Life Without a Higgs

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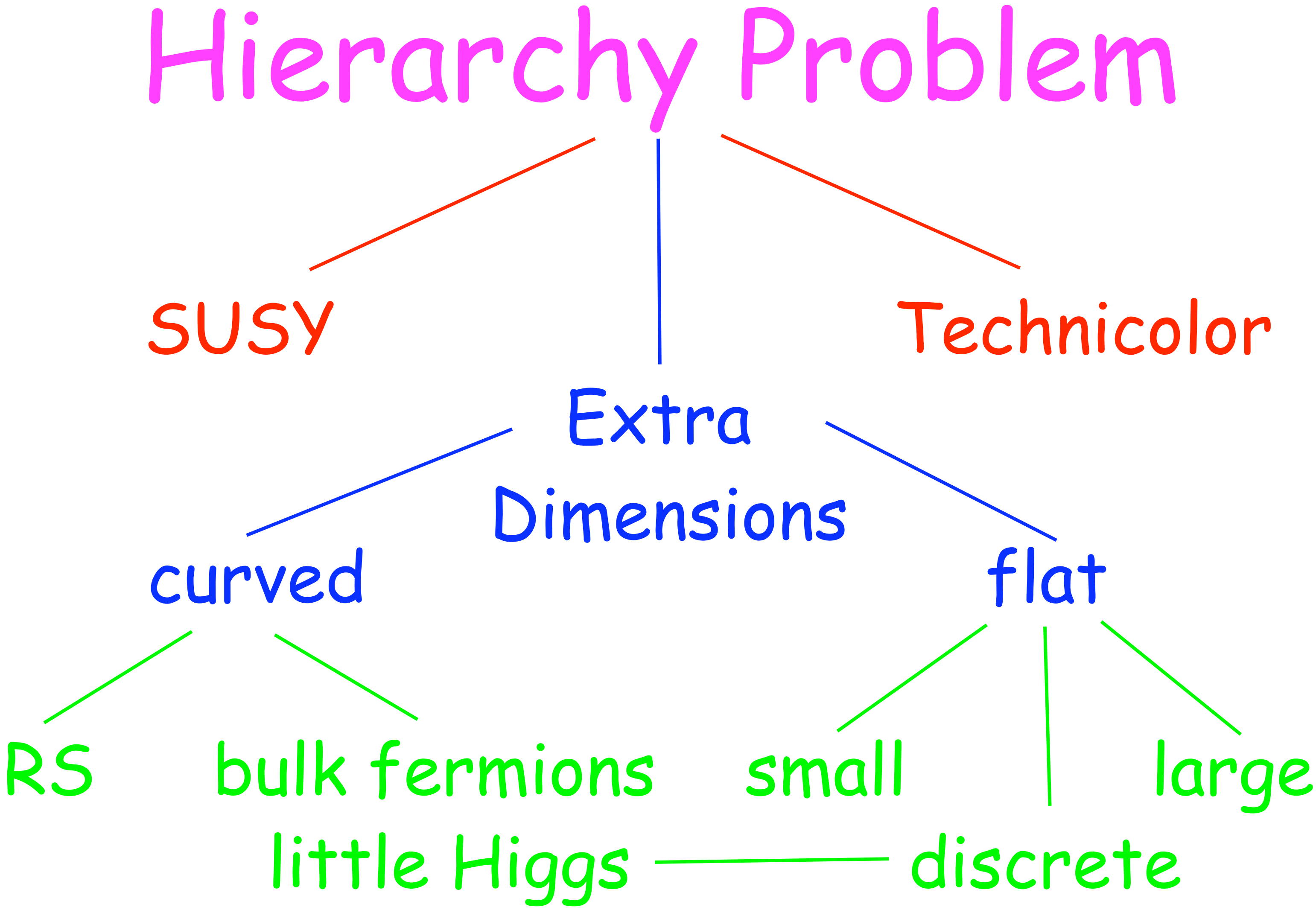
Outline

- Motivation
- General Boundary Conditions
- Unitarity of WW Scattering
- Model of EWSB without a Higgs
- Conclusions

Hierarchy Problem

SUSY

Technicolor


Hierarchy Problem
SUSY
Technicolor
Extra
Dimensions
curved
flat
RS
bulk fermions
small
large
little Higgs
discrete

Can we break Electroweak Symmetry with Boundary Conditions?

- how do we reduce the rank of the gauge group?
- is WW scattering unitary?
- why is $M_W \neq M_Z$?
- why is $\rho = 1$?
- how do we get quark and lepton masses??
- precision electroweak measurements???

5D Scalar Theory

$$\begin{aligned}\mathcal{S} &= \int d^4x \int_0^R dy \left[\tfrac{1}{2}\partial^N\phi\partial_N\phi - V(\phi)\right] \\ &\quad - \int_{y=0} d^4x \, \tfrac{1}{2}\phi^2 \, M_1^2 - \int_{y=R} d^4x \, \tfrac{1}{2}\phi^2 \, M_2^2\end{aligned}$$

$$\begin{aligned}\delta\mathcal{S} &= -\int d^4x \int_0^R dy \, \delta\phi \left(\Box_5\phi + \frac{\partial V}{\partial\phi}\right) - \\ &\quad \int d^4x \left(\delta\phi \left(\partial_5\phi + M_1^2\,\phi\right)\Big|_R - \delta\phi\left(\partial_5\phi + M_2^2\,\phi\right)\Big|_0\right) = 0\end{aligned}$$

$$\delta\phi\left(\partial_5\phi + M_1^2\,\phi\right)\Big|_{0,R} = 0$$

Consistent BC's:

$$\begin{array}{ll} (i) & \left(\partial_5\phi + M_i^2\,\phi\right)\big|_{y=0,R} = 0 \\ (ii) & \phi\big|_{y=0,R} = \text{ const.} \end{array}$$

5D Gauge Theory

$$\mathcal{S} = -\int d^4x \int_0^R dy \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} F^a_{5\nu} F^{a5\nu} + \frac{1}{2\xi} (\partial_\mu A^{a\mu} - \xi \partial_5 A^a_5)^2$$

$$\delta\mathcal{S} =$$

$$\int d^5x \left(\partial_M F^{aM\nu} - g f^{abc} F^{b\nu}_M A^{Mc} + \frac{1}{\xi} \partial^\nu \partial . A^a - \partial^\nu \partial_5 A^a_5 \right) \delta A^a_\nu$$

$$- \int d^5x \left(\partial^\sigma F^a_{\sigma 5} - g f^{abc} F^b_{\sigma 5} A^{c\sigma} + \partial_5 \partial . A^a - \xi \partial_5^2 A^a_5 \right) \delta A^a_5$$

$$+ \frac{1}{2} \int d^4x \, [F^a_{\nu 5} \, \delta A^{a\nu}] + \int d^4x \, [(\partial_\sigma A^{a\sigma} + \xi \partial_5 A^a_5) \delta A^a_5]$$

Boundary pieces:

$$F^a_{\nu 5} \, \delta A^{a\nu}{}_{|0,R} = 0$$

$$(\partial_\sigma A^{a\sigma} + \xi \partial_5 A^a_5) \delta A^a_{5|0,R} = 0$$

Consistent BC's:

(i) $A^a_{\mu|} = 0, A^a_{5|} = const.$

(ii) $A^a_{\mu|} = 0, \partial_5 A^a_{5|} = 0$

(iii) $\partial_5 A^a_{\mu|} = 0, A^a_{5|} = const.$

Boundary Higgs

$$\mathcal{S} = -\int d^4x \int dy\, \mathcal{L}_{bulk} + \int d^4x \left(\tfrac{1}{2} D_\mu \Phi_i D^\mu \Phi_i - V(\Phi)\right)$$

$$\Phi_i = \langle \Phi_i \rangle + \phi_i, \qquad X_i^a = T_{ij}^a \langle \Phi_j \rangle$$

$$\begin{aligned}
\mathcal{S} &= \int d^4x \int dy\, \mathcal{L}_{bulk} \\
&+ \int d^4x \left(\tfrac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i + g^2 X_i^a X_i^b A^a_{\mu}{}_{|_0} A^{\mu b}{}_{|_0} + g\partial^\mu \phi_i X_i^a A^a_{\mu}{}_{|_0}\right) \\
&+ \int d^4x \left(-\tfrac{1}{2\xi}(\partial_\mu A^{\mu a}{}_{|_0} - \xi g X_i^a \phi_i)^2 - \tfrac{1}{2} M_{ij}^2 \phi_i \phi_j\right)
\end{aligned}$$

BC's:

$$\begin{aligned}
&(F^a_{\nu 5} + g^2 X_i^a X_i^b A^b_\nu + \tfrac{1}{\xi}\partial_\nu \partial_\mu A^{a\mu})\, \delta A^{a\nu}{}_{|0} = 0 \\
&(\partial_\sigma A^{a\sigma} - \xi \partial_5 A^a_5)\delta A^a_{5|0} = 0 \\
&(\partial_\mu \partial^\mu \phi_i + \xi g^2 X_i^a X_j^a \phi_j + M_{ij}^2)\delta \chi_i = 0
\end{aligned}$$

Unitary limit: $\partial_y A^a_\mu|_{0,R} = g^2 X_i^a X_i^b A^b_\mu|_0$

Dirichlet and Neumann are special cases

KK Modes

$$A^a_\mu(x,y) = \sum_n a_\mu f^a_n(y) e^{ip_n x}, \text{ where } p_n^2 = M_n^2$$

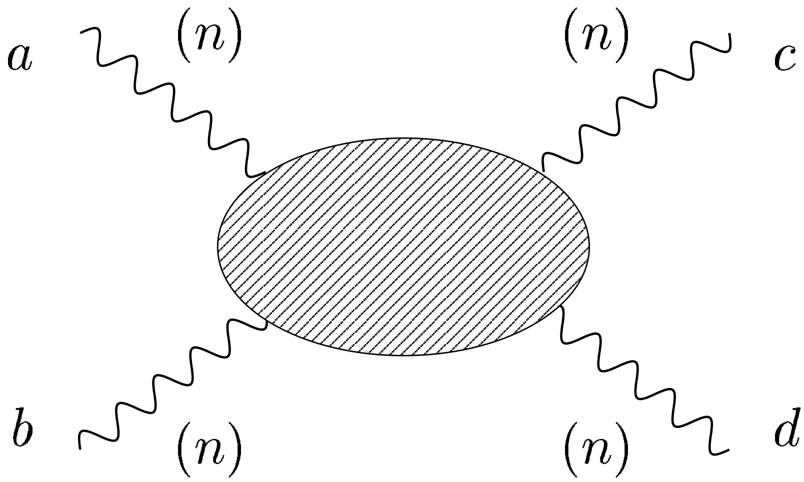
$$f^{a\,\prime\prime}_n(y) + M_n^2 f^a_n(y) = 0, \quad f^{a\,\prime}_{n|0,R} = V^{ab}_{0,R} f^b_{n|0,R}$$

$$g_{cubic} \quad \longrightarrow \quad g^{abc}_{mnk} = g \int dy f^a_m(y) f^b_n(y) f^c_k(y)$$

$$g^2_{quartic} \quad \longrightarrow \quad g^{2\,abcd}_{mnkl} = g^2 \int dy f^a_m(y) f^b_n(y) f^c_k(y) f^d_l(y)$$

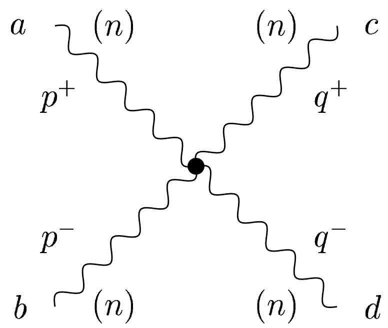
Scattering Amplitude

incoming: $p_\mu = (E, 0, 0, \pm\sqrt{E^2 - M_n^2})$
outgoing: $k_\mu = (E, \pm\sqrt{E^2 - M_n^2}\sin\theta, 0, \pm\sqrt{E^2 - M_n^2}\cos\theta)$
longitudinal polarization: $\epsilon_\mu = (\frac{|\vec{p}|}{M}, \frac{E}{M}\frac{\vec{p}}{|\vec{p}|})$

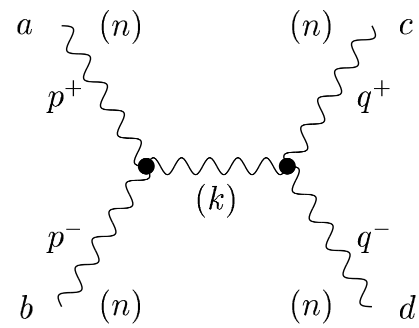


$$\mathcal{A} = A^{(4)}\frac{E^4}{M_n^4} + A^{(2)}\frac{E^2}{M_n^2} + A^{(0)} + \ldots$$

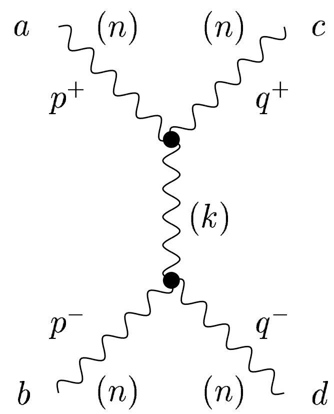
WW Scattering via KK bosons



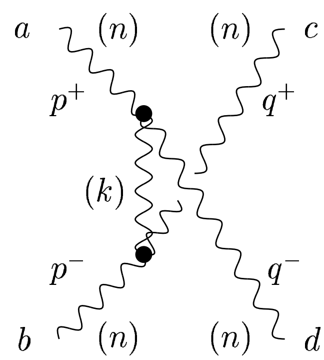
contact interaction



s channel exchange



t channel exchange



u channel exchange

Scattering Amplitude

E^4 term:

$$A^{(4)} = i\left(g^2_{nnnn} - \sum_k g^2_{nnk}\right)\left(f^{abe}f^{cde}(3+6c_\theta - c_\theta^2) + 2(3-c_\theta^2)f^{ace}f^{bde}\right)$$

E^2 term is:

$$\begin{aligned} A^{(2)} &= \tfrac{i}{M_n^2} f^{ace}f^{bde}\left(4g^2_{nnnn}M_n^2 - 3\sum_k g^2_{nnk}M_k^2\right) \\ &\quad - \tfrac{i}{2M_n^2} f^{abe}f^{cde}\left(4g^2_{nnnn}M_n^2 - 3\sum_k g^2_{nnk}M_k^2 \right. \\ &\quad \left. + \left(12g^2_{nnnn}M_n^2 + \sum_k g^2_{nnk}(3M_k^2 - 16M_n^2)\right)c_\theta\right) \end{aligned}$$

if $g^2_{nnnn} - \sum_k g^2_{nnk} = 0$ then

$$A^{(2)} = \tfrac{i}{M_n^2}\left(4g^2_{nnnn}M_n^2 - 3\sum_k g^2_{nnk}M_k^2\right)\left(f^{ace}f^{bde} - \sin^2\tfrac{\theta}{2}\ f^{abe}f^{cde}\right)$$

Cancellation

$$g^2_{nnnn} = \sum_k g^2_{nnk}$$

$$\int_0^R dy\, f_n^4(y) = \sum_k \int_0^R dy \int_0^R dz\;\, f_n^2(y) f_n^2(z) f_k(y) f_k(z)$$

true for BC's that maintain hermiticity of ∂_y^2
because of completeness:

$$\sum_k f_k(y) f_k(z) = \delta(y-z)$$

E^2 terms are more subtle:

$$\sum_k M_k^2 \int_0^R dy \int_0^R dz\, f_n^2(y) f_n^2(z) f_k(y) f_k(z) = \frac{4}{3}\, M_n^2 \int_0^R dy\, f_n^4(y)$$

$$\begin{aligned} \sum_k\; M_k^2\;\; \left(\int dy f_n^2(y) f_k(y)\right)^2 \quad &= \tfrac{4}{3} M_n^2 \int dy f_n^4(y) - \tfrac{2}{3}[f_n^3 f_n'] \\ &+2 \sum_k [f_n f_n' f_k] \int dy f_n^2(y) f_k(y) \\ &- \sum_k [f_n^2 f_k'] \int dy f_n^2(y) f_k(y) \end{aligned}$$

for Dirichlet or Neumann BC's the E^2 terms cancel

Mixed Boundary Condition

$$\partial_5 A_\mu^a(x,0) = 0, \quad \partial_5 A_\mu^a(x,R) = V A_\mu^a(x,R)$$
$$A_5^a(x,0) = 0, \quad A_5^a(x,R) = 0$$

$$A_\mu^a(x,y) = \sum_{k=1}^{\infty} f_k(y) A_\mu^{(k)}(x), \quad \text{with } f_k(y) = a_k \cos(M_k y)$$

$$a_k = \frac{\sqrt{2}}{\sin(M_k R)\sqrt{R(1+M_k^2/V^2)-1/V}}$$

$$M_k \tan(M_k R) = -V$$

$$\sum_k M_k^2 \left(\int dy f_n^2 f_k\right)^2 = \frac{4}{3} M_n^2 \int dy f_n^4 + \frac{1}{3} V f_n^4(R)$$

$$A^{(2)} = \frac{ig^2}{M_n^2} V f_n^4(R) \left(-\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} \sin^2\theta/2 + \delta^{ad}\delta^{bc} \cos^2\theta/2\right)$$

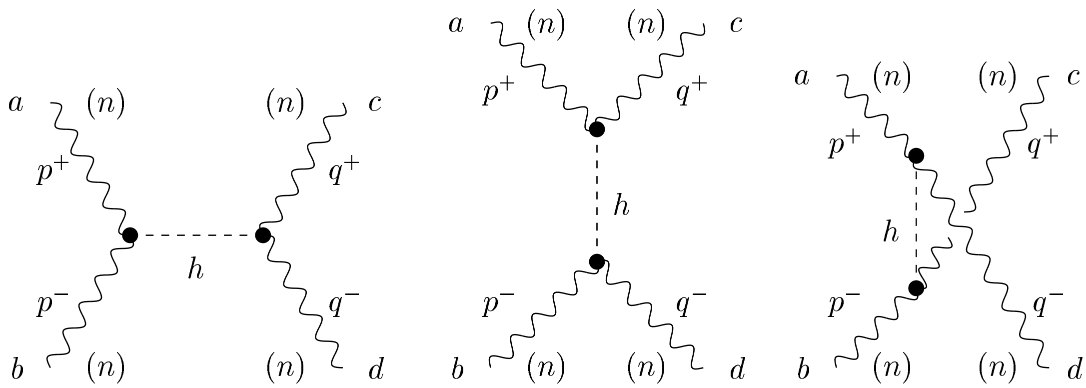
Boundary Higgs Field

$$\langle H \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v \end{array} \right)$$

$$\mathcal{L}_{\text{boundary}} = \int d^4x \frac{1}{8} g^2 v^2 A_\mu^2(x, R)$$

boundary terms: $\left(\partial_5 A_\mu \, \delta A^\mu + \frac{1}{4} g^2 v^2 A_\mu \, \delta A^\mu\right)_{|R}$

mixed BC: $\partial_5 A_\mu^a(x, R) = V A_\mu^a(x, R)$, with $V = -\frac{1}{4} g^2 v^2$



s channel   t channel   u channel

$$A^{(2)} = \frac{i g^4 v^2}{4 M_n^2} f_n^4(R) \left(-\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} \sin^2\theta/2 + \delta^{ad}\delta^{bc} cos^2\theta/2\right)$$

Decoupling the Higgs

$$A^{(2)} = \frac{ig^2}{M_n^2} V f_n^4(R) \left(-\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd}\, \sin^2\theta/2 + \delta^{ad}\delta^{bc}\, \cos^2\theta/2\right)$$

$$f_k(R) = a_k \cos(M_k R)$$

for $V \gg 1/R$

$$M_k \sim \frac{2k+1}{2R}\left(1 + \frac{1}{RV} + \ldots\right), \quad k = 0, 1, 2\ldots$$

$$V f_n^4(R) \sim \frac{(2n+1)^4 \pi^4}{4R^6 V^3}$$

Higgs does not contribute to scattering

Towards a Realistic Model

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

BC's:

$$\text{at } z = R: \quad \begin{cases} \partial_z A_\mu^{L\,a} = 0, \;\; A_\mu^{R\,1,2} = 0 \\ \partial_z \left(g_5 B_\mu + \tilde{g}_5 A_\mu^{R\,3}\right) = 0, \; \tilde{g}_5 B_\mu - g_5 A_\mu^{R\,3} = 0 \\ A_5^{L\,a} = 0, \;\; A_5^{R\,a} = 0, \;\; B_5 = 0 \end{cases}$$

$$\text{at } z = R': \quad \begin{cases} \partial_z \left(A_\mu^{L\,a} + A_\mu^{R\,a}\right) = 0, \;\; \partial_z B_\mu = 0 \\ A_\mu^{L\,a} - A_\mu^{R\,a} = 0, \\ A_5^{+\,a} = 0, \;\; \partial_z A_5^{-\,a} = 0, \;\; B_5 = 0 \end{cases}$$

at $z = R'$, $F_{\nu 5}^{La} + F_{\nu 5}^{Ra} = 0$

KK Modes

$$
\begin{aligned}
B_\mu(x,z) &= g_5\, a_0 A_\mu(x) + \textstyle\sum_{k=1}^{\infty} \psi_k^{(B)}(z)\, Z_\mu^{(k)}(x)\,, \\
A_\mu^{L\,3}(x,z) &= \tilde{g}_5\, a_0 A_\mu(x) + \textstyle\sum_{k=1}^{\infty} \psi_k^{(L3)}(z)\, Z_\mu^{(k)}(x)\,, \\
A_\mu^{R\,3}(x,z) &= \tilde{g}_5\, a_0 A_\mu(x) + \textstyle\sum_{k=1}^{\infty} \psi_k^{(R3)}(z)\, Z_\mu^{(k)}(x)\,, \\
A_\mu^{L\,\pm}(x,z) &= \textstyle\sum_{k=1}^{\infty} \psi_k^{(L\pm)}(z)\, W_\mu^{(k)\,\pm}(x)\,, \\
A_\mu^{R\,\pm}(x,z) &= \textstyle\sum_{k=1}^{\infty} \psi_k^{(R\pm)}(z)\, W_\mu^{(k)\,\pm}(x)\,.
\end{aligned}
$$

$$\psi_k^{(A)}(z) = z\left(a_k^{(A)} J_1(q_k z) + b_k^{(A)} Y_1(q_k z)\right)$$

$$\text{spectrum}: (R_0 - \tilde{R}_0)(R_1 - \tilde{R}_1) + (\tilde{R}_1 - R_0)(\tilde{R}_0 - R_1) = 0$$

$$\text{where } R_i \equiv \frac{Y_i(MR)}{J_i(MR)},\ \ \tilde{R}_i \equiv \frac{Y_i(MR')}{J_i(MR')}$$

$$M_W^2 = \frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}$$

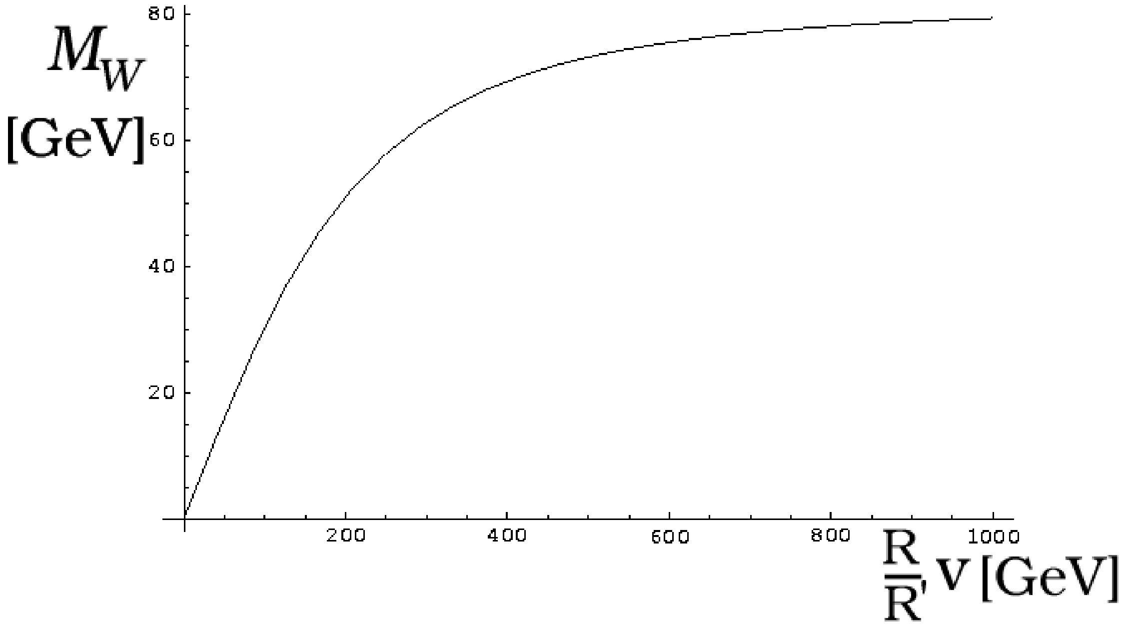
$$M_Z^2 = \frac{g_5^2 + 2\tilde{g}_5^2}{g_5^2 + \tilde{g}_5^2}\, \frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}$$

Finite VEV

BC's for a finite VEV:

$$\text{at } z = R' : \quad \begin{cases} \partial_z \left(A_\mu^{L\,a} + A_\mu^{R\,a} \right) = 0, \\ \partial_z \left(A_\mu^{L\,a} - A_\mu^{R\,a} \right) = -\frac{g_5^2 v^2}{2} \left(A_\mu^{L\,a} - A_\mu^{R\,a} \right) \end{cases}$$

for small v: $M_W^2 = \frac{g^2 v^2}{4} \frac{R^2}{R'^2}$



for $R' = 2 \cdot 10^{-3}$ GeV^{-1}, $R = 10^{-19}$ GeV^{-1}

Resonances

using $R = 10^{-19}\ \mathrm{GeV}^{-1}$, $R' = 2 \cdot 10^{-3}\,\mathrm{GeV}^{-1}$ gives

- $M_W \approx 80$ GeV

  uncanceled E^2 amplitudes blow up at 1.8 TeV

- KK excitations: $M_2^W \sim M_2^Z \sim M_2^\gamma \sim$ 1.2 TeV

- next set of resonances arise at $M_3^W \sim M_3^Z \sim$ 1.9 TeV

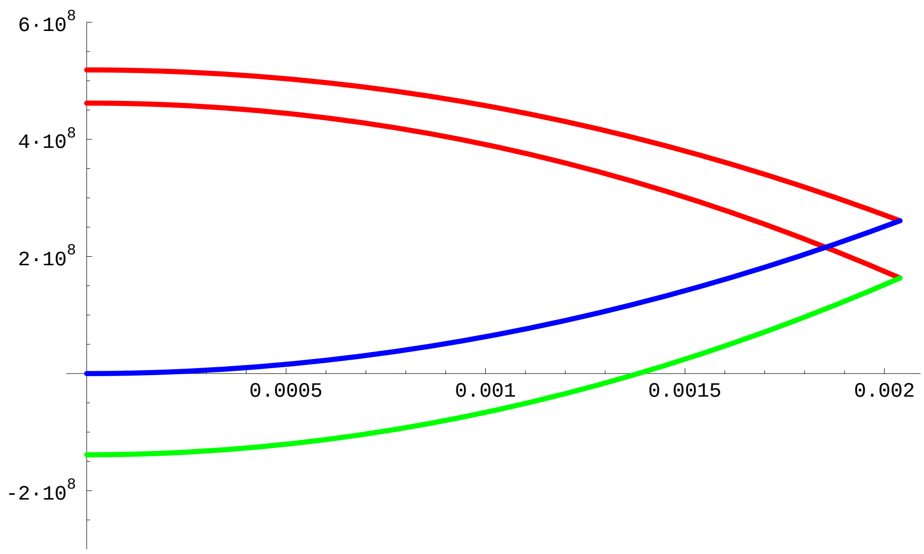
Wavefunctions

$$\psi(z) \approx c_0 + M^2 \left(c_1 z^2 - \frac{c_0}{2} z^2 \log(z/R) \right) + \mathcal{O}(M^4 z^4)$$

$$\int_R^{R'} dz \, \left(\frac{R}{z}\right) \psi(z)^2 \approx R \, c_0^2 \log\left(\frac{R'}{R}\right)$$

$$c_0^{(L\pm)} = c_\pm \;, \quad c_0^{(R\pm)} \approx 0 \;,$$

$$c_0^{(L3)} = c \;, \quad c_0^{(R3)} \approx -c \, \frac{\tilde{g}_5^2}{g_5^2+\tilde{g}_5^2} \;, \quad c_0^{(B)} \approx -c \, \frac{g_5 \, \tilde{g}_5}{g_5^2+\tilde{g}_5^2}$$



SM Gauge Couplings

$$\tilde{g}_5 g_5 a_0 Q\gamma_\mu + g_5 \psi_1^{L\pm}(R) T_\pm W_\mu^\pm$$
$$+ \left(g_5 \psi_1^{(L3)}(R) T_3 + \tilde{g}_5 \psi_1^{(B)}(R) \frac{Y}{2} \right) Z_\mu$$

$$\begin{aligned} g^2 &= \frac{g_5^2 \psi_1^{(L\pm)}(R)^2}{\int_R^{R'} dz \frac{R}{z} (\psi_1^{(L\pm)}(R)^2 + \psi_1^{(R\pm)}(R)^2)} = \frac{g_5^2}{R\log(R'/R)} \\ e^2 &= \frac{\tilde{g}_5^2 g_5^2 a_0^2}{\int_R^{R'} dz \left(\frac{R}{z}\right)(2\tilde{g}_5^2 + g_5^2) a_0^2} = \frac{g_5^2 \tilde{g}_5^2}{(g_5^2 + 2\tilde{g}_5^2) R\log(R'/R)} \end{aligned}$$

the Z couplings are also reproduced:

$$\begin{aligned} g^2 \cos\theta_W^2 &= \frac{g_5^2 \psi^{(L3)}(R)^2}{\int_R^{R'} dz \left(\frac{R}{z}\right)(\psi^{(L3)}(R)^2 + \psi^{(R3)}(R)^2 + \psi^{(B)}(R)^2)} \\ &= \frac{g_5^2}{R\log(R'/R)} \frac{g_5^2 + \tilde{g}_5^2}{g_5^2 + 2\tilde{g}_5^2}, \\ g'^2 &= \frac{g_5^2 \tilde{g}_5^2}{(g_5^2 + \tilde{g}_5^2) R\log(R'/R)}, \\ \sin\theta_W &= \frac{\tilde{g}_5}{\sqrt{g_5^2 + 2\tilde{g}_5^2}} = \frac{g'}{\sqrt{g^2 + g'^2}}. \end{aligned}$$

Hence $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \approx 1$, $S \approx \mathcal{O}(1)$

Fermion Boundary Conditions

$$\chi_{L,R} \;=\; z^{\frac{5}{2}}\left[A_{L,R}J_{\frac{1}{2}+c_{L,R}}(m_n z)+B_{L,R}J_{-\frac{1}{2}-c_{L,R}}(m_n z)\right]$$

$$\psi_{L,R} \;=\; z^{\frac{5}{2}}\left[A_{L,R}J_{\frac{1}{2}-c_{L,R}}(m_n z)+B_{L,R}J_{-\frac{1}{2}+c_{L,R}}(m_n z)\right]$$

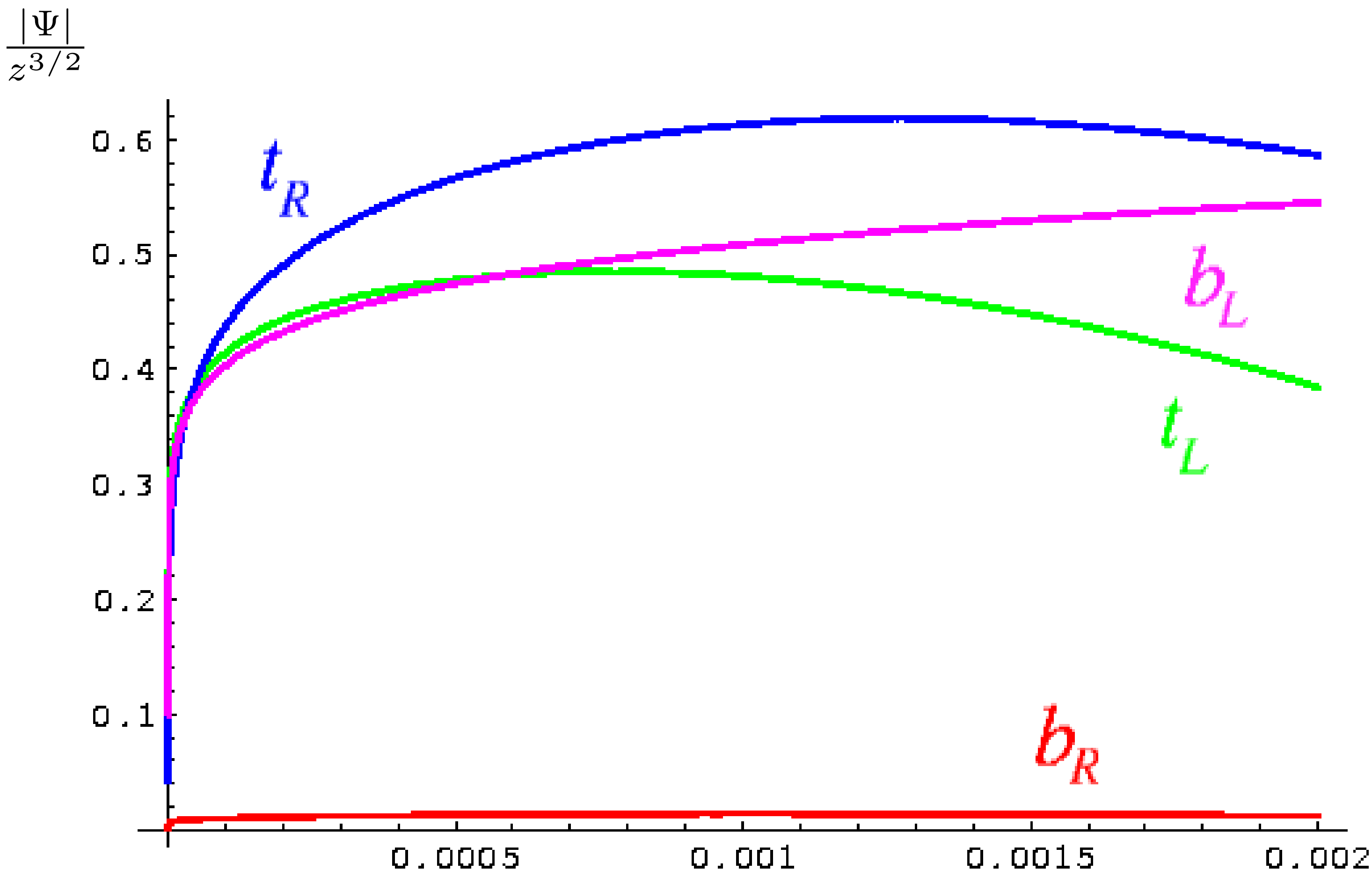
$$S_{TeV}=\int d^4x\,\left(\frac{R}{z}\right)^4 M_D R'\left[\psi_R\chi_L+\bar{\chi}_L\bar{\psi}_R+\psi_L\chi_R+\bar{\chi}_R\bar{\psi}_L\right]$$

$$\begin{aligned} S_{Pl} = \quad & \int d^4x - i\bar{\xi}\bar{\sigma}^\mu\partial_\mu\xi - i\eta\sigma^\mu\partial_\mu\bar{\eta}+f\left(\eta\xi+\bar{\xi}\bar{\eta}\right) \\ & +M\sqrt{R}\left(\psi_R\xi+\bar{\xi}\bar{\psi}_R\right) \end{aligned}$$

Taking $c_L \approx 0.4$, $c_R \approx -\frac{1}{3}$, $M_D = 900$, $M_t = 0$, $M_b = 10^{18}$ and $f = 3 \cdot 10^{13}$ GeV gives

$$\begin{gathered} m_t \approx 170\,\mathrm{GeV} \\ m_b \approx 4.5\,\mathrm{GeV} \end{gathered}$$

Fermion Wavefunctions



Precision Electroweak

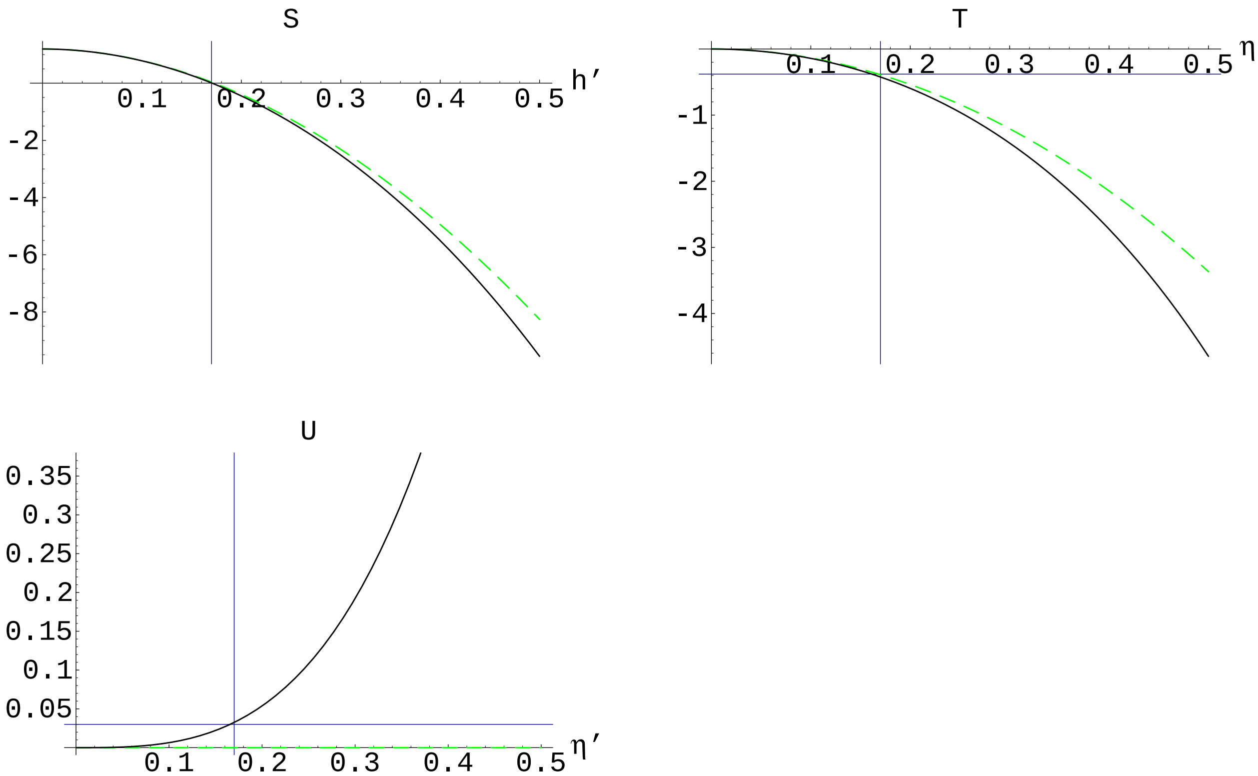
$$\mathcal{L}_{brane} = -\frac{R'}{R}\frac{\tau'}{4}B_{\mu\nu}B^{\mu\nu}\delta(z-R')$$

one resonance drops from 1.2 TeV to 300 GeV

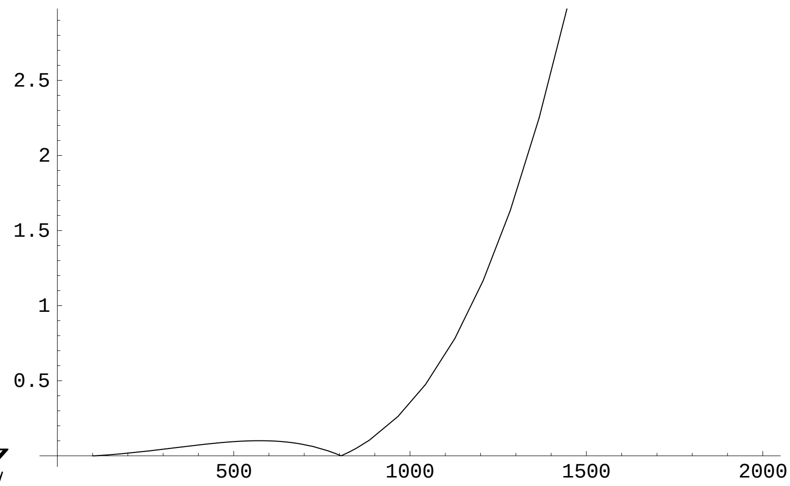
wavefunction renormalization:

$$\begin{aligned} Z_W &= 1 - g^2\Pi'_{11} \\ Z_Z &= 1 - (g^2 + g'^{\,2})\Pi'_{33} \end{aligned}$$

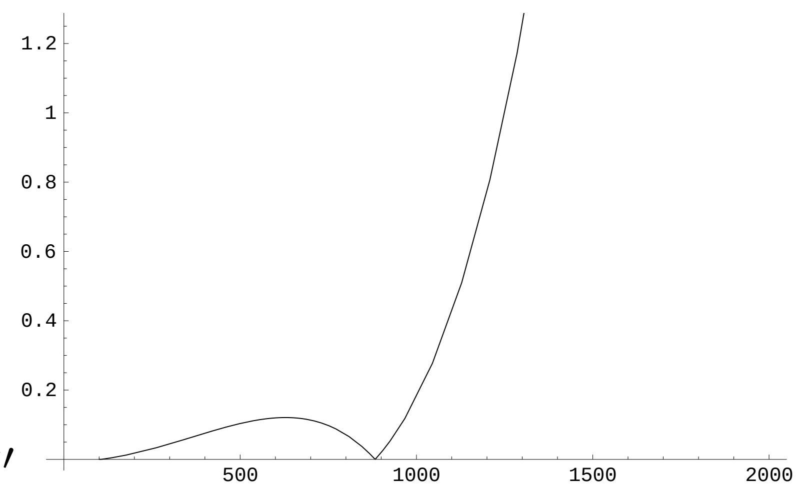
$$\begin{aligned} S &\approx \frac{6\pi}{g^2\log\frac{R'}{R}} - \frac{8\pi}{g^2}\left(1 - \left(\frac{g'}{g}\right)^2\right)\frac{\tau'^2}{(R\log R'/R)^2} \\ T &\approx -\frac{2\pi}{g^2}\left(1 - \left(\frac{g'}{g}\right)^4\right)\frac{\tau'^2}{(R\log R'/R)^2} \\ U &\approx 0 \end{aligned}$$


S
h′
0.1
0.2
0.3
0.4
0.5
-2
-4
-6
-8
T
η′
0.1
0.2
0.3
0.4
0.5
-1
-2
-3
-4
U
0.35
0.3
0.25
0.2
0.15
0.1
0.05
η′
0.1
0.2
0.3
0.4
0.5

J=0 Partial Wave Scattering

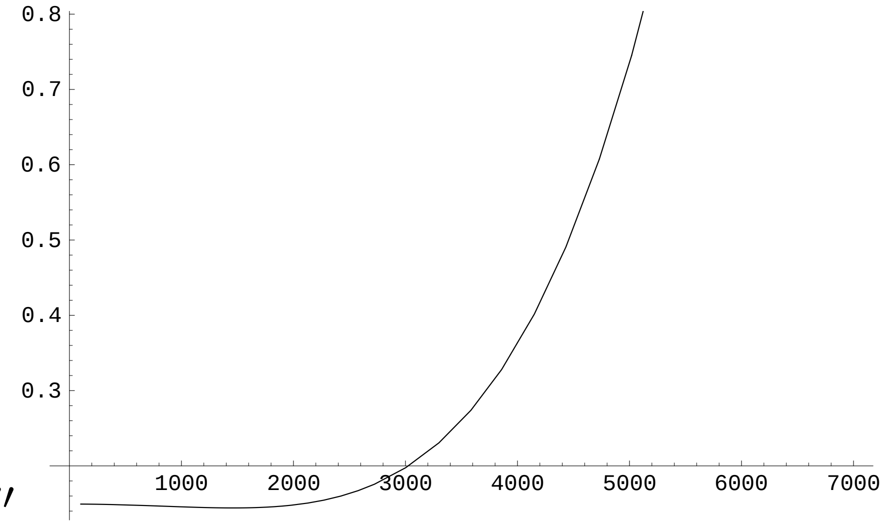


$\gamma + Z$

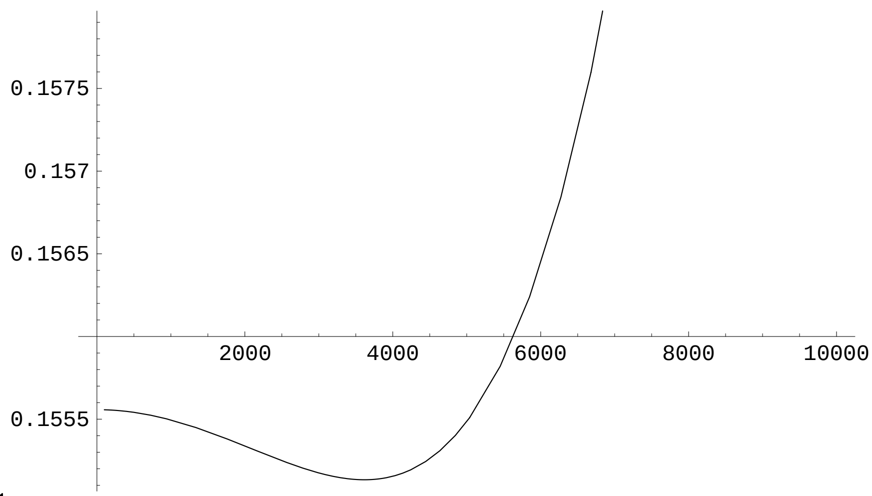


$\gamma + Z + Z'$

J=0 Partial Wave Scattering



$\gamma + Z + Z''$



$\gamma + Z + 9Z'\mathrm{s}$

Conclusions

- BC's can be used to break electroweak symmetry
- WW scattering can be unitarized without a Higgs via KK modes
- nevertheless M_W/M_Z depends on gauge couplings
- models with custodial symmetry exist
- quarks and leptons can also obtain masses from BC's
- precision electroweak measurements can be satisfied