Lattice QCD & *B* Physics

Andreas Kronfeld

Aspen Winter Conference

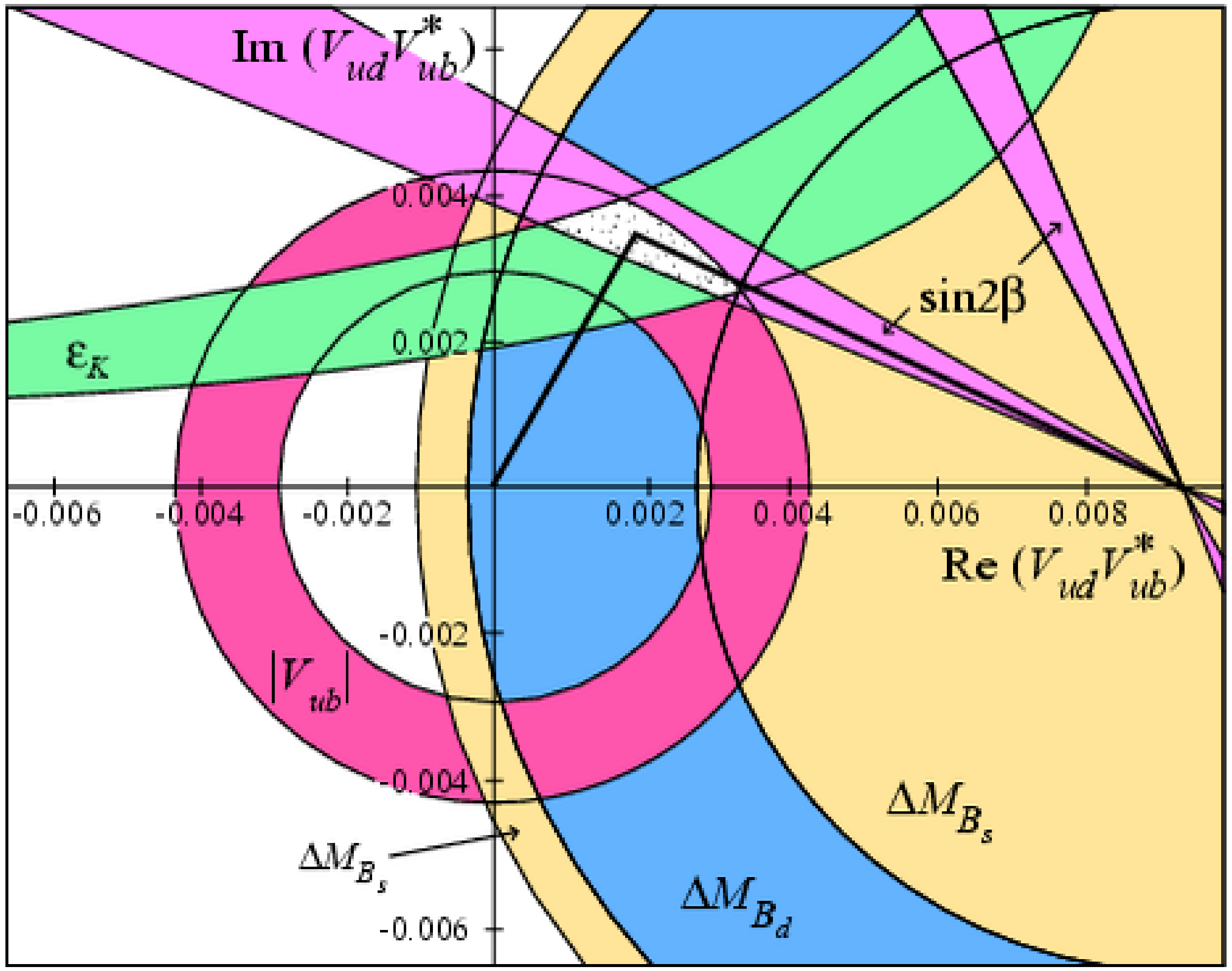
February 2-7, 2004

Fermi National Accelerator Laboratory

THE MATRIX

Unitarity Triangle

- Accuracy limited by QCD.
- Are *theoretical* error bands reliable?
- To diagnose new physics?



PDG 2002

Lattice & CKM

- "Standard UT fit is now entirely in the hands of Lattice QCD (up to, perhaps, $|V_{ub}|$)"

  Martin Beneke (Lattice 2001, Berlin)

-

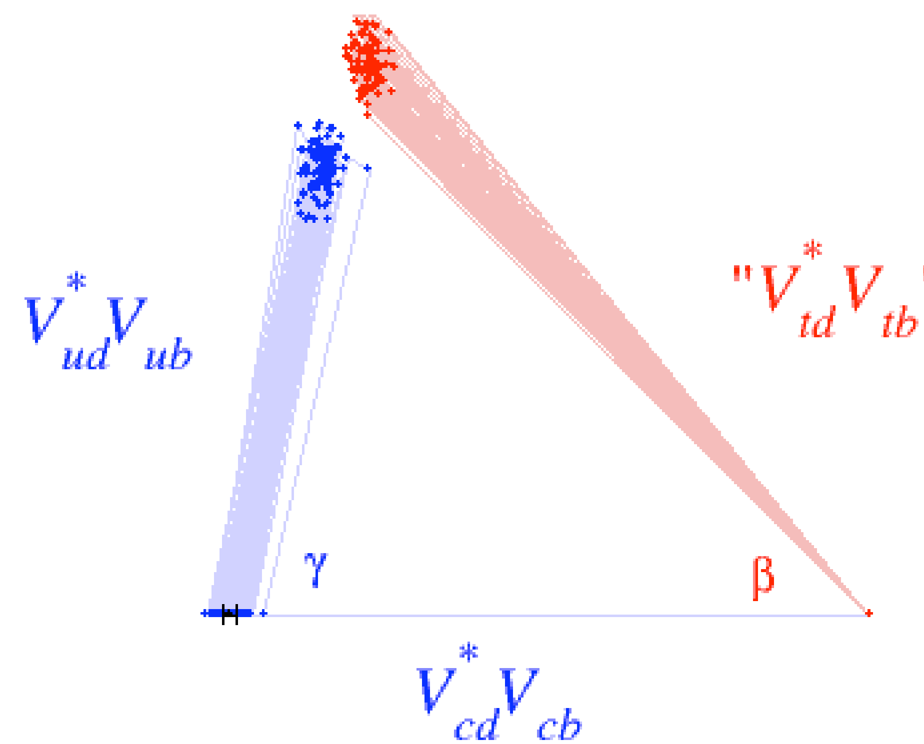
- Are there non-Standard (non-KM) sources of *CP*V in *B* and *K* mixing? In rare decays?

-

- "Like trying to fix a watch in a snow storm!"

