non-visible source of matter



Cosmic Microwave Background WMAP

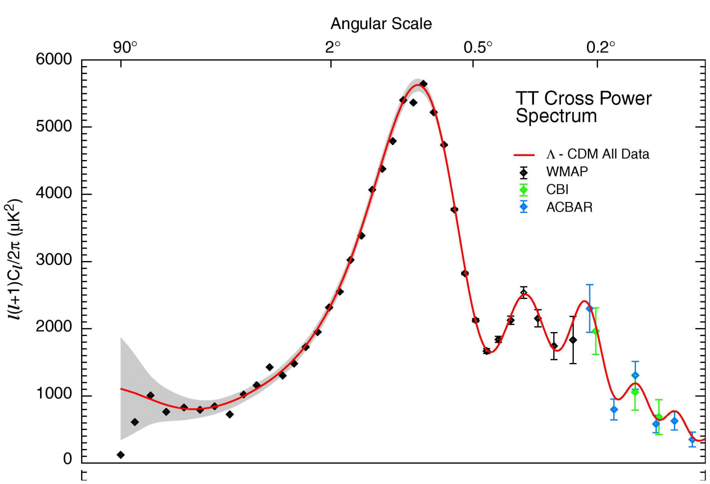
$h = 0.71 \pm 0.04$

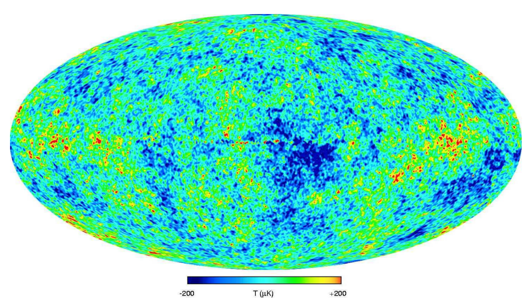
$\Omega_M h^2 = 0.135 \pm 0.009$

$\Omega_B h^2 = 0.0224 \pm 0.0009$

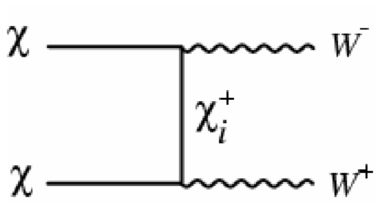
$\Omega_{tot} = 1.02 \pm 0.02$

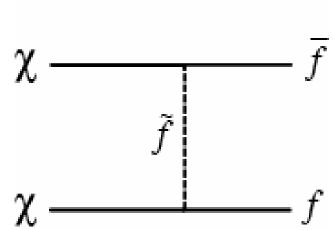
$$\Omega_X = \frac{\rho_X}{\rho_C}$$

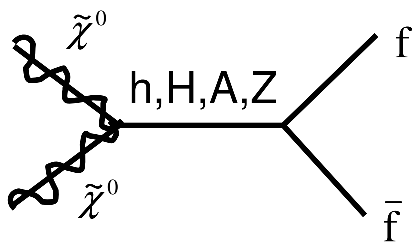


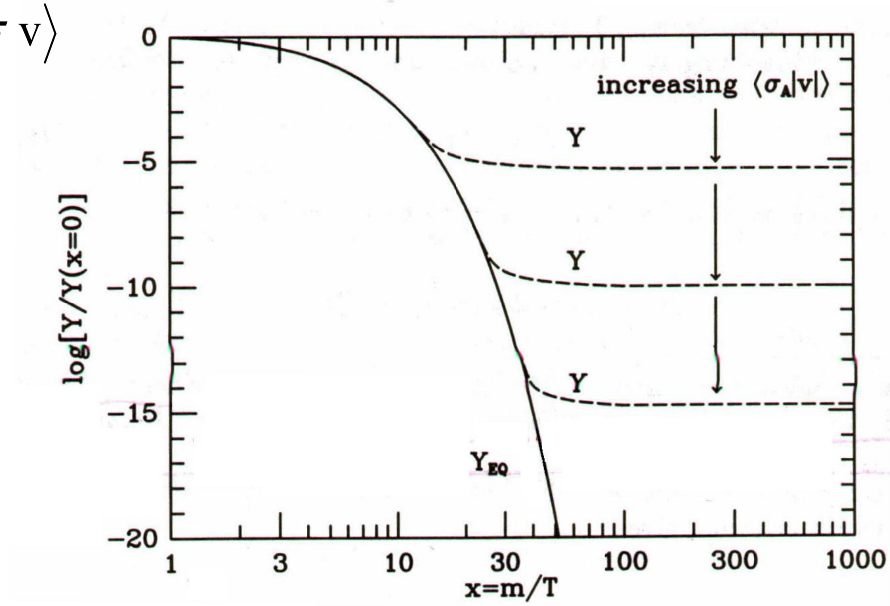


Relic density is inversely proportional to the thermally averaged $\tilde{\chi}^0 \tilde{\chi}^0$ annihilation cross section $\langle \sigma v \rangle$

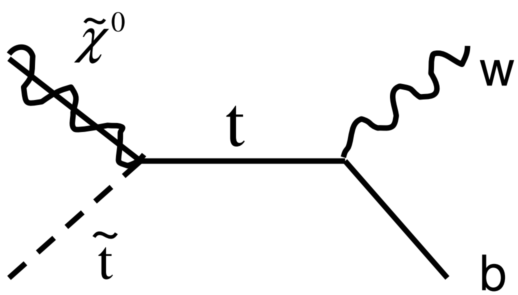


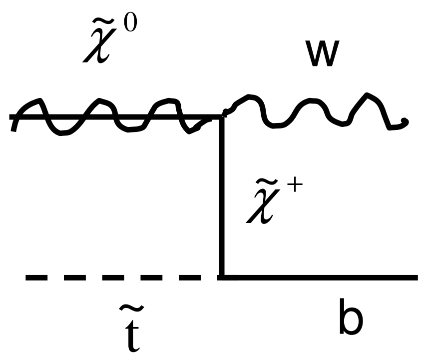






If any other SUSY particle has mass close to the neutralino LSP, it may substantially affect the relic density via co-annihilation





If stops NLSP

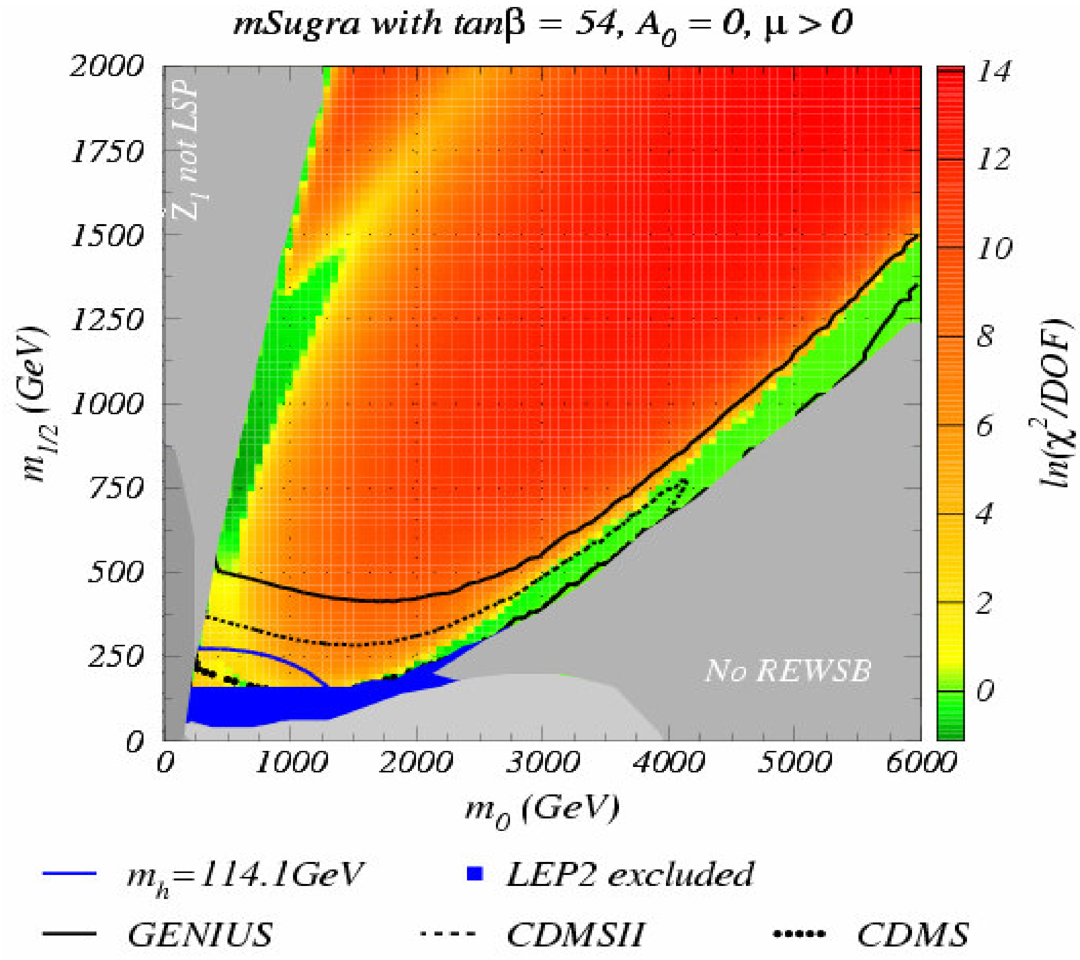
neutralino-stop

co-annihilation

Relevant cross sections and masses

- Relic density is inversely proportional to the annihilation cross section.
- What are the cross sections necessary to get a reasonable relic density ?
- It can be shown that, neutral, stable, weakly interacting particles, with masses of the order of the weak scale, lead to the proper relic density !
- This provides one of the strongest motivations for new physics at the weak scale ! It is based on FACTS and not on OPINIONS.
- Getting an acceptable relic density imposes a constraint on models of new physics. In SUSY theories, DM canidate is lightest neutralino and annihilation cross section depends on spectrum of SUSY particles !

Dark Matter in Minimal Supergravity Models



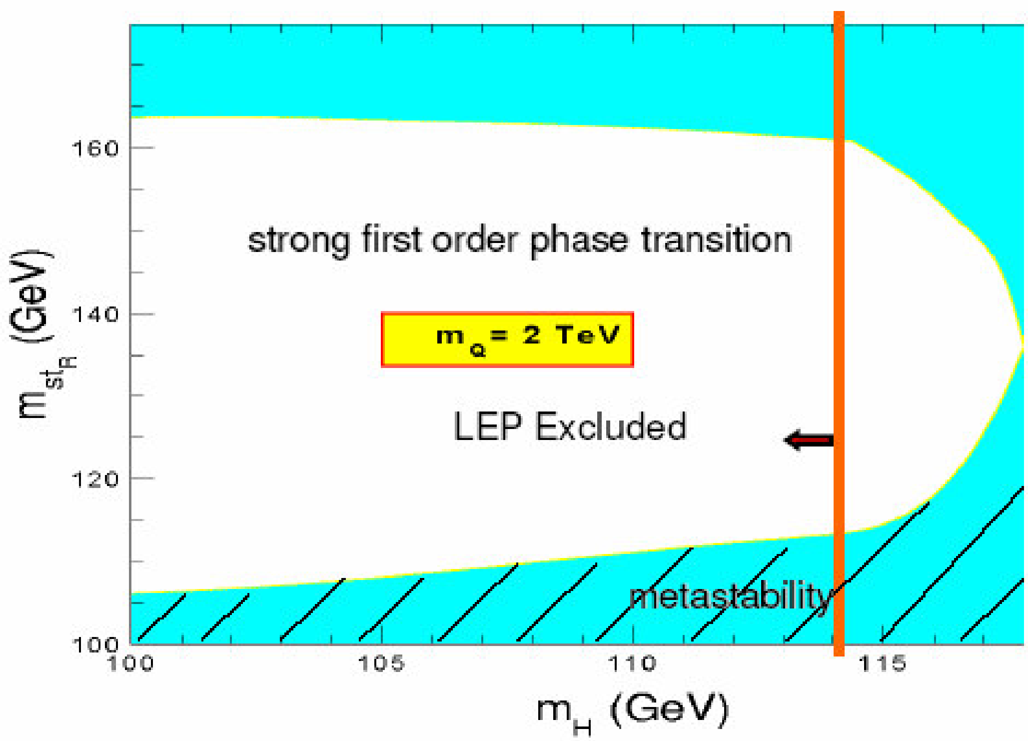
⋆ Provides a solution to the Matter–Anti-matter Asymmetry

$$\eta = \frac{n_B}{n_\gamma} = 2.6810^{-8}\Omega_B h^2 \simeq 6 \times 10^{-10}$$

SUSY opens the window for Baryogenesis at the electroweak scale (ruled out in the SM)

- Higgs associated with mass below 120 GeV and SM-like properties

M.Carena, M.Quiros, M.Seco, C.E.M.Wagner



- Also requires:
  - Gaugino and Higgsino masses on the order of the electroweak scale
  - CP-odd Higgs masses < 500 GeV
  - New CP violating phases which are constrained by EDM's.
- other stop heavy, $m_{\tilde{t}_L} \simeq 1$ TeV, to respect limit $m_h \geq 114$ GeV.

Electroweak Baryogenesis: *only testable model of baryon asymmetry generation*

Light Stops: Motivation

- In low energy supersymmetry models, light stops are induced as a consequence of large mixing or large negative radiative effects.
- They are required for the realization of the mechanism of electroweak baryogenesis in the MSSM
- Signatures of a light stop at the Tevatron collider depend strongly on the chargino and neutralino spectrum as well as on the nature of supersymmetry breaking

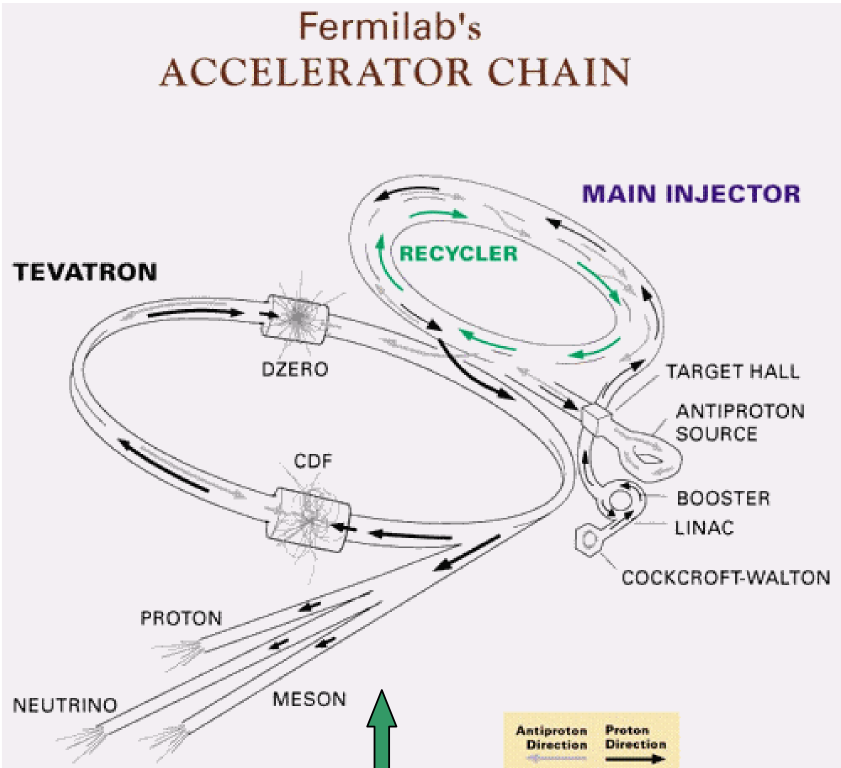
Stop mass matrix

- Radiative corrections affect mostly the hierarchy of diagonal masses in stop mass matrix

$$M^2 = \begin{bmatrix} \mathrm{m}_Q^2 + \mathrm{m}_t^2 & \mathrm{m}_t\,(\mathrm{A}_t - \mu\ \mathrm{cotan}\beta) \\ \mathrm{m}_t\,(\mathrm{A}_t - \mu\ \mathrm{cotan}\beta) & \mathrm{m}_U^2 + \mathrm{m}_t^2 \end{bmatrix}$$

- Large stop mixing induced by off-diagonal elements in stop mass matrix

Searching for New Physics

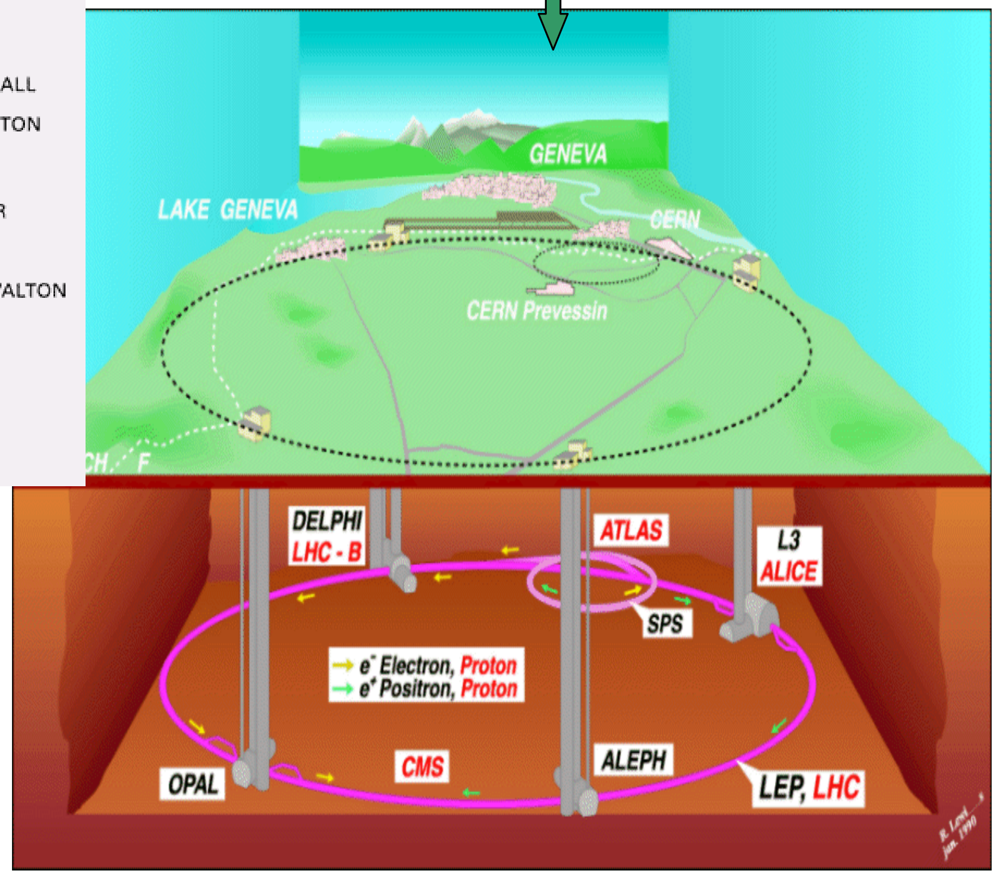


The Tevatron at Fermilab

$p\bar{p}$ at $\sqrt{s} = 2\,\mathrm{TeV}$ (2001 - 2009)

Large Hadron Collider (LHC) at CERN

pp at $\sqrt{s} = 14\,\mathrm{TeV}$ (2007/8 - 2015/20)



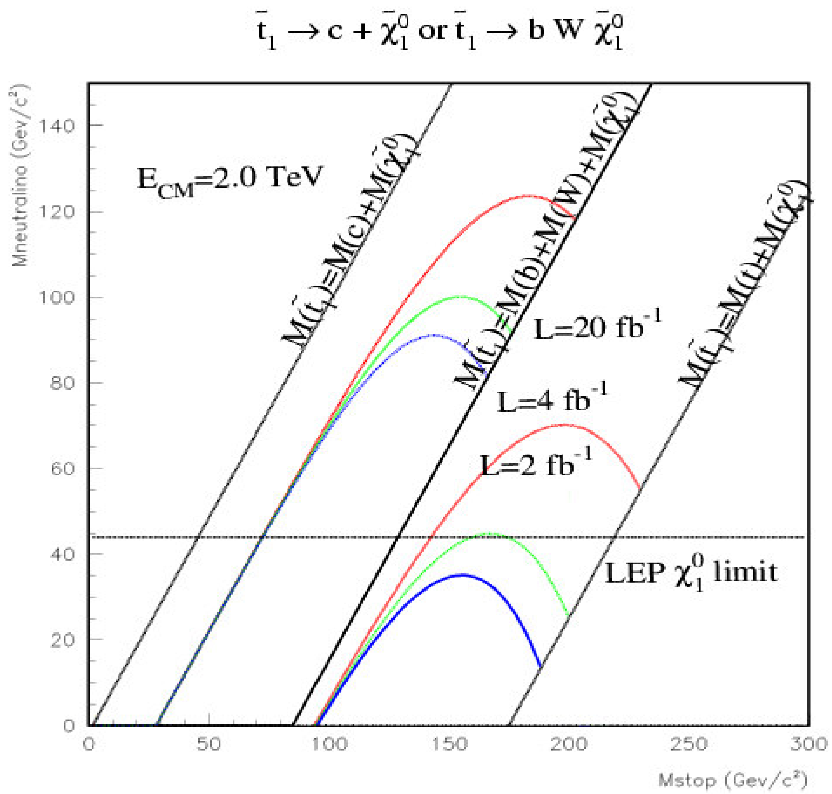
Tevatron Stop Reach when two body decay channel is dominant

Main signature:

2 or more jets plus missing energy

2 or more Jets with $E_T > 15\,\text{GeV}$

Missing $E_T > 35\,\text{GeV}$



Demina, Lykken, Matchev,Nomerotsky '99

Stop-Neutralino Mass Difference: Information from the Cosmos

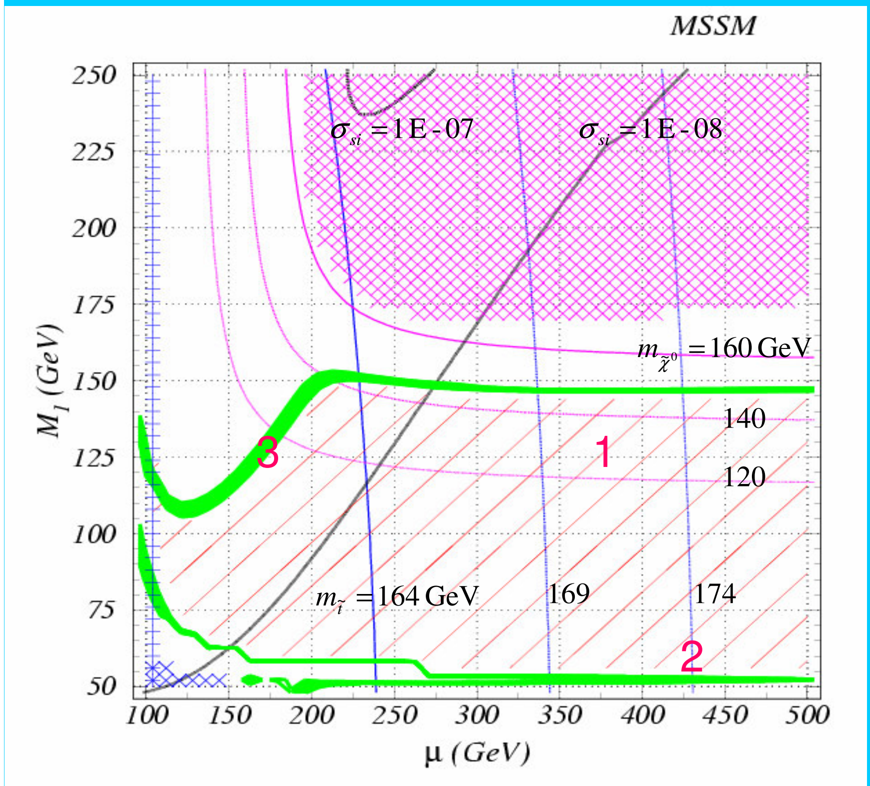
- If the neutralino provides the observed dark matter relic density, then it must be stable and lighter than the light stop.

- Relic density depends on size of neutralino annihilation cross section.

  If only stops, charginos and neutralinos are light, there are three main annihilation channels:

1. Coannihilation of neutralino with light stop. Small mass difference.
2. s-channel annihilation via light CP-even Higgs boson
3. s-channel annihilation via heavy CP-even Higgs boson and CP-odd Higgs boson

Dark Matter and Electroweak Baryogenesis



Three interesting regions with neutralino relic density compatible with WMAP obs.

$0.095 < \Omega_{\mathrm{CDM}} h^2 < 0.129$ (green areas)

1. Mass difference about 20-30 GeV neutralino-stop co-annihilation
2. s-channel neutralino annihilation via light Higgs boson

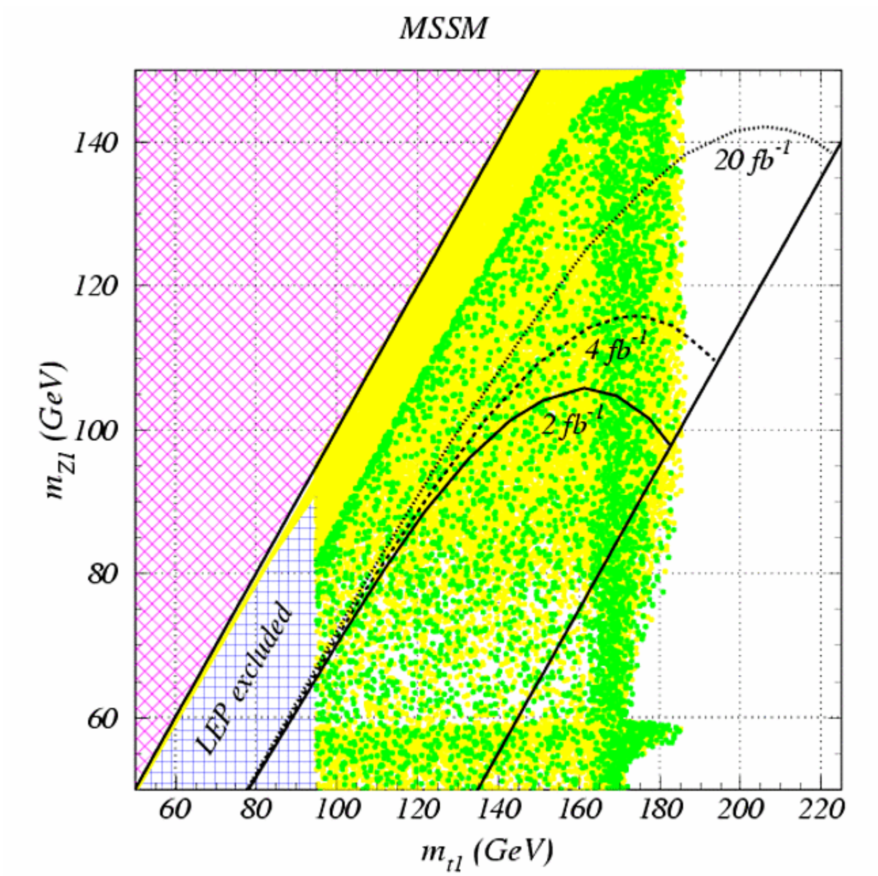
$$m_{\tilde{\chi}^0} \approx m_h/2$$

3. Annihilation via Z boson exchange

M. Carena, C. Balazs and C.W. 04

Tevatron Run II reach for stops probes
Dark Matter and Baryogenesis at the Electroweak scale!

Dots show scan over SUSY space with neutralino relic density compatible with WMAP observations $0.095 \leq \Omega_{CDM} h^2 \leq 0.129$ and with electroweak Baryogenesis



Lines show reach at the Tevatron for different total luminosities for dominant decay mode $\tilde{t}_1 \to c\,\tilde{\chi}$.

If stop-neutralino mass difference is below 30 GeV:

$\longrightarrow$ trigger on $\not\!\!E_T$ crucial

$\longrightarrow$ co-annihilation region difficult at Tevatron or any hadron collider

Balazs, Carena & C.W.

A definite test of this scenario at the LC

Large Hadron Collider (LHC)

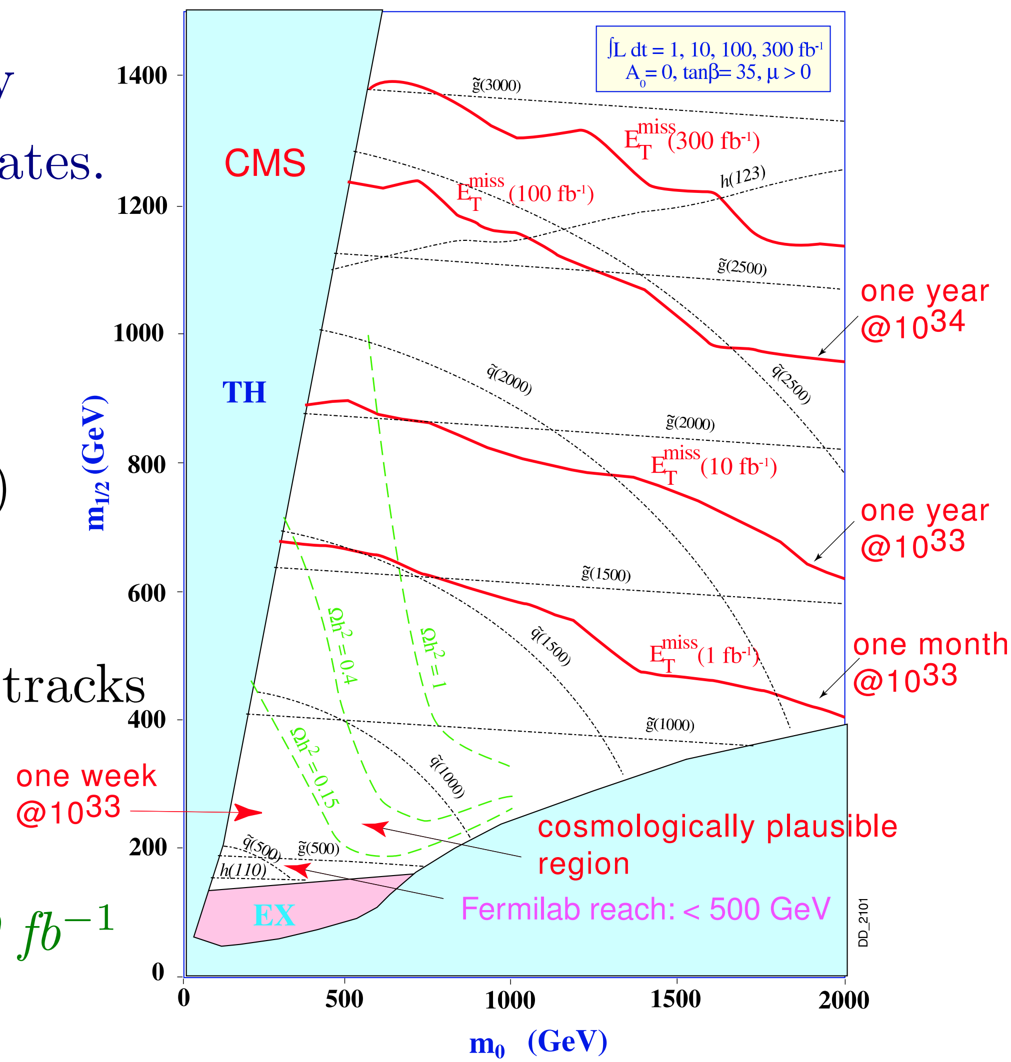
CERN, Geneva, Switzerland

SUSY

LHC: SUSY particles, especially strongly interacting ones, are produced at large rates.

- most likely types of signatures:
  - ‘mSUGRA’ type – high E_T jets and $\not\!\!E_T$ (maybe leptons)
  - ‘GMSB’ type – hard photons & $\not\!\!E_T$; heavily ionizing tracks

reach: $M_{\tilde q}$ and $M_{\tilde g}$ up to $\sim$2 TeV with 10 fb^{-1}



If low-energy SUSY is there, we expect to see some of its signature(s) by the end of this decade.

Linear Collider (LC)

Location Unknown

Supersymmetry at a LC

(a) Measurements of SUSY particles masses

$\Longrightarrow$ sleptons, charginos, neutralinos
with an accuracy of 1% or less

If any visible SUSY particle produced,

$\longrightarrow \delta M_{\tilde{\chi}_1^0} \sim 1\% \Longrightarrow$ important for LHC meas.

(b) Measurement of SUSY parameters

- $\tilde{\chi}_i^{\pm}$, $\tilde{\chi}_i^0$ production & decay
  $\longrightarrow$ param. of mixing mass matrix to 1%
  $\longrightarrow$ determine composition in terms of SUSY partners of γ, Z, W, H
- slepton and squark mixing angles from cross sections with polarized beams

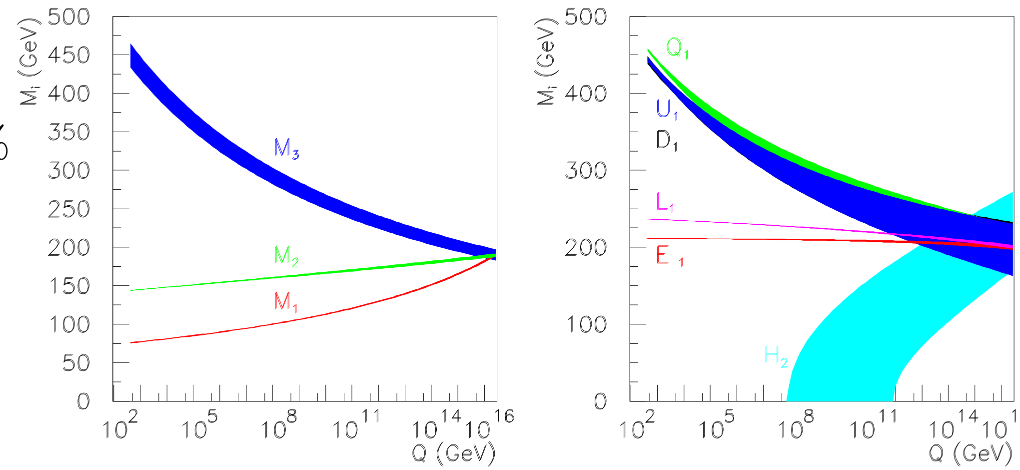
(c) Spin of SUSY particles:

Simplicity of production reactions allows spin determination from angular distributions

Precise SUSY measurements at LC
+ LHC input on gluinos/squarks

$\Longrightarrow$ allow for precise extrapolation of SUSY parameters at high energies

Test type of SUSY theory at high energies.



TeV scale Physics can provide our first glimpse of the Planck scale regime!!

Linear Collider and the Cosmos

- Weak-interacting particles with weak-scale masses naturally provide $\Omega_{\rm DM}$.

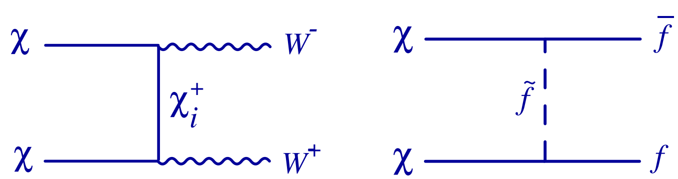
$\Rightarrow$ A coincidence or DM provides fundamental motivation for new particles at EW scale.

$\star$ Understanding what DM is made of demands Collider & Astrophysical/Cosmological input.

- If the LSP is found to be a stable neutralino

$\rightarrow$ accurate meas. of $\tilde{\chi}_1^0$ mass & composition

$\Longrightarrow$ Comput. of $\tilde{\chi}_1^0\tilde{\chi}_1^0$ annih. cross section



$\Longrightarrow$ determined thermal relic density assuming standard evolution of the universe

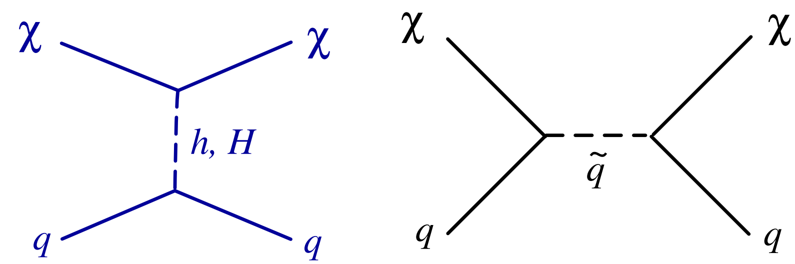
- comparing this result with $\Omega_{\rm DM}$ from Astrophysical/Cosmological input

$\Longrightarrow$ new insights into history of our universe

- Dark Matter Detection:

• Direct: depends on $\tilde{\chi}_1^0\, N$ scattering

$\longrightarrow$ input from both collider and conventional DM experiments



• Indirect: through annih. decay products ($\tilde{\chi}_1^0\,\tilde{\chi}_1^0 \rightarrow \gamma$'s in galactic center, e^+'s in halo, anti-protons, ν's in centers of Earth & Sun)

$\Longrightarrow$ $\tilde{\chi}_1^0\, N$ scattering not necessarily in one-to-one correspondence with DM detection rates

$\Longrightarrow$ LC will provide important info about DM halo densities and velocity distributions.

Alternative SUSY Breaking scenario: Gauge Mediation

Supersymmetry Breaking is transmitted via gauge interactions

Particle Masses depend on the strength of their gauge interactions.

Spectrum of supersymmetric particles very similar to the case of the Minimal Supergravity Model for large values of $M_{1/2}$:

Sparticle Masses

$\frac{M_i}{M_j} = \frac{\alpha_i}{\alpha_j}$

$\frac{m_{\tilde{q}}}{m_{\tilde{l}}} \simeq \frac{\alpha_3}{\alpha_i}$

Lightest SM–Sparticle tends to be a Bino or a Higgsino

Gauge-Mediated Low-energy SUSY Breaking Scenarios

- Special feature $\longrightarrow$ LSP: light (gravitino) Goldstino:

$$m_{\tilde{G}} \sim \frac{F}{M_{Pl}} \simeq 10^{-6} - 10^{-9} \mathrm{GeV}$$

If R-parity conserved, heavy particles cascade to lighter ones and

NLSP $\longrightarrow$ SM partner + $\tilde{G}$

- Signatures:

decay length $\quad L \sim 10^{-2} \mathrm{cm} \left(\frac{m_{\tilde{G}}}{10^{-9} \mathrm{GeV}} \right)^2 \times \left(\frac{100 \mathrm{GeV}}{M_{\mathrm{NLSP}}} \right)^5$

$\star$ NLSP can have prompt decays:

Signature of SUSY pair: 2 hard photons, (H's, Z's) + $\not{E}_T$ from $\tilde{G}$

$\star$ macroscopic decay length but within the detector:

displaced photons; high ionizing track with a kink to a minimum ionizing track

(smoking gun of low energy SUSY)

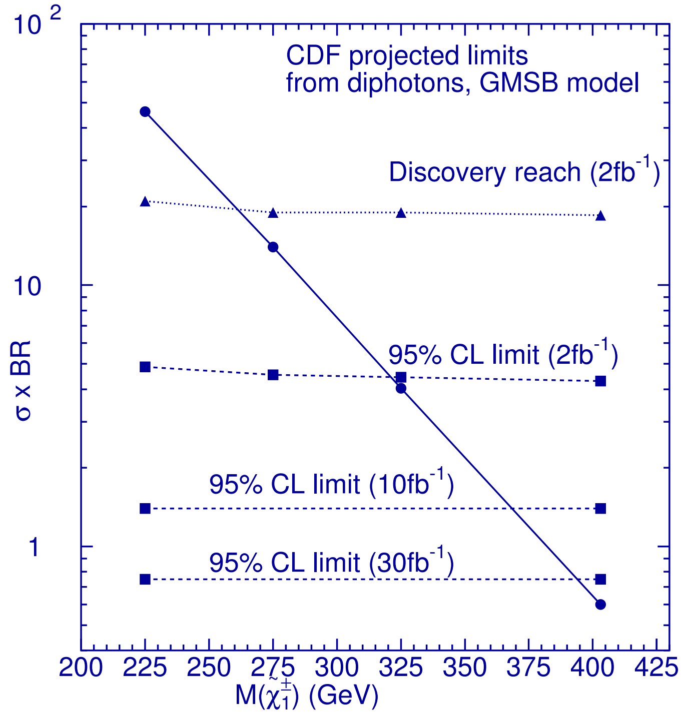
$\star$ decay well outside the detector: $\not{E}_T$ like SUGRA

Gauge-Mediated Tevatron Reach

- Bino-like NLSP: $\tilde{\chi}_1^0 \to \gamma \tilde{G}$

  Signal: $\gamma\gamma X \not{E}_T$

  X = ℓ's and/or jets



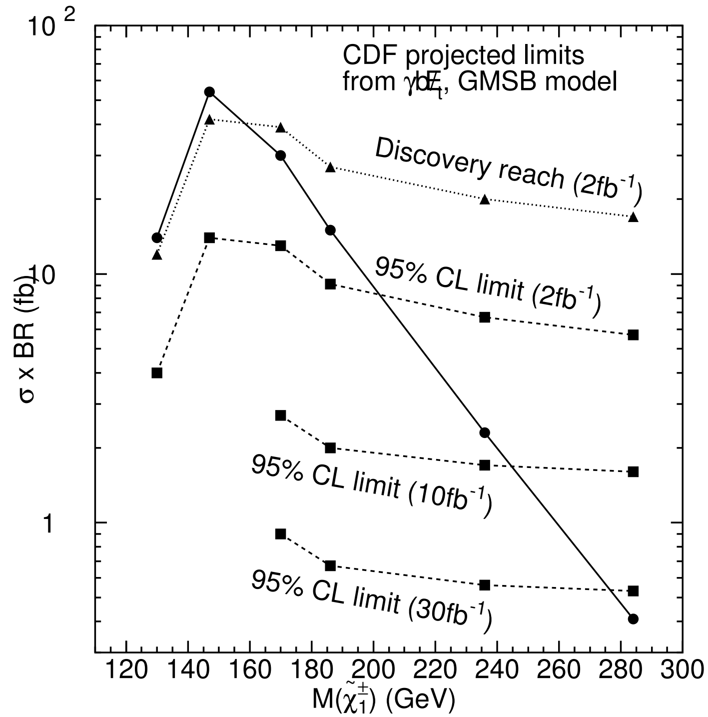
$M_{\tilde{\chi}^\pm} \sim 325$ GeV *(exclusion)* &

~ 260 GeV *(discovery)*

- Higgsino-like NLSP: $\tilde{\chi}_1^0 \to (h, Z, \gamma)\tilde{G}$

  Signal: γ b $\not{E}_T X$

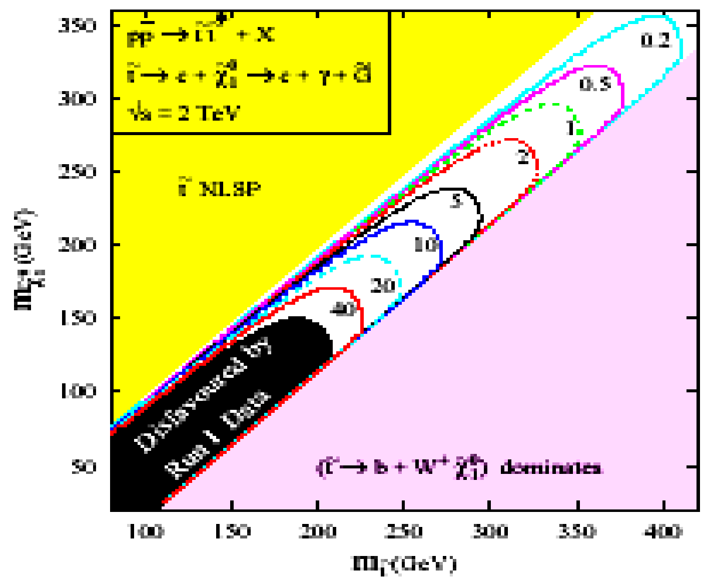
  diboson signatures $(Z \to \ell\ell/\text{jj};\ h \to b\bar{b})\not{E}_T$



$M_{\tilde{\chi}_1^\pm}$ sensitivity ~ 200 GeV for 2 fb^{-1}

Tevatron stop searches in low-energy SUSY breaking models

Carena, Choudhury,Diaz, Logan and C.W. '02



Extra photon and large missing energy helpful in stop detection

Cross sections for stop pair production in fb, with $\tilde{t} \longrightarrow c\gamma\tilde{G}$ and Signal/selection $jj\gamma\gamma \not{E}_T$

$\int \mathcal{L}$	σ_S 5σ	Max. $m_{\tilde{t}}$ (2 body)
2 fb^{-1}	6 fb	290 GeV
4 fb^{-1}	3.5 fb	315 GeV

Extra Dimensions

Fields in Extra Dimensions

- Any extra Dimension should be compact.
- Let's denote by x^μ our ordinary dimensions.
- Extra dimensions: y^M
- Take the circular topology of extra dimensions and require that after a turn, the wave function comes back to its original value

$$\Phi(x, y_i + 2\pi R_i) = \Phi(x, y_i), \qquad \Phi(x, y) = \frac{1}{\sqrt{V_d}} \sum_n \tilde{\Phi}_n(x) \exp\left(in\frac{y}{R}\right) \tag{48}$$

Kaluza Klein Modes

- Simple case: $d = 1$,

$$S = \int d^4x dy (\partial_A \Phi)^* \partial_A \Phi = \sum_n \int d^4x \left[\partial_\mu \tilde{\Phi}_n^* \partial^\mu \tilde{\Phi}_n + \frac{n^2}{R^2} \tilde{\Phi}_n^* \tilde{\Phi}_n \right] \tag{49}$$

- From the pont of view of a four dimensional observer, we have a tower of massive excitations !
- These excitations is what are called Kaluza Klein modes.
- In many extra dimensions, one can generalize the argument and the masses of the KK modes are

$$(M_{KK}^{n_1, n_2, \ldots n_d})^2 = \sum_i \left(\frac{n_i}{R_i} \right)^2 \tag{50}$$

Lowering the Planck Scale

- Idea: We live in a four dimensional wall, but there are extra dimensions and only gravity can penetrate into them.
- Problem: If gravity can penetrate intro the extra dimensions, Newton law will be modified

$$\vec{F} = \frac{m_1 m_2 \hat{r}}{\left(M_{Pl}^{\rm fund}\right)^{2+d} r^{2+d}} \tag{51}$$

- $M_{Pl}^{\rm fund}$ = Fundamental Planck Scale. Behaviour valid for $r \ll R$. For $r \gg R$, instead

$$\vec{F} = \frac{m_1 m_2 \hat{r}}{\left(M_{Pl}^{\rm fund}\right)^{2+d} r^2 R^d} \tag{52}$$

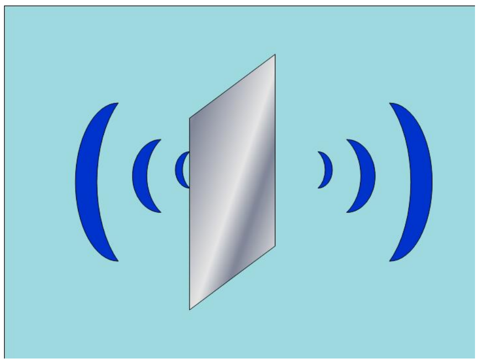
- Hence,

$$M_{Pl}^2 = \left(M_{Pl}^{\rm fund}\right)^{2+d} R^d \tag{53}$$

Gravity in Extra Dimensions (ED)

Gravity in ED $\Longrightarrow$ fundamental scale, pushed down to electroweak scale by geometry

Metric: $ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$ $\Longrightarrow$Solution to 5d Einstein eqs.



k=0 (flat)

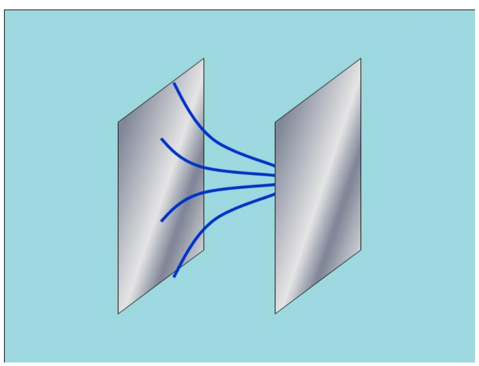
gravity flux in ED $\Longrightarrow$ Newton's law modified:

$$M_{Pl}^2 = (M_{Pl}^{\text{fund.}})^{2+d}\, R^d$$

this lowers the fundamental Planck scale,

$\Longrightarrow$ depending on the size & number of ED.

$M_{Pl}^{\text{fund.}} \simeq 1$ TeV $\Longrightarrow$ R= 1 mm, 10^{-12} cm if d = 2,6



k$\neq$0 (warped ED)

$$M_{Pl}^2 = \frac{(M_{Pl}^{\text{fund.}})^3}{2k}(1 - e^{-2kL})$$

fundamental scales: $M_{Pl} \sim M_{Pl}^{\text{fund.}} \sim v \sim k$

$\Longrightarrow$ Physical Higgs v.e.v. suppressed by e^{-kL}

$\Longrightarrow$ $\tilde{v} = v\, e^{-kL} \simeq m_Z$ if $kL \approx 34$

How can we probe ED from our 4D wall (brane)?

Flat case $(k = 0)$: 4-D effective theory:

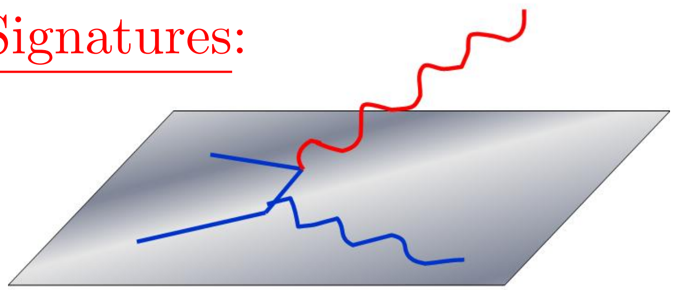
SM particles + gravitons + tower of new particles:

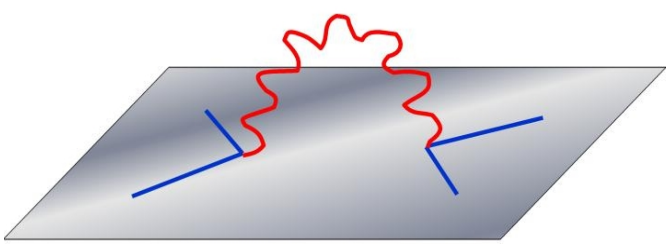
Kaluza Klein (KK) excited states with the same quantum numbers as the graviton and/or the SM particles

Mass of the KK modes $\Longrightarrow$ $E^2 - \vec{p}^{\,2} = p_d^2 = \sum_{i=1,d} \frac{n_i^2}{R^2} = M_{G_{\vec{n}}}^2$

imbalance between measured energies and momentum in 4-D

Signatures:





- Coupling of gravitons to matter with $1/M_{Pl}$ strength
  $R^{-1} \simeq 10^{-2}$ GeV $(d = 6)$;
  $1/R \simeq 10^{-4}$ eV $(d = 2)$;

(a) Emission of KK graviton states: $G_n \Leftrightarrow \not{E}_T$
(gravitons appear as continuous mass distribution)

(b) Graviton exchange $2 \rightarrow 2$ scattering
deviations from SM cross sections

Effective Gravitational Constant

- In general, the Planck Scale may be defined as the one at which the gravity interactions become strong.

$$\vec{F} = \left(\frac{E}{M_{Pl}}\right)^2 N_{KK}(M_{KK} < E)\frac{\hat{r}}{r^2} \tag{54}$$

- $\left(\frac{E}{M_{Pl}}\right)^2 N_{KK}(M_{KK} < E)$ may be interpreted like an effective gravitational constant
- In four dimension, the effective constant becomes of order one for energies of the order of the physical Planck scale.
- What happens for $d \geq 1$

Fundamental Planck Scale

- The number of KK modes at energies below a given one can be easily computed.
- In $d = 1$, for instance, the KK masses are n/R and hence,

$$N_{KK}(M_{KK} < E) = E \times R \tag{55}$$

- It is simple to convince yourself that, for d extra dimensions, one gets

$$N_{KK}(M_{KK} < E) = (E \times R)^d \tag{56}$$

- Hence, the interactions becomes strong at

$$\frac{E^2}{M_{Pl}^2} R^d E^d = 1 \rightarrow M_{Pl}^2 = \left(M_{Pl}^{\mathrm{fund}}\right)^{2+d} R^d \tag{57}$$

- That is the same result we obtain before, by other methods.

Size of flat Extra Dimensions

- Let's assume that the fundamental Planck scale is of the order of 1 TeV, to solve the hierarchy problem.

$$M_{Pl}^2 = (1\mathrm{TeV})^{2+d} R^d \tag{58}$$

- Then, the value of R is given by

$$R = 10^{32/d} 10^{-17} \mathrm{cm} \tag{59}$$

- For $d = 1$ we get $R = 10^{15}$ cm $\rightarrow$ Excluded
- For $d = 2$ we get $R \simeq 1$ mm $\rightarrow$ Allowed !
- For $d = 6$ we get $R \simeq 10^{-12}$ cm.
- The scenario is allowed for $d \geq 2$

Effective Cross Sections

- Let us consider the emission of gravitons in the collision of electrons and positrons (protons and antiprotons).
- Final state will be γ + Missing energy (jets + Missing Energy)
- Each graviton extremely weakly coupled but cross section will be given by the sum of the individual KK graviton production cross section, scaling with $N_K K$.
- Again, the effective gravitational constant appears and we get

$$\sigma \simeq \frac{1}{M_{Pl}^2} \frac{E^2 \left(E^d R^d\right)}{M_{Pl}^2} \tag{60}$$

$$\sigma \simeq \frac{1}{s} \left(\frac{\sqrt{s}}{M_{Pl}^{\text{fund}}} \right)^{2+d} \tag{61}$$

Warped Case

- Graviton KK modes have 1/TeV coupling strength to SM fields and masses starting with a few hundred GeV.
- KK graviton states produced as resonances.
- One can rewrite the warp factor and the massive graviton couplings in terms of mass parameters as:

$$\exp(-kL) = \frac{m_n}{k x_n}$$
$$\Lambda_\pi \simeq \frac{\bar{M}_{Pl} m_1}{k x_1} \tag{62}$$

with $x_1 \simeq 3.8$, $x_n \simeq x_1 + (n-1)\pi$.

- Calling $\eta = k/\bar{M}_{Pl}$, one gets that the graviton width is

$$\Gamma(G^n) \simeq m_1 \eta^2 \frac{x_n^3}{x_1} \tag{63}$$

Flat Extra Dimensions

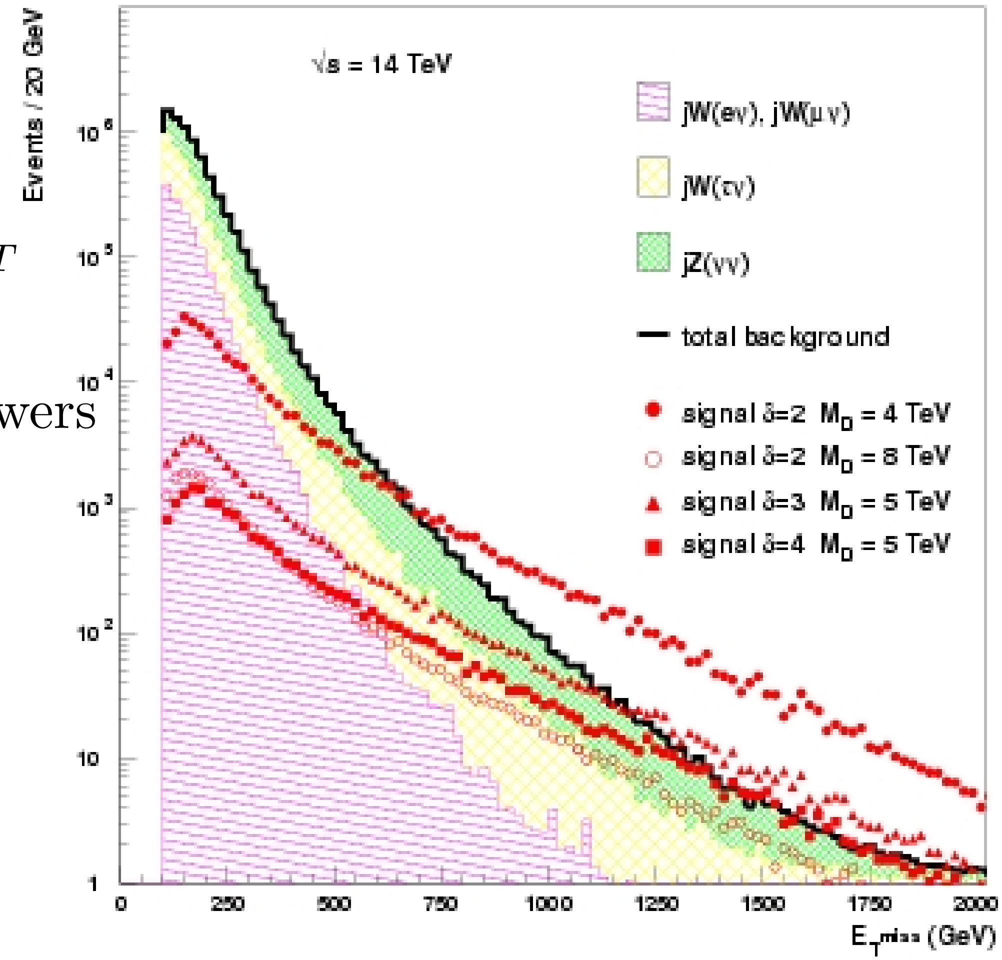
- Emission of KK graviton states

$$p\bar{p} \to g\,G_N\;(G_N \to \not{E}_T) \longrightarrow \text{jet}+\not{E}_T$$

Cross section summed over full KK towers

$$\Longrightarrow \qquad \sigma/\sigma_{SM} \propto (\sqrt{s}/M_{\rm Pl}^{\rm fund})^{2+d}$$

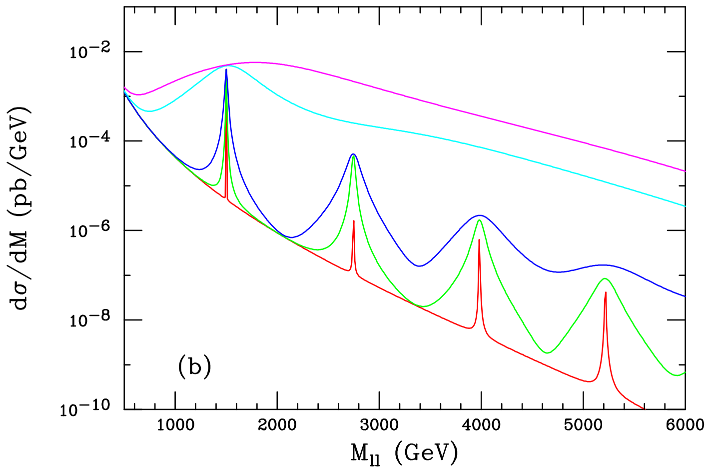
Emitted graviton appears as a continuous mass distribution.



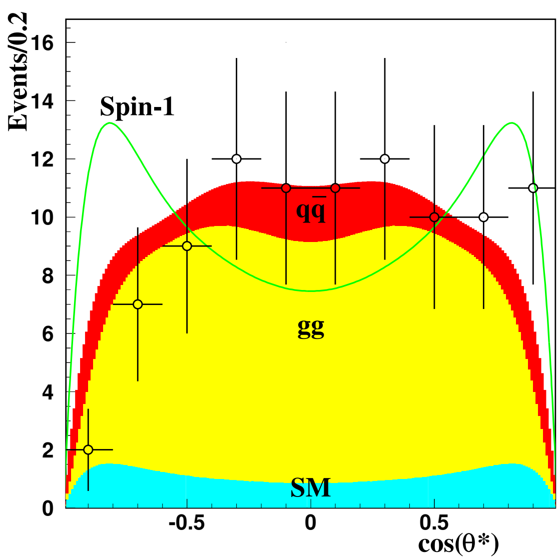
Discovery reach for fundamental Planck scales on the order of 5–10 TeV (depending on $d = 4,3,2$)

• Warped Extra Dimensions

Narrow graviton resonances: $pp \to G_N \to e^+e^-$



From top to bottom: $k/M_{Pl} = 1, 0.5, 0.1, 0.05, 0.01$

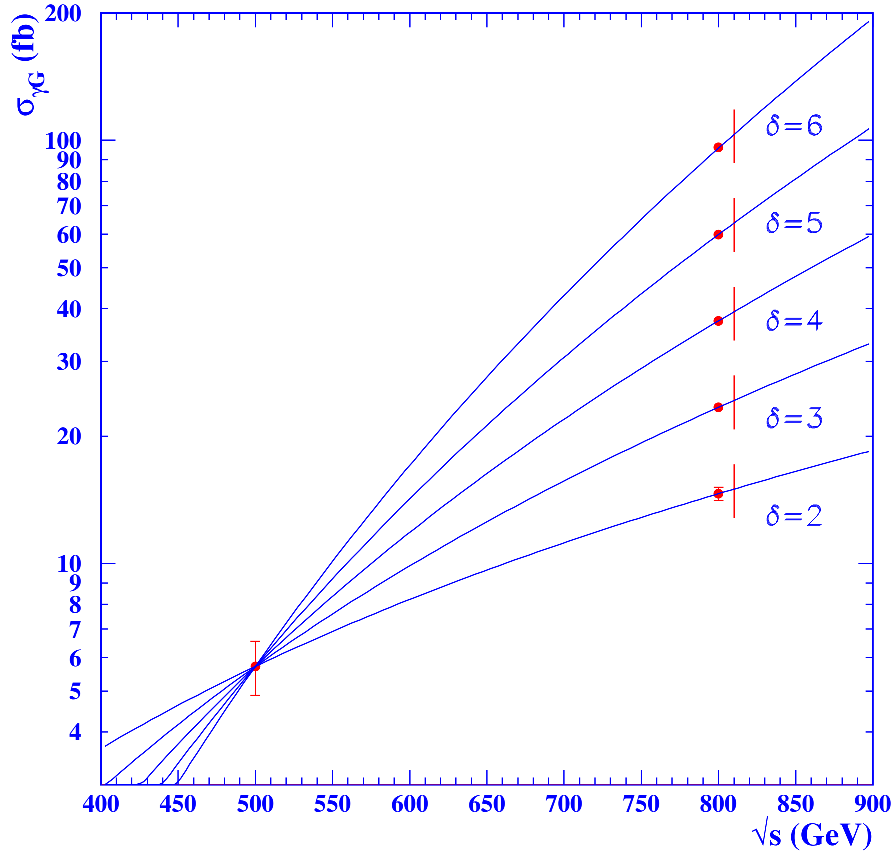


⋆ Angular distributions reveal spin of resonance

Extra Dimensions

Flat ED:

graviton emission: $e^+e^- \to \gamma\, G_N$



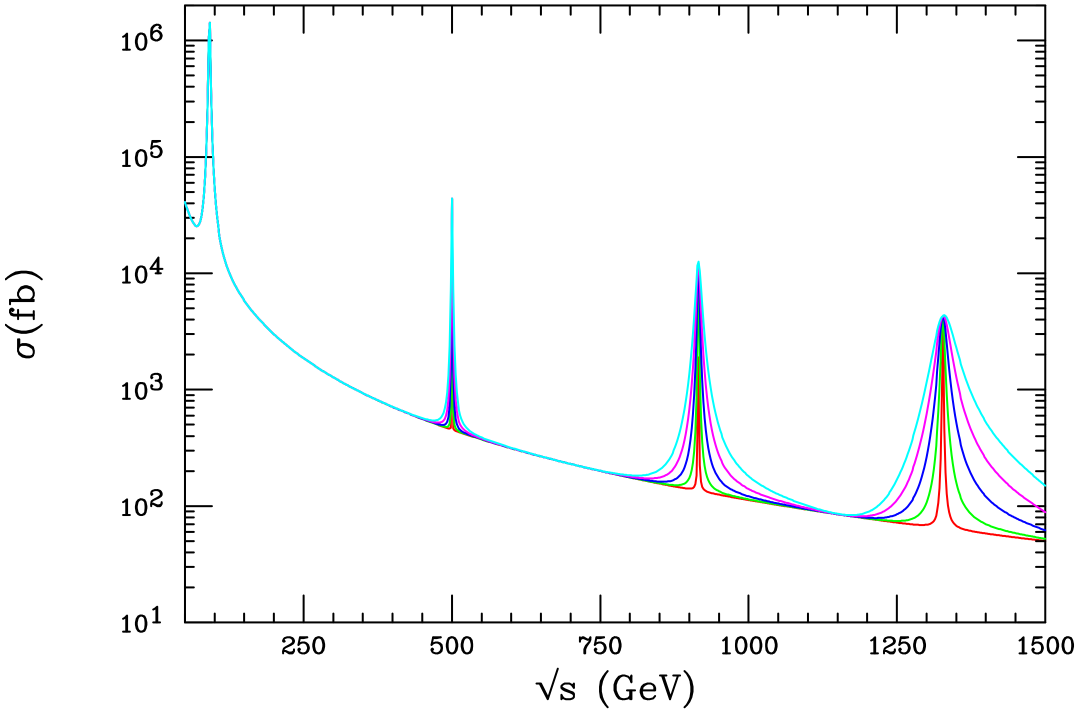
- Varying $\sqrt{s}$ one can determine values of fundamental parameters: $M_{\rm Pl}^{\rm fund}$ & δ

graviton exchange in $2 \to 2$ processes:

- deviations for $e^+e^- \to f\bar{f}$ or new decays with hh or $\gamma\gamma$
- ability to determine spin-2 nature

Warped ED:

• Given sufficient center-of-mass energy, KK graviton states produced as resonances:



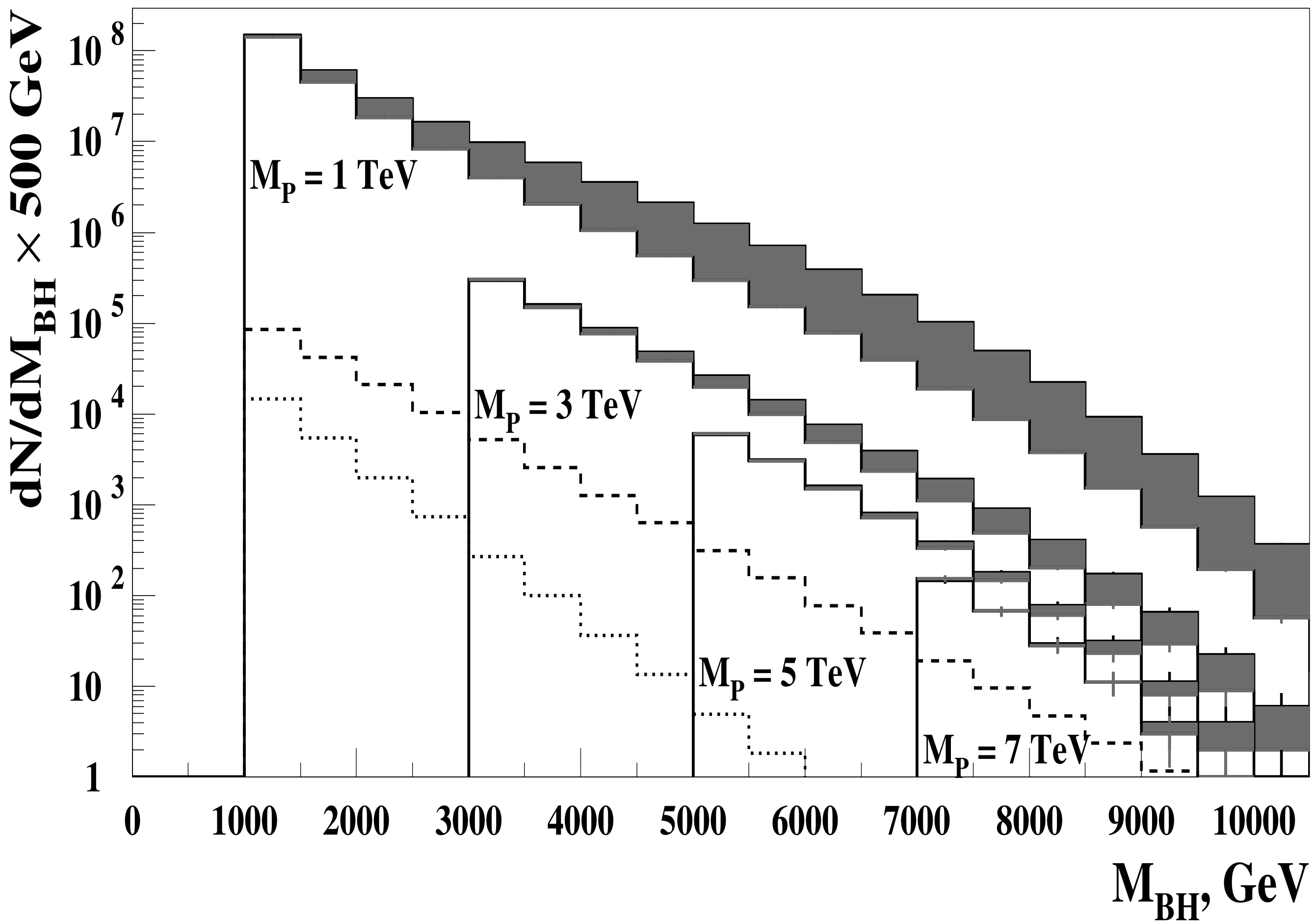
$\sigma(e^+e^- \to \mu^+\mu^-)$ as a function of $\sqrt{s}$, including KK graviton exchange,

$m_1 = 500$ GeV, $k/M_{Pl} = 0.01$–0.05 range.

Black Hole Production ?

- Two partons with center of mass energy $\sqrt{s} = M_{BH}$, with $M_{BH} > M_{Pl}^{fund}$ collide with a impact parameter that may be smaller than the Schwarzschild radius.
  $R_S \simeq \frac{1}{M_{Pl}^{fund}} \left(\frac{M_{BH}}{M_{Pl}^{fund}} \right)^{\frac{1}{d+1}}$
- Under these conditions, a blackhole may form
- If $M_{Pl}^{fund} \simeq 1$ TeV $\rightarrow$ more than 10^7 BH per year at the LHC (assuming that a black hole will be formed whenever two partons have energies above M_{Pl}).
- Decay dictaded by blackhole radiation, with a temperature of order $1/R_S$. Signal is a spray of SM particles in equal abundances: hard leptons and photons.
- At LHC, limited space for trans-Planckian region and quantum gravity.

Black Hole production at the LHC



Dimopoulos and Lansberg; Thomas and Giddings '01

Sensitivity up to $M_{Pl}^{\text{fund}} \simeq 5 - 10$ TeV for 100 fb^{-1}.

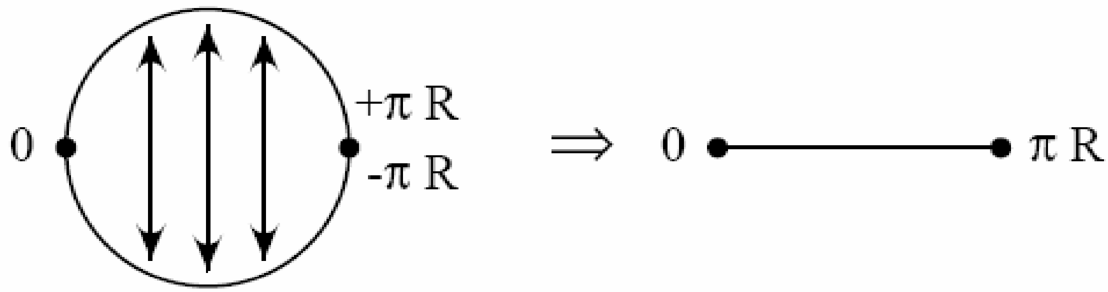
Universal Extra Dimensions

Most natural extension of four dimensional description:

- All particles live in all dimensions, including quarks, leptons, Higgs bosons, gauge bosons and gravitons.
- Universality implies a translational invariance along the extra dimension, and thus conservation of the component of momentum in the that direction.
- This implies that a KK state with $n \neq 0$, carrying non-zero momentum in the extra dimension, cannot decay into standard, zero modes.
- The lightest KK particle is stable, being a good dark matter candidate.
- Other intersting properties that arise in six dimensions are natural proton stability and an explanation of the number of generations

Orbifold

- Massless 5d spinors have 4 components, leading to mirror fermions at low energies.
- If extra dimension is compactified in a circle, no standard chiral theory may be obtained.
- Chiral theories may be obtained by invoking orbifold boundary conditions, projecting out unwanted degrees of freedom.
- Fold the extra dimension, identifying y with $-y$



- Boundary Conditions:
  $\Psi(-y) = \gamma_5 \Psi(y)$
  $V_\mu(-y) = V_\mu(y), V_5(-y) = -V_5(y)$

KK Decomposition

- We expand fields in KK modes:

$$\Phi(x^\mu, y) = \sum_n f^n(y)\Phi^n(x^\mu) \tag{6}$$

- Flat, universal extra dimension:
  - Even fields (A_μ, Ψ_L) have zero modes:

$$\Phi(x^\mu, y) = \sqrt{\frac{1}{\pi R}}\Phi^0(x^\mu) + \sum_{n\geq 1}\sqrt{\frac{2}{\pi R}}\cos\left(\frac{ny}{R}\right)\Phi^n(x^\mu) \tag{7}$$

  - Odd fields (A_5, Ψ_R) don't:

$$\Phi(x^\mu, y) = \sum_{n\geq 1}\sqrt{\frac{2}{\pi R}}\sin\left(\frac{ny}{R}\right)\Phi^n(x^\mu) \tag{8}$$

  - KK masses (before EWSB): n/R

- KK fermions are Dirac, with vector-like interactions.
- In a chiral theory, the left- and right-handed zero modes each have a *separate* tower of KK modes.
- This is somewhat like SUSY, with each SM particle accompanied by partner fields.

KK Parity

- Conservation of KK number is broken to conservation of KK parity: $(-1)^n$.
- KK-parity requires odd KK modes to couple in pairs:
- The lightest first-level KK mode is stable.
- First level KK modes must be pair-produced.
- The Lightest Kaluza-Klein Particle plays a crucial role in phenomenology, similar to the LSP of SUSY:
- All relic KK particles decay to LKPs.
- Any first level KK particle produced in a collider decays to zero modes and an LKP.
- KK parity is also present with boundary fields, provided the same fields live on both boundaries.

Identity of the LKP

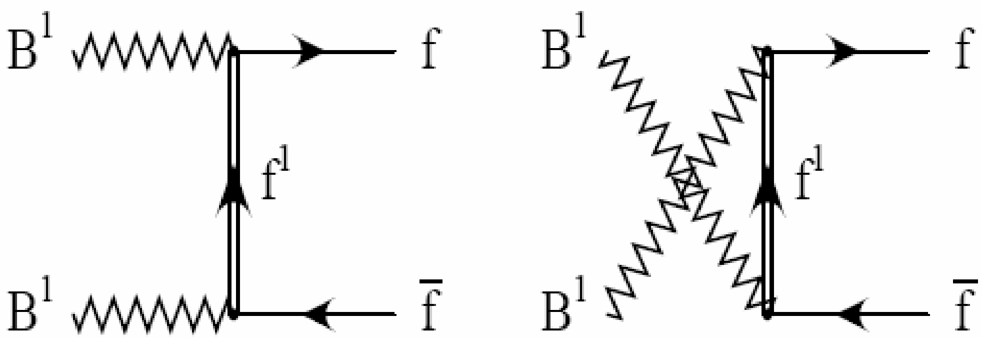
- Boundary terms play a role analogous to soft SUSY-breaking masses, determining masses and couplings for the entire KK tower.
- If we imagine the boundary terms are zero at the cut-off scale, they will be induced at loop size:

$$\delta M^2 \quad \sim \quad \frac{1}{R^2}\frac{\alpha}{4\pi}\log\left(\Lambda R\right) \tag{9}$$

- This prescription is kind of like specializing to mSUGRA from the MSSM.
- Since $\alpha_1 \ll \alpha_2 \ll \alpha_3$, we can imagine that the smallest corrections are to the U(1) gauge boson:
- Since $\delta M \sim 1/R \gg M_W$, the LKP is (almost) purely a KK mode of the U(1) gauge boson: $B_\mu^{(1)}$.
- Following this line of reasoning, the NLKP is the right-handed electron: $e_R^{(1)}$.

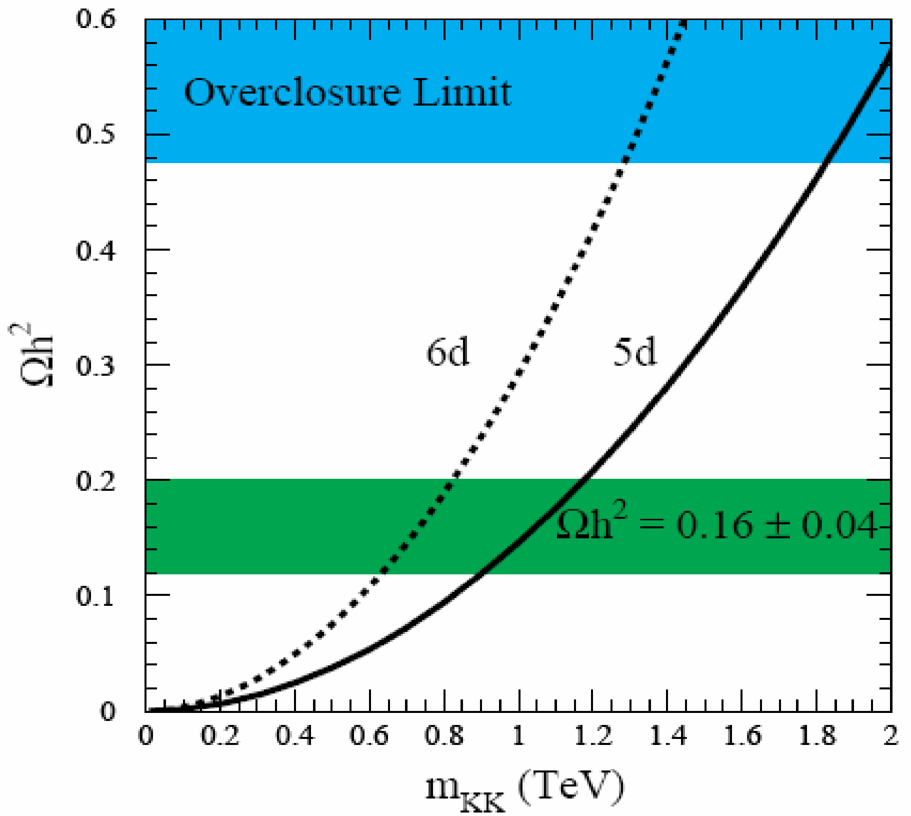
Dark Matter

- Relic Density depends strongly on annihilation cross section.
- In the case of universal extra dimensions, dominant annihilation diagram is given by interchange of first tower of KK particles.



- Whenever the KK mode of the right-handed leptons is close enough in mass to the LKP, coannihilation should be also taken into account

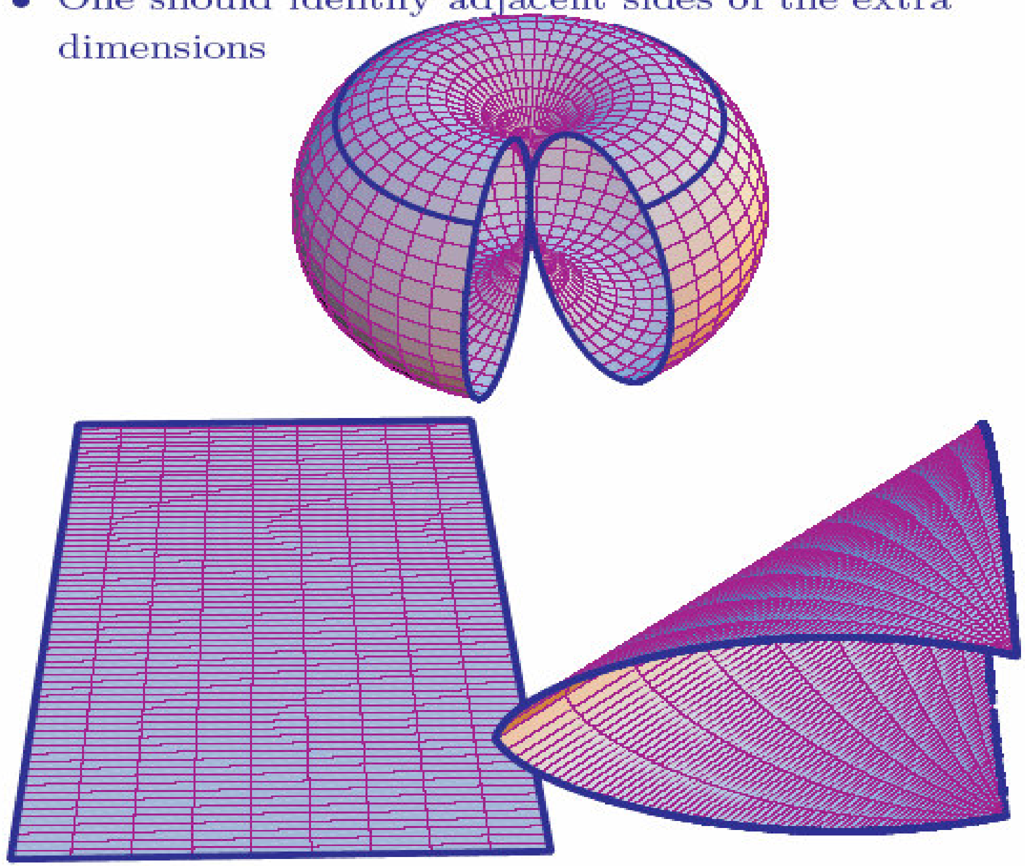
Relic Density: Results



Universal extra dimensions of the order of 500 GeV–1 TeV
preferred for the LKP to be a good dark matter candidate

Two Universal extra Dimensions

- As happens in the case of one extra dimensions, compactification in a circle (torus in this case) does not lead to chiral fermions
- One should identify adjacent sides of the extra dimensions



Magic of 6 Dimensions

- Cancellations of global gauge anomalies demands that the number of generations $N = 0 \bmod 3$
- 6d Gauge and Lorentz invariance also demand that all operators fulfill the condition

$$3\Delta B + \Delta L = 0 \tag{10}$$

Proton stability

- In the SM, proton is stable.
- Simplest operators lead to the decay of

$$p \longrightarrow e^+\pi^0, \qquad \text{withstrength} \simeq \left(\frac{m_p}{M}\right)^4 \tag{11}$$

This leads to a bound on $M \geq 10^{16}$ GeV

- In six dimensions, the constraint on ΔB and ΔL specified above, implies that the dominant decay is $p \longrightarrow e^- \nu_e \nu_\mu \pi^+ \pi^+$
  with strength $\simeq \left(\frac{1}{MR}\right)^{12} \left(\frac{m_P}{M}\right)^{10}$
  Consistent with experimental bounds even if $M \simeq$ few TeV.

Strongly Interacting Scenarios

Strongly Coupled EWSB Dynamics

(a) Models which do not require a Higgs Boson

$\Longrightarrow$ Strong interactions at the TeV scale: Technicolor,

- New gauge interaction which is asymp. free and becomes strong at scales of order 1 TeV $\rightarrow$ new fermions (technifermions) feel this interaction and form condensates $\rightarrow$ EWSB

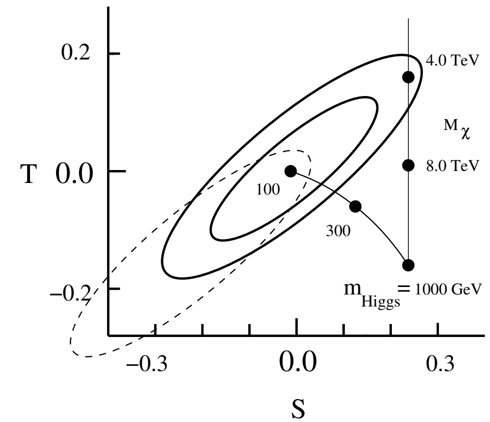
Robust prediction:

vector resonance with mass $\leq$ 2 TeV (to unitarize the $W_L^- W_L^+ \rightarrow W_L^- W_L^+$ amplitude)

(b) Strong interactions above TeV scale give rise to bound states

$\Longrightarrow$ Composite Higgs Models

Top-condendate models: effective four-Fermi interactions that induce bound states with the same quantum numbers than a Higgs, and condensation of such bound state $\rightarrow$ EWSB



Top quark seesaw theory:

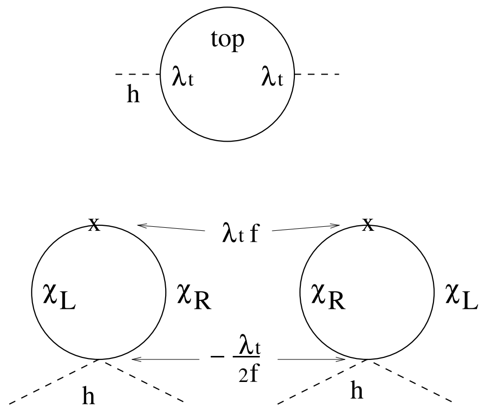
- Higgs is a bound state of left-handed top and right-handed component of a new vector-like fermion: $m_H \simeq 500$ GeV.
- New contributions from additional quarks bring agreement with precision electroweak measurements.

(c) Little Higgs Models:

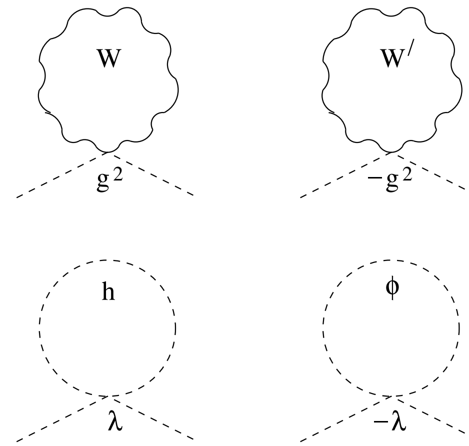
• Higgs is a pseudo-Goldstone boson from a spontaneous global symmetry breaking at scale 10–30 TeV. $\Longrightarrow$ New Dynamics needed above that scale.

• Global symmetry explicitly broken by gauge and Yukawa interactions, however, no single int. breaks all the symmetries, hence protecting the Higgs mass
Higgs acquires mass only radiatively at the electroweak scale.

• Non-linearly realized symmetry yields cancellation of quadratically divergent quantum corrections between fields of the same spin.



A fermion loop cancels a fermion loop.



The gauge and Higgs loops are cancelled by diagrams with new bosons in loops.

Cancellation of quadratic divergences works at one loop.
$\Longrightarrow$ new fermionic partners for SM quarks and leptons and new gauge boson partners for SM gauge fields at the TeV scale.

- LHC should discover some of them;
- LC: precision measurement of heavy gauge boson couplings to fermions via polarized cross sections and asymmetries.

Many possible Signatures of strongly coupled EWSB:

• Strong WW scattering • Anomalous gauge couplings

• Extra scalars $\rightarrow$ composites of underlying strongly coupled fermions

• Extra Fermions • Heavy vector bosons

• Extended Higgs sector at TeV scale or below $\rightarrow$ mixing can bring the SM-like Higgs down to 200 GeV.

No compelling model exists that can be called the Standard Model of Strongly coupled EWSB

Why is it so difficult?

$\star$ The mechanism of fermion mass generation is ackward (not simple as in the simplest Higgs model) and it is distinct from the gauge bosons mass generation mechanism

$\star$ In most models, the energy scale associated with the flavor dynamics is rather close to the scale of EWSB $\longrightarrow$ need to address the origin of EWSB and flavor in the same overall picture

$\star$ No clear connection to fundamental physics at high energy. Gauge coupling unification must be regarded as accidental.

$\star$ Strongly-coupled systems are hard to treat theoretically $\Longrightarrow$ explicit computations are often very difficult