CDF/ANAL/HEAVYFLAVOR/CDFR/1492
June 27, 1991

Fragmentation Issues in the Bottom Quark Cross Section Measurement

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1 Introduction

When we measure b-quark production cross section using the semileptonic decay electrons, the B-hadron or b-quark spectrum is not directly accessible. We can convert the electron rate to the quark rate, once the kinematical relations between the two is obtained using the Monte Carlo events. The fragmentation of the b-quark into the B-hadron is obviously important since it determines the b-quark's energy-momentum fraction transferred to the B-hadron.

This note discusses the issues related to the fragmentation of the bottom quark in measuring its production cross section. We first summarize the Peterson model and the e^+e^- experiment results of the heavy quark fragmentation. Next we discuss the adequate values of the parameter to be used as an input to a particular Monte Carlo, ISAJET. Finally we investigate the effect on the electron rates due to the uncertainty in the fragmentation.

2 The Peterson model

Since it is impossible to describe the hadronization (fragmentation) of partons starting from the first principles, we have to rely on some models. As we shall see later, the fragmentation of heavy quarks is known to be "hard"; the hadron containing the heavy quark carries a large fraction of the energy-momentum of the parent quark. This is quite contrary to the light quarks and the gluon, which produce many soft particles. This has been understood with simple kinematical arguments [1]. Later Peterson *et al.* [2] derived an analytic form of the fragmentation function $D(z)$ for heavy quarks as

$$D(z) = \frac{N}{z\,[\,1-(1/z)-\epsilon/(1-z)\,]^2},$$

where z is the fraction of the parent quark energy-momentum carried by the hadron, and N is a normalization constant. The function includes only one parameter ϵ, which is approximately

$$\epsilon \simeq \frac{m_q^2}{m_Q^2},$$

where $m_q(m_Q)$ is the mass of the light (heavy) quark. In practice it is to be determined experimentally for each quark species. The parameter controls the "hardness" of the fragmentation; the smaller the ϵ value, the harder the fragmentation. The quark mass dependence of the ϵ parameter predicts a harder fragmentation for a heavier quark. The energy-momentum fraction z is defined as

$$z = \frac{(E + p_{\|})_{\text{hadron}}}{(E + p)_{\text{quark}}}, \tag{1}$$

where $(E + p_{\|})_{\text{hadron}}$ is the sum of the energy and the momentum component parallel to the quark direction, and $(E + p)_{\text{quark}}$ is the energy-momentum of the quark after the gluon and photon radiation in the final state.

Although it is very simple and includes no dynamical arguments, the Peterson function is widely accepted as a standard from of the heavy quark fragmentation function.

3 e^+e^- experiment results

The fragmentation of heavy quarks has been measured in various e^+e^- experiments at PEP, PETRA and LEP. Since it is hard to completely reconstruct the hadronic decays of B-hadrons, the measurement of the b-quark fragmentation is done through its decay leptons, whose kinematic distribution reflect that of the parent hadron, and thus the fragmentation of the parent quark. Here z value is not obtained on an event-by-event basis, and a typical way of determining the b-quark fragmentation is to fit the observed lepton spectrum with the Monte Carlo distributions where ϵ is varied as a free parameter.

Here complications arise. First, different experiments quote fragmentation values using different definition of the energy-momentum fraction. Some use the energy-momentum fraction with respect to the beam energy, instead of the energy-momentum of the quark just before the fragmentation. This definition is usually referred to as x, and has been popular because it is more easily accessible experimentally, once a heavy hadron, for example D^{*+} meson, is fully reconstructed. x is not identical to z because of the existence of the initial and final state photon and the final state gluon radiation. By definition x is smaller than z.

Second complication is a purely technical one associated with the Monte Carlo event generation. In Monte Carlo events the $< z >$ value reconstructed using the final hadron and the parton energies and momenta is not in general identical to what we expect from the Peterson function with a given ϵ. Generally the reconstructed z value is larger than the Peterson function's value. As we see later, this is also the case with the ISAJET Monte Carlo. In the Monte Carlo generators which employ the independent fragmentation model, such as ISAJET, there exists a problem of the energy-momentum non-conservation. This happens because massive jets of physical hadrons are created from the massless parton. In order to remedy this, the final state hadron momenta are re-scaled so that they balance the parton momentum, after all hadronizations are complete. Due to this procedure, the original kinematical relations between the parton and the hadrons, z for example, is only approximately retained.

Taking these effects into account, Chrin [3] presents a compilation of the PEP/PETRA results as

$$< z >_b = 0.83 \pm 0.01 \pm 0.02,$$
$$< z >_c = 0.67 \pm 0.02 \pm 0.02,$$

where z is the momentum fraction defined by Eq. (1). These values correspond to the ϵ parameter values of

$$\epsilon_b = 0.006^{+0.001+0.002}_{-0.001-0.002},$$
$$\epsilon_c = 0.06^{+0.02+0.02}_{-0.01-0.01}.$$

The first error is statistical and the second error is systematic. More recently, a measurement by ALEPH [4] at LEP gives

$$\epsilon_b = 0.006^{+0.004}_{-0.003},$$

where the quoted error includes both the statistics and systematic effects. These are shown in Figure 1. These experiments cover the kinematical region of our interest, between about 15 and 40 GeV/c in transverse momentum. The results implies an independence of the quark energy.

4 Choice of ϵ parameter value for ISAJET

Now the question is what ϵ value we should use as an input to ISAJET. As noted earlier, the Monte Carlo generators do not in general reproduce the $< z >$ value of the Peterson function. Figure 2 shows the $< z >$ as a function of the ϵ parameter for the ISAJET b-jets with the average transverse momentum (p_T) of 26 GeV/c. We observe that the ISAJET $< z >$ value is larger than the Peterson function value. The same tendency is observed in other Monte Carlo generators [3].

Moreover, the relation turns out to be dependent on the quark momentum, as shown in Figure 3. We note that this effect is of no physics origins; it is an artifact caused while ISAJET is trying to restore the energy-momentum conservation between the parton and the physical hadrons.

Now we want the Monte Carlo events with $< z >$ value of 0.83, independent of the quark momentum. We modify ISAJET fragmentation routines so that they give a constant $< z >$ behavior as a function of the jet momentum, and provide with the ϵ parameter value so as to give the desired $< z >$ value. In practice, this is achieved by changing the input ϵ parameter on a jet-by-jet basis knowing the jet momentum. Figure 4 shows the input ϵ value necessary for ISAJET to give the $< z >$ value of 0.83 ± 0.03, as a function of the b-quark p_T. It seems that the ISAJET $< z >$ approaches asymptotically to the Peterson function's $< z >$ value as the quark p_T goes to infinity. The effect of the quark/hadron mass differences certainly vanishes in the infinite momentum frame.

p_T^e	$p_T^{min}(b)$	Relative electron rates		
		$< z >= 0.80$	$< z >= 0.83$	$< z >= 0.86$
10 - 15	15	0.877 ± 0.015	1.0	1.131 ± 0.017
15 - 20	23	0.841 ± 0.024	1.0	1.152 ± 0.028
20 - 25	32	0.823 ± 0.039	1.0	1.155 ± 0.043

Table 1: Electron rates for three b-quark fragmentation values.

5 Effect on the electron rates

Here we investigate the effect on the electron rates. We use 0.83 as a nominal value for $< z >$ and 0.03 as its uncertainty.

5.1 Analytic calculation

First we present results of an analytic calculation, which does not involve the difficulties mentioned earlier. For a given b-quark p_T, we generate the B-hadron momentum according to the Peterson function. The electron momentum is generated in the B-meson rest frame as measured by CLEO, and then Lorentz-boosted to the laboratory frame of the B-hadron. We convolute them with the parent b-quark p_T spectrum calculated by Nason, Dawson and Ellis (NDE) [5].

The electron spectrum thus obtained is shown in Figure 5 together with the NDE b-quark spectrum. Only b-quarks with p_T of 10 GeV or above are used. Two solid lines correspond to the bounds on $< z >$, 0.80 and 0.86. The shape of the electron spectrum remains almost the same by the change in the fragmentation value, while the absolute rate changes about $\pm 12\%$ for the electrons in the p_T range of our interest, between 10 and 25 GeV/c.

Figure 6 and 7 shows the electron spectrum when the parent b-quark spectrum is changed by a factor of 2 in every 10 GeV/c p_T interval. We observe that the electron spectrum is rather sensitive to the parent b-quark spectrum.

5.2 Full Monte Carlo calculation

In order to include the possible effects on the electron finding efficiencies, we generate b-jets with the ISAJET Monte Carlo with the modification applied, and simulating the detector responses and calculate the electron rates after the identification cuts are applied. Here we calculate the ratio of the electron rate to the b-quark rate defined by

$$(\sigma_e/\sigma_b)_{\rm MC} = \frac{N_{e^-}}{N_b},$$

where N_{e^-} is the number of electrons (not including positrons) passing the same geometrical, kinematical and identification cuts as in data, and N_b is the number of all b-quarks produced in the kinematic ranges (p_T and rapidity) and decaying into electron channel.

We use three kinematic ranges;

$$
\begin{aligned}
10 < p_T^e < 15\,\mathrm{GeV}/c, \quad p_T^b > 15\,\mathrm{GeV}/c. \\
15 < p_T^e < 20\,\mathrm{GeV}/c, \quad p_T^b > 23\,\mathrm{GeV}/c. \\
20 < p_T^e < 25\,\mathrm{GeV}/c, \quad p_T^b > 32\,\mathrm{GeV}/c.
\end{aligned}
$$

In calculating the quantity, we generate the b-quarks with a flat p_T spectrum rather than that by ISAJET, and later convolute them with the p_T spectrum by NDE.

The relative electron rates for the three three fragmentation values are given in Table 1 and in Figure 8. A modest change in the electron rates are observed. We take 15% as the uncertainty in the b-quark cross section due to this fragmentation effect.

References

[1] M. Suzuki, Phys. Lett. 71B (1977) 139.
J. D. Bjorken, Phys. Rev. D17, (1978) 171.

[2] C. Peterson, D. Schlatter, I. Schmitt and P. M. Zerwas, Phys. Rev. D27 (1983) 105.

[3] J. Chrin, Z. Phys. C36 (1987) 165.
See also, S. Bethke, Z. Phys. C29 (1085) 175; W. Bartel *et al.*, Z. Phys. C33 (1987) 339.

[4] ALEPH Collaboration, D. Decamp *et al.*, Phys. Lett. B244 (1990) 551.

[5] P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B327 (1989) 49.

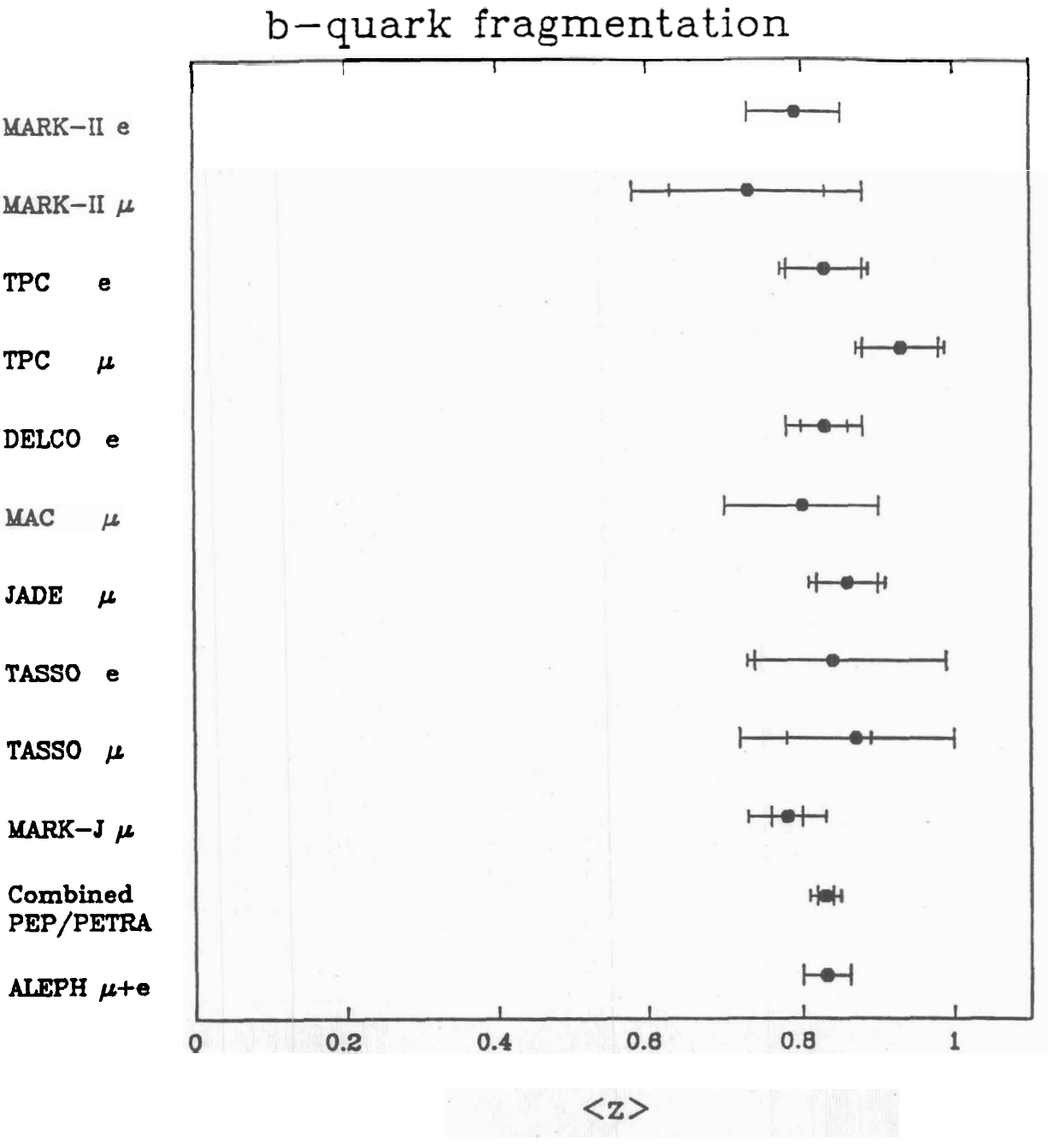

Figure 1: The b-quark fragmentation measurements from the various e^+e^- experiments at PEP, PETRA and LEP.

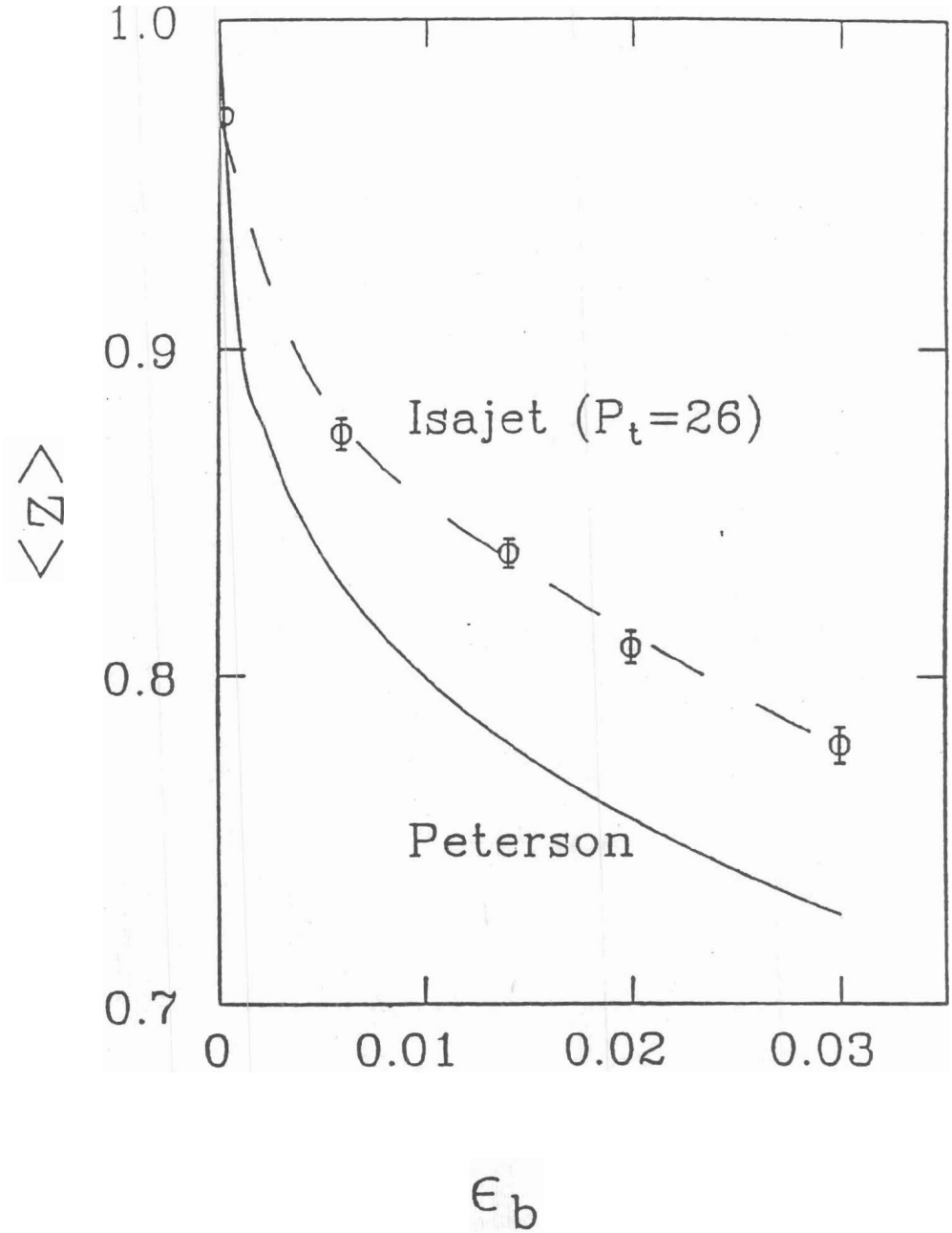

Figure 2: The average z value for ISAJET b-jets as a function of the input ϵ. Also shown is the relation expected from the Peterson function.

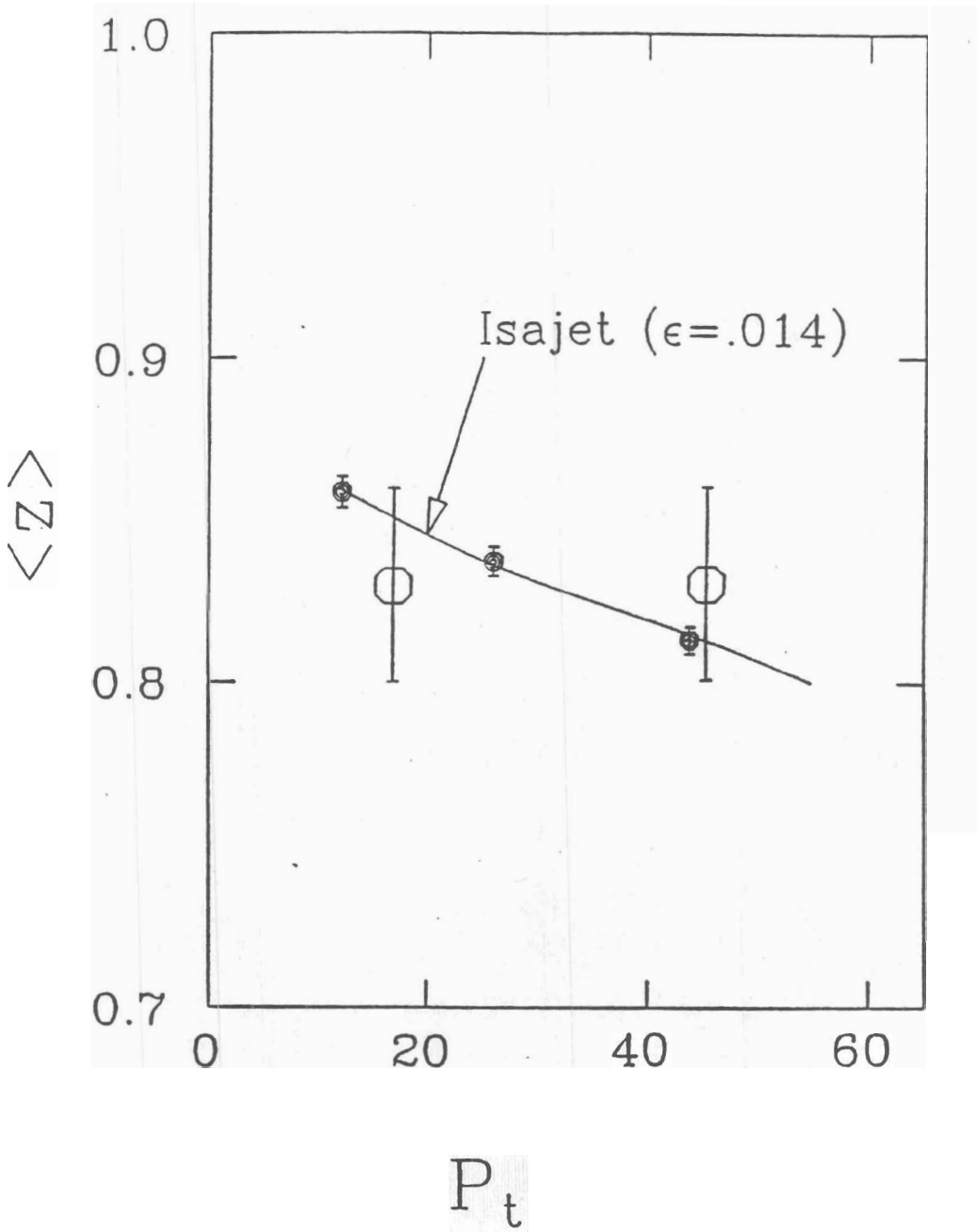

Figure 3: The reconstructed $< z >$ values for ISAJET b-jets for a fixed input ϵ parameter value of 0.014. Open circles are the e^+e^- experiment measurements.