- $|V_{cd}|$ from f_D, $f_+^{D\to\pi}(E_\pi)$
- $|V_{cb}|$ from $\mathcal{F}^{B\to D^*}(1)$
  lattice QCD error already ~4%
- $|V_{ub}|$ from $f_+^{B\to\pi}(E_\pi)$
- $|V_{ud}|$ from $F_1^{n\to p}$
- $|V_{td}|$ "from" $f_B^2 B_B$

gold-plated in lattice QCD (as defined below)



Lattice QCD

A Multi-Scale Problem

- QCD is a multi-scale problem
  - Λ: the characteristic scale of the strong interaction
  - m_q: light quark masses $m_q \ll \Lambda$: good for u, d, (s)
  - m_Q: heavy quark masses $m_Q \gg \Lambda$: good for t, b, (c)
  - a^{-1}: ultraviolet cutoff, always needed in QFT
  - L^{-1}: infrared cutoff, often helpful in QFT

- Ken Wilson said, integrate the functional integral numerically (with finesse and brute force):

$$\int \mathcal{D}A\,\mathcal{D}\psi\mathcal{D}\bar\psi\;\bar\psi_u\gamma_5\psi_d(x)\bar\psi_d\gamma_5\psi_u(y)\;e^{-S_g-\bar\psi M\psi} = \int \mathcal{D}A\;\mathrm{tr}[G_d(x,y)\gamma_5 G_u(y,x)\gamma_5]\,\det M\,e^{-S_g}$$

$M = [\not{D} + m]_{\mathrm{lat}}$ $\qquad$ S_g = lattice gauge action

- $G = M^{-1}$ (quark propagators): expensive
- det M (sea quark loops): very expensive

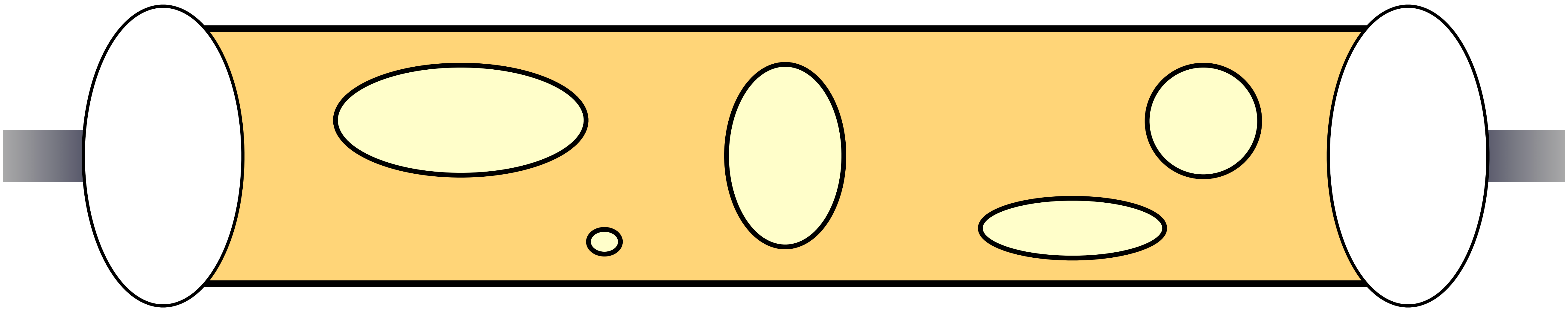
Systematics

- MC treats Λ exactly, up to statistical errors.
- Control systematics with effective field theories:
  - $m_q = rm_s > m_d$: chiral perturbation theory (χPT)
  - $L < \infty$: general EFT of hadrons
  - $a \neq 0$: Symanzik effective field theory
  - $m_Q a \sim 1$: HQET, NRQCD [hep-lat/0310063]
- verify with numerical data, then extrapolate

Andreas Kronfeld

Quenched Approximation

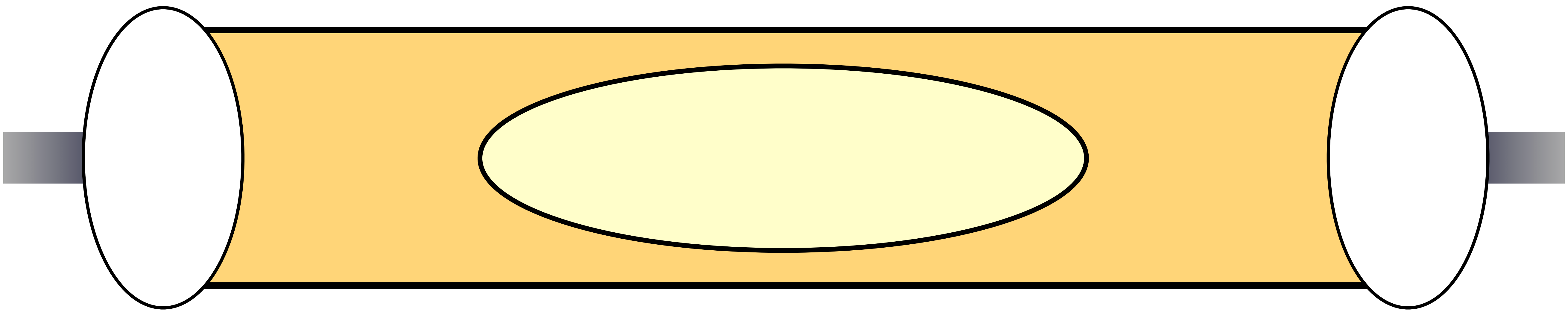
Full QCD has (expensive) quark loops.



- Replace det M with 1, and compensate by shifting bare gauge coupling and bare masses. “Dielectric”.
- Arguably OK if all light quarks had mass $m_q \sim \Lambda$.
- Where we *were*, not where we’re going.

Chiral Extrapolation

Virtual quark loops: $B \to \left\{ \begin{array}{c} B^*\pi \\ B_s^* K \\ B_{(s)}^*\eta \end{array} \right\} \to B.$



- Loops yield non-analytic behavior, *e.g.*, $m_\pi^2 \ln m_\pi^2$.
- Extrapolation needs small enough m_q.

Lattice Fermions

- Naïve: 16 species per field, called "tastes".
- Wilson: 1 taste (flavor), but hard chiral symmetry breaking ⇒ fine tuning ⇒ $m_q > 0.7 m_s$ [JLQCD].
- Staggered: still 4 tastes per field, but remnant of chiral symmetry ⇒ $m_q > 0.15 m_s$ [MILC].
- Ginsparg-Wilson (domain wall or overlap): flavor simple, full chiral symmetry. More expensive—but where we'll end up someday.

The Berlin Wall

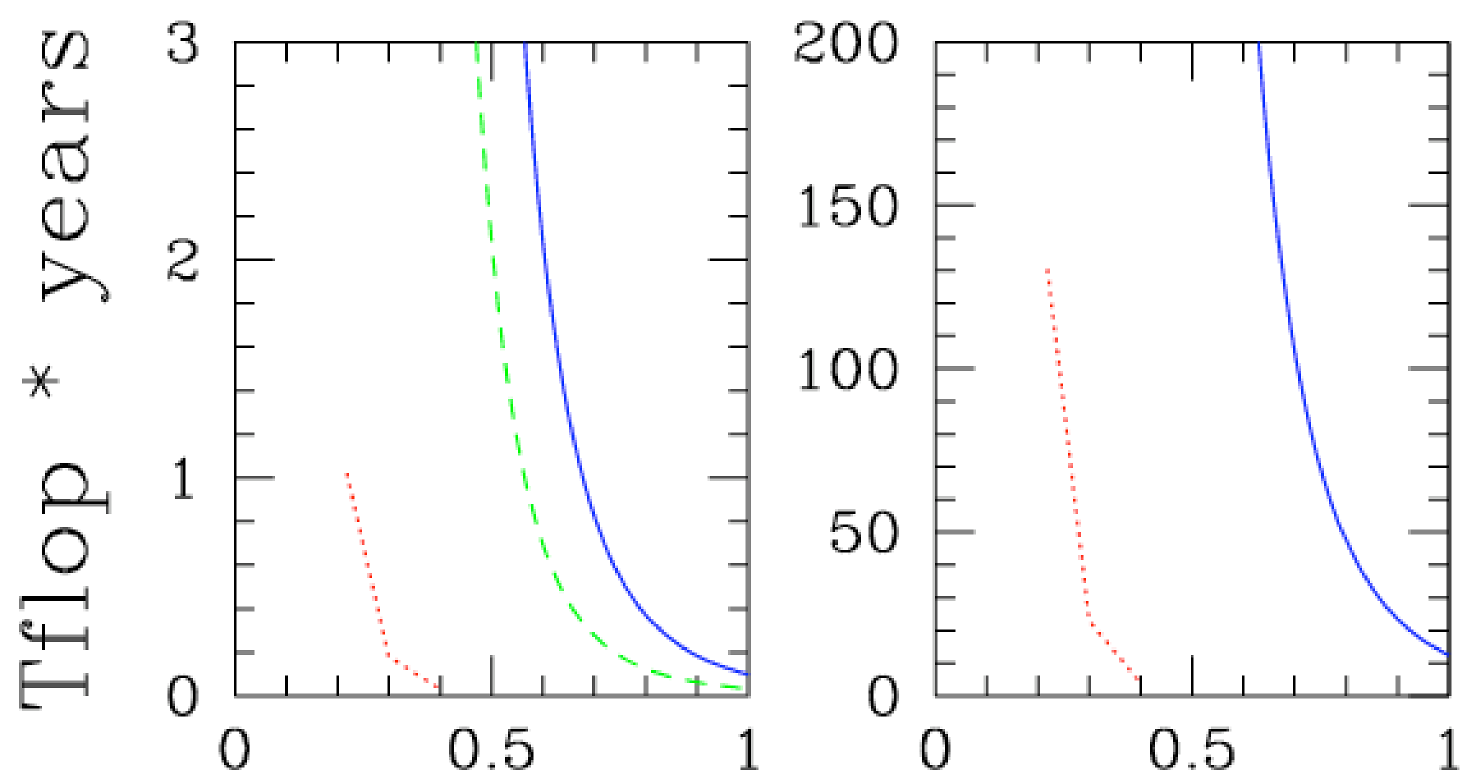
$$\texttt{cost} \propto \left(\frac{m_{\rm V}^2}{m_{\rm PS}^2}\right)^3 L^{4+1}\, a^{-(4+3)}$$

- cost for Wilson

  ≡ 3 times faster

- cost for staggered

- Plot from Jansen, who had input from Ukawa & Gottlieb
  hep-lat/0311039



$m_{\rm PS}/m_{\rm V}$

a = 1/11 fm
measured

$m_{\rm PS}/m_{\rm V}$

a = 1/22 fm
extrapolated

Staggered Quarks

- Staggered fermions have always been fast.
- Discretization effects $O(a^2)$, but "large".
- Traced to "taste-changing" interactions.
- Systematically removed by Orginos & Toussaint:
  - the "Fat7 action"
- Remaining $O(a^2)$ removed by Lepage
  - the "asqtad action": $O(\alpha_s a^2)$, $O(a^4)$ and "small".

Gold-plated Quantities

- Some quantities are under much better control:
  - 1 hadron in the initial state & 0 or 1 in the final state;
  - stable, or narrow and not too close to threshold.
- *Chiral extrapolation must also be under control!*
- D^*, $D_s(J^+)$, … not gold-plated, but not bad.
- (almost) elastic ρ, Δ, $K \to \pi\pi$ much, much harder.
- Inelastic $B \to \pi\pi$ conceptually beyond us.

Unquenched vs. Quenched

The MILC Ensembles

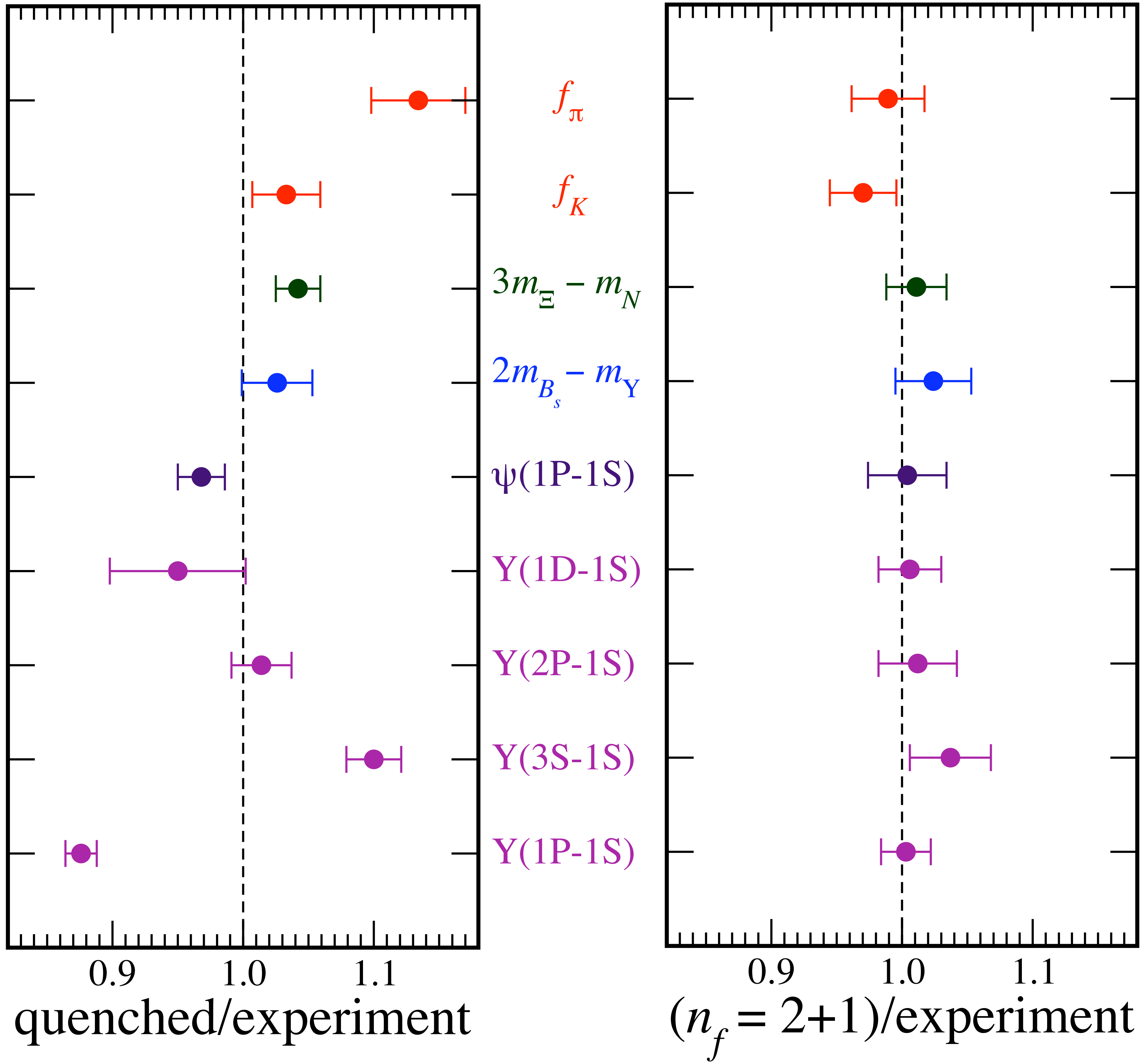
- MILC Collaboration = dozen or so physicists at Arizona, UCSB, Colorado, APS, Indiana, Pacific, Utah, Washington U. (St. Louis)
- Improved staggered quarks (asqtad action)
- Sea quark loops (det M) for 2 + 1 flavors
- a = 1/8, 1/11 fm
- Many (valence and sea) m_q down to $0.15m_s$
- Several hundred lattice gauge fields per ensemble

- Freely available over the internet
- Several groups started looking at light hadrons (MILC), hadrons with bottom quarks (HPQCD), hadrons with charmed quarks (Fermilab).
- All of the QCD scale was being probed.
- A consistent picture emerged: after tuning $1 + n_f$ parameters, we checked a 9 other mass splittings and decay constants.

Tune Bare Couplings

- pick $g_0^2(a)$ and use Δm_Υ(2S-1S) to deduce a
  - not very sensitive to quark masses, even m_b
- light (u, d) and strange masses tuned to (m_π^2, m_K^2)
- charmed mass tuned to (spin-averaged) m_{Ds}
- bottom mass tuned to m_Υ(1S)
- Useful to compare quenched vs. unquenched.

Davies et al., hep-lat/0304004



What About the Merovingian?

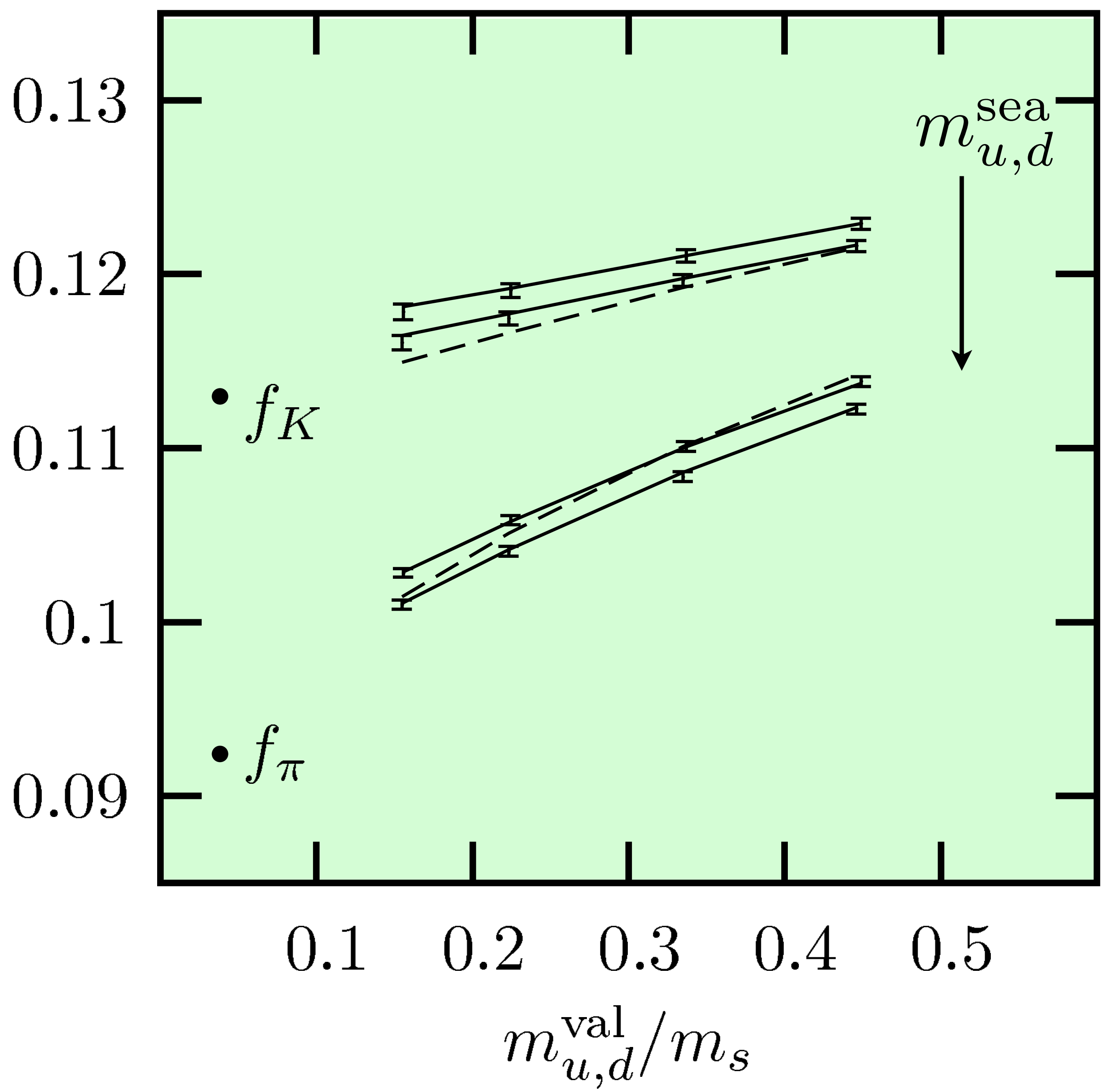
- Because staggered quarks come in four tastes, we have used $[\det_4 M]^{1/4}$ for $\det_1(\not{D} + m)$.
- $\det_4 M^{1/4}$ looks non-local and, hence, terrifying.
- However:
  - Correct in perturbation theory.
  - Chiral anomalies incorporated correctly.
  - Long-distances well described by a version of χPT designed to handle it.
- "Not proven," but several positive indications.

Summary So Far

- Lattice QCD with improved staggered quarks agrees with Nature for 5+9 gold-plated quantities.
- Only improved staggered fermions can achieve the following (in the near term):
  - 2+1 flavors of sea quark
  - light enough quarks for chiral perturbation theory
- Very promising for B, D, and K physics.

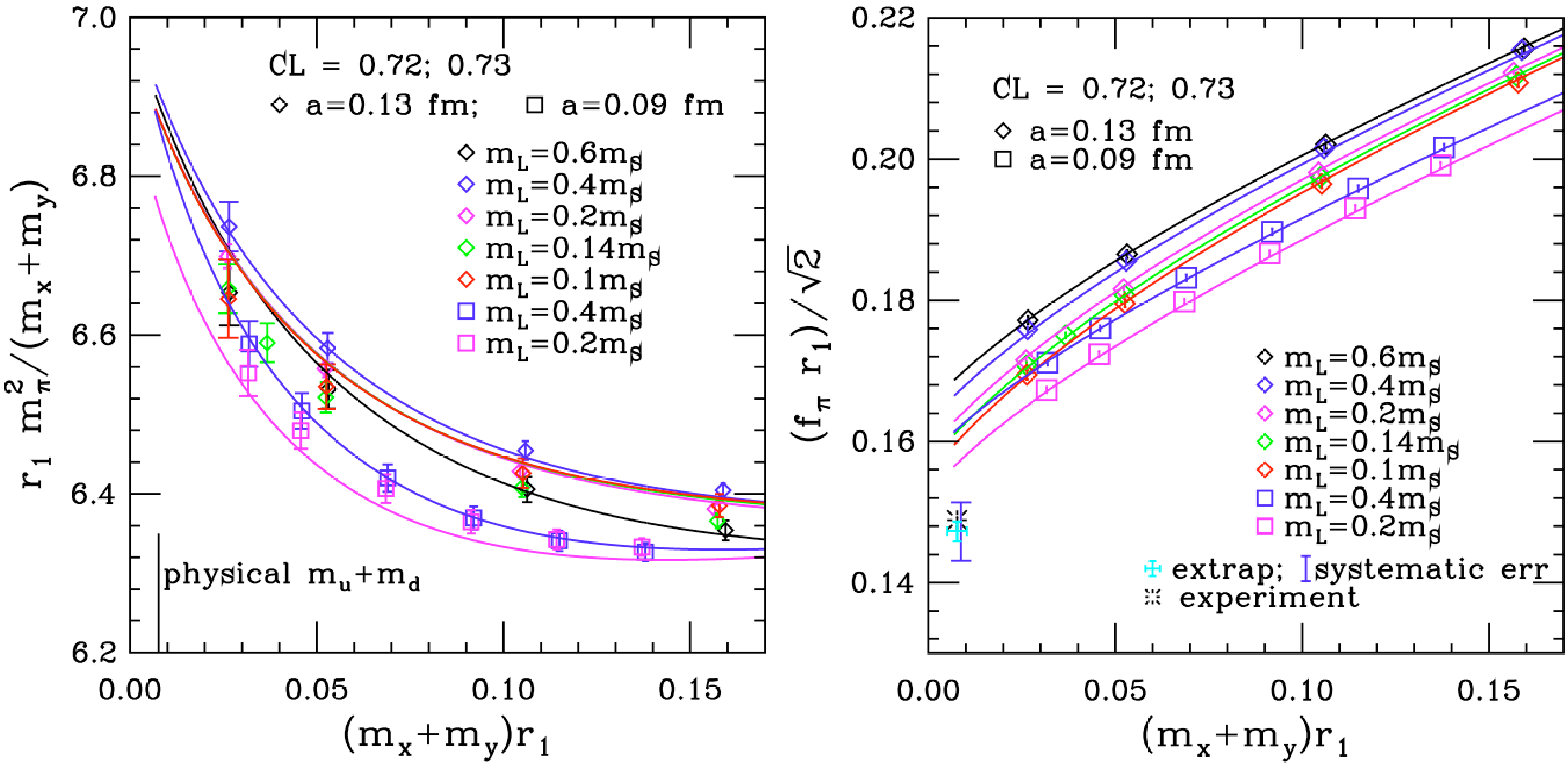
f_π and f_K

Chiral Extrapolation



- Dots at 0.04 are experimental.
- Error bars are lattice QCD.
- Linear extrap (by eye).
- Gasser-Leutwyler χlog gets closer (solid).
- Sharpe-Shoresh χlog even closer (dashed).

- Finally, χPT can be modified to incorporate the 4 tastes and the 1/4 root [Aubin & Bernard].



One fit to all quark mass combos & both lattice spacings!

- Four extrapolations:
  - linear
  - continuum χPT, assuming $m_q^{\mathrm{val}} = m_q^{\mathrm{sea}}$
  - continuum χPT, with $m_q^{\mathrm{val}} \neq m_q^{\mathrm{sea}}$
  - χPT with taste-symmetry breaking and ....
- Successively more accurate.
- Hard to reconcile with a non-local underlying theory.

Tests with D Decays

- Most of the challenges for lattice B physics hold for D physics too.
- CKM UT fit needs decay constant f_B and form factor $f_+^{B\to\pi}(E_\pi)$
- Corresponding f_D and $f_+^{D\to\pi}(E_\pi)$ poorly known, but CLEO-c will measure them soon.

- CKM drops out of

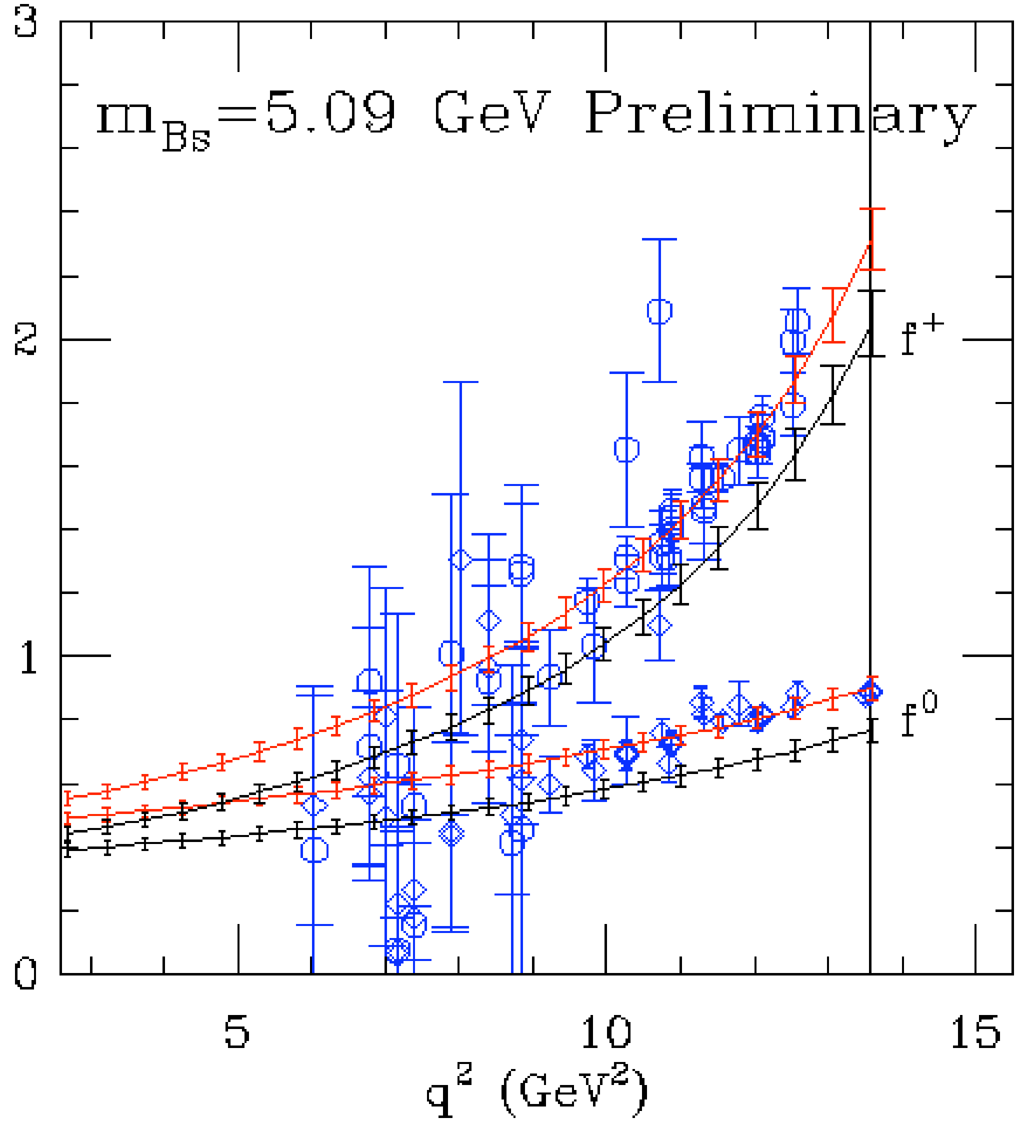
$$\frac{1}{\Gamma_{D_s \to l\nu}} \frac{d\Gamma_{D \to K l\nu}}{dE_K} \propto \left| \frac{f_+^{D \to K}(E_K)}{f_{D_s}} \right|^2$$

$$\frac{1}{\Gamma_{D \to l\nu}} \frac{d\Gamma_{D \to \pi l\nu}}{dE_\pi} \propto \left| \frac{f_+^{D \to \pi}(E_\pi)}{f_D} \right|^2$$

- Pure tests of non-perturbative QCD
- Functions of energy
- Lattice calculations by Summer?
- CLEO-*c* measurements by Autumn?

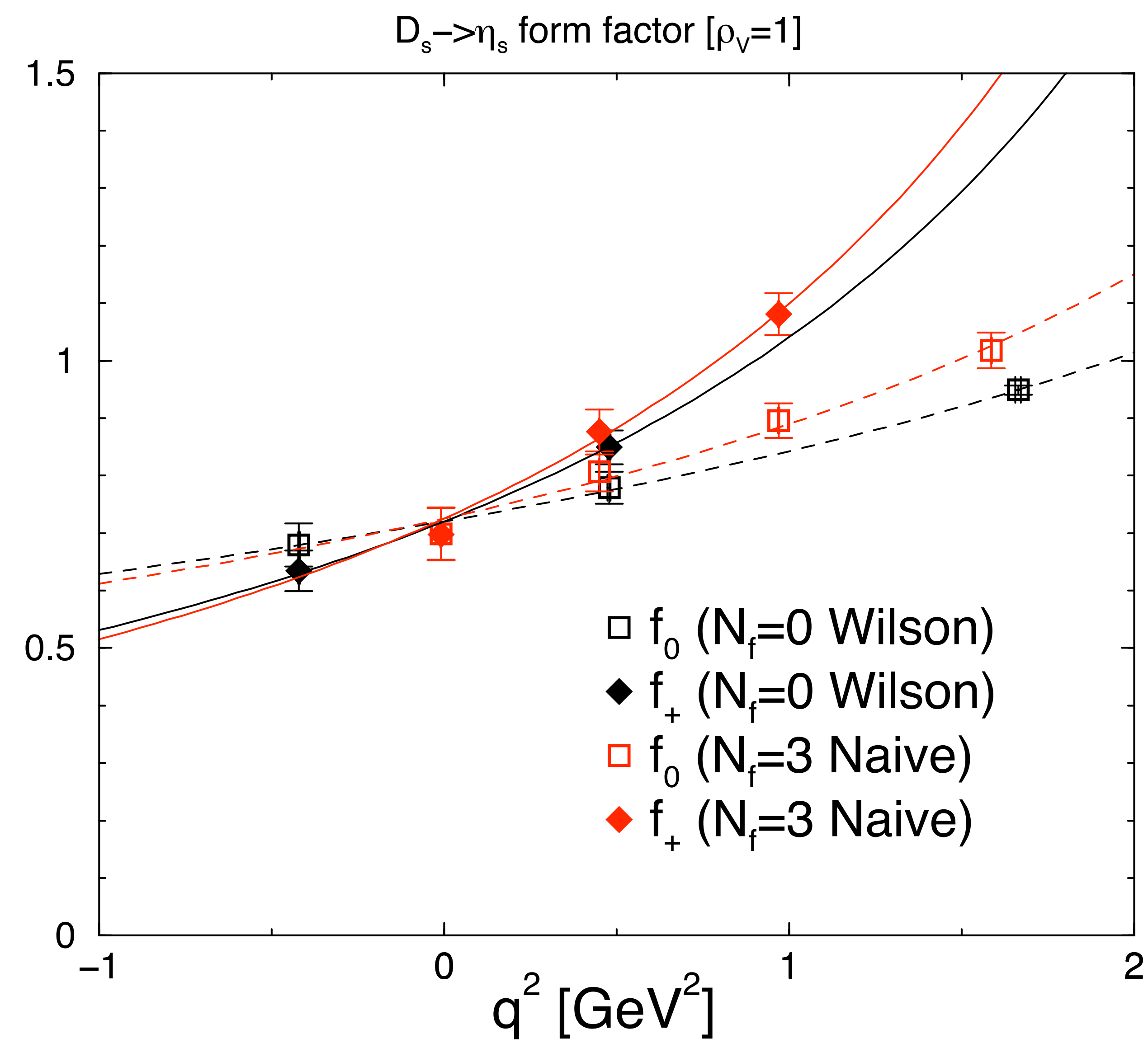
$B, D \to \pi$

Improved Wilson light quark



$B_s \to \eta_s$ DeTar, hep-lat/0309055

Improved staggered light quark



$D_s \to \eta_s$ Okamoto, hep-lat/0309107

f_B and B_q^0-$\overline{B_q^0}$ Mixing

Andreas Kronfeld

Mixing in SM

- In the Standard Model:

$$\begin{aligned}\Delta m_q &= \frac{G_F^2 m_W^2 S_0}{16\pi^2 m_{B_q^0}} |V_{tb}^* V_{tq}|^2 \eta_B \mathcal{M}_q \\ \mathcal{M}_q &= \langle \bar{B}_q^0 | [\bar{b}\gamma^\mu(1-\gamma_5)q][\bar{b}\gamma_\mu(1-\gamma_5)q] | B_q^0 \rangle \\ &= \tfrac{8}{3} m_{B_q^0}^2 f_{B_q^0}^2 B_{B_q^0}\end{aligned}$$

  - omitting non-Standard contributions!

- Δm_d is precisely measured.
- Δm_s will be measured sooner or later.

Andreas Kronfeld

Quotable Quantities

- f_{B_s}, B_{B_s}: no π loops ⇒ gentle chiral extrap
- B_{B_d}, $\xi_B^2 = B_{B_s}/B_{B_d}$: small chiral log

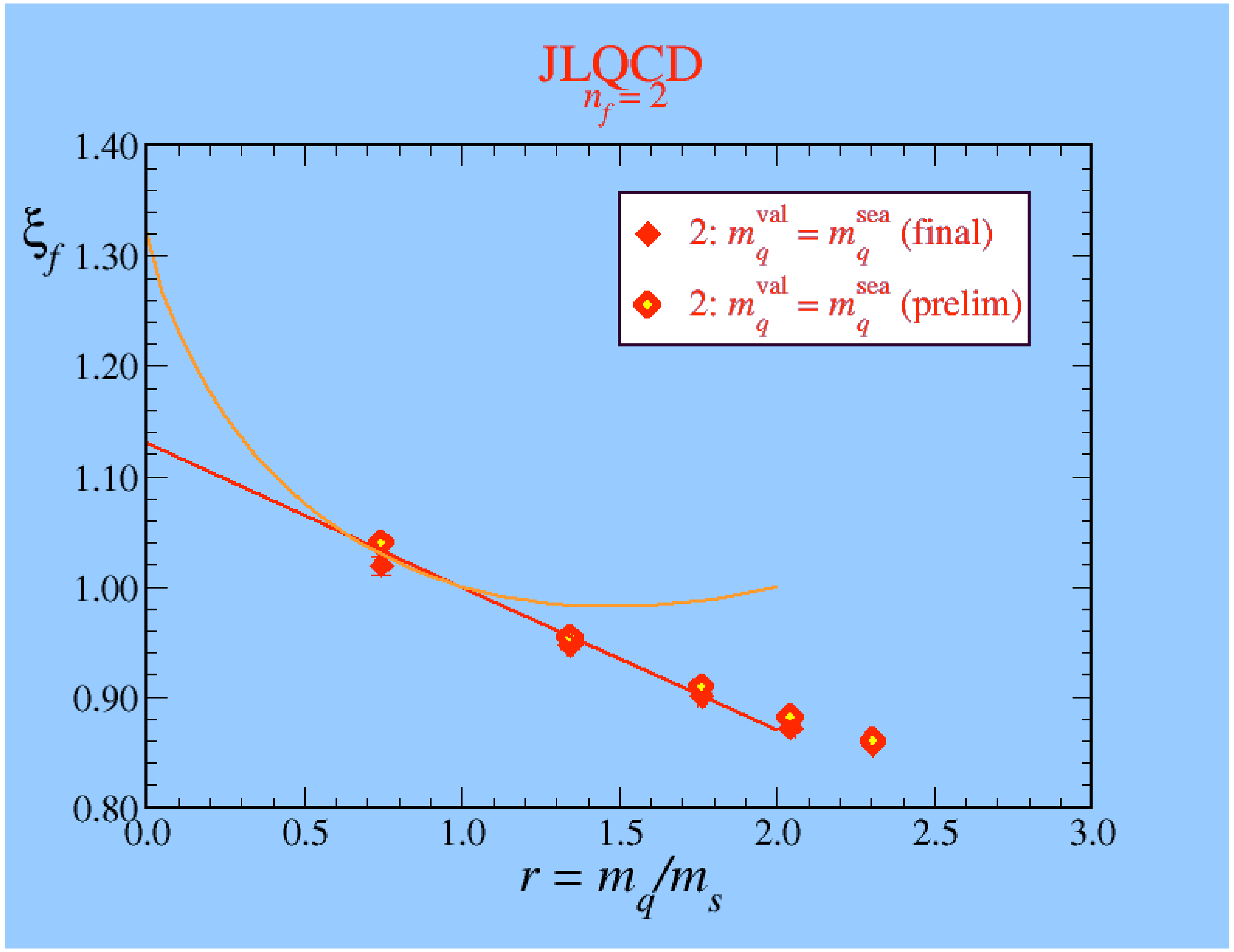
≡ $$(1-3g^2)[m_\pi^2 \ln m_\pi^2]/(4\pi f_\pi)^2, \quad g^2 \approx 0.35.$$

- $\xi_f = f_{B_s}/f_{B_d}$, $\xi = \xi_f \xi_B$: "only" uncertainty is chiral extrapolation
- $\Xi = \xi f_\pi / f_K$: chiral logs mostly cancel

Bećirević, Fajfer, Prelovšek, and Zupan, hep-ph/0211271

- For the Standard CKM fit, the most important combination is ξ.
- At one stage thought to be precise: 1.15 ± 0.5.
- Wrong: omitted chiral log $\sim m_\pi^2 \ln(m_\pi^2)$.
- (Despite warnings from Booth, Sharpe & Zhang.)

χlog vs linear

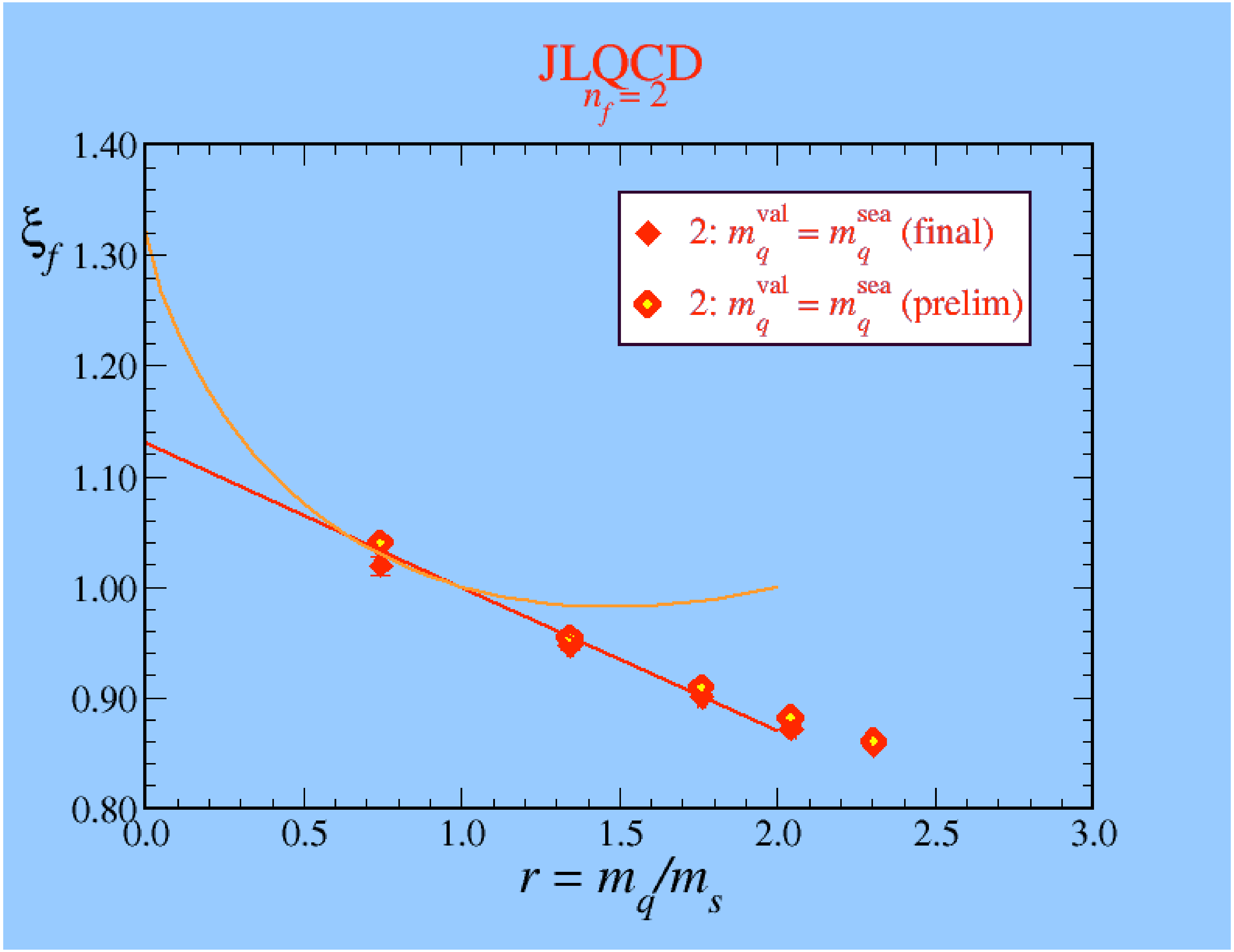


The plot compares JLQCD's linear fit with one that feeds their slope into the χlog expression. ASK & Ryan, hep-ph/0206058

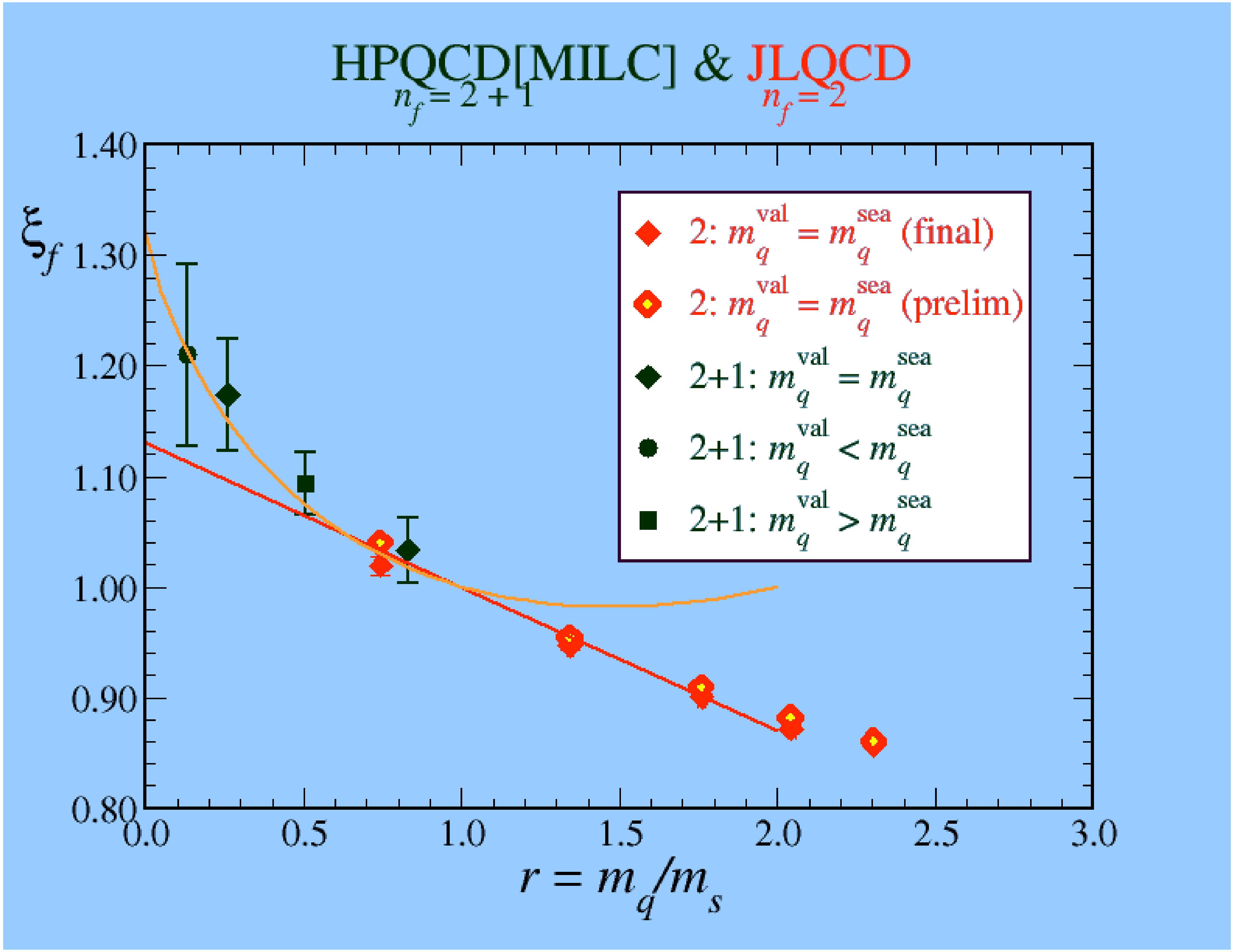
Other Ansätze lie between these two.

Thanks to N. Yamada, S. Hashimoto, and T. Onogi

Now add 2+1 (MILC) results from Wingate (HPQCD)



S. Aoki et al. [JlQCD], hep-lat/0307039 → PRL



M. Wingate et al. [HPQCD], hep-lat/0309092 (conf)

Best Estimates, 2003

$$\left.\begin{aligned}\hat{B}_{B_s} &= 1.31 \pm 0.10 \\ \hat{B}_{B_d} &= 1.26 \pm 0.10\end{aligned}\right\} \xi_B = 1.022 \pm 0.018$$ [JLQCD]

$$f_{B_s} = 240 \pm 35\ \mathrm{MeV}$$

[JLQCD + HPQCD]

$$\xi = 1.25 \pm 0.10$$

From ASK, hep-lat/0310063
(Review talk at Lattice 2003)

B_c

- The B_c^+ $(\bar{b}c)$ was observed in 1998 by CDF
  - $m_{B_c} = 6.4 \pm 0.5$ GeV (semi-leptonic decay has ν)
  - more precise mass expected soon

- Lattice calculation: this as golden as anything
  - $m_{B_c} - \frac{1}{2}(m_\psi + m_\Upsilon)$ = binding energy difference
  - $m_{B_c} - (\bar{m}_{D_s} + \bar{m}_{B_s})$ = binding energy difference

HPQCD [Glasgow/Fermilab]

- with quarkonium baseline (preliminary)
  - $m_{B_c} = 6.307 \pm 0.002^{+0.000}_{-0.010}$ GeV
  - systematic dominated by the B_c Darwin correction
- with heavy-light baseline (preliminary)
  - $m_{B_c} = 6.253 \pm 0.017^{+0.030\sim 50}_{-0.000}$ GeV
  - systematic dominated by the D_s Darwin correction
- Further study of m^{sea} & a dependence underway

Conclusions

- In B (and other quark flavor) physics, you would like to learn about quarks, but you measure hadrons.
- Lattice QCD seems (at last) to be able to help.
- Improved staggered quarks make it possible to
  - restore sea quarks
  - control chiral extrapolation

- Tests of lattice QCD are coming up soon:
  - compare B_c mass to CDF
  - compare D decay properties to CLEO-c
- Other applications of lattice field theory abound:
  - (low) moments of parton densities, which constrain high x region
  - other strongly interacting models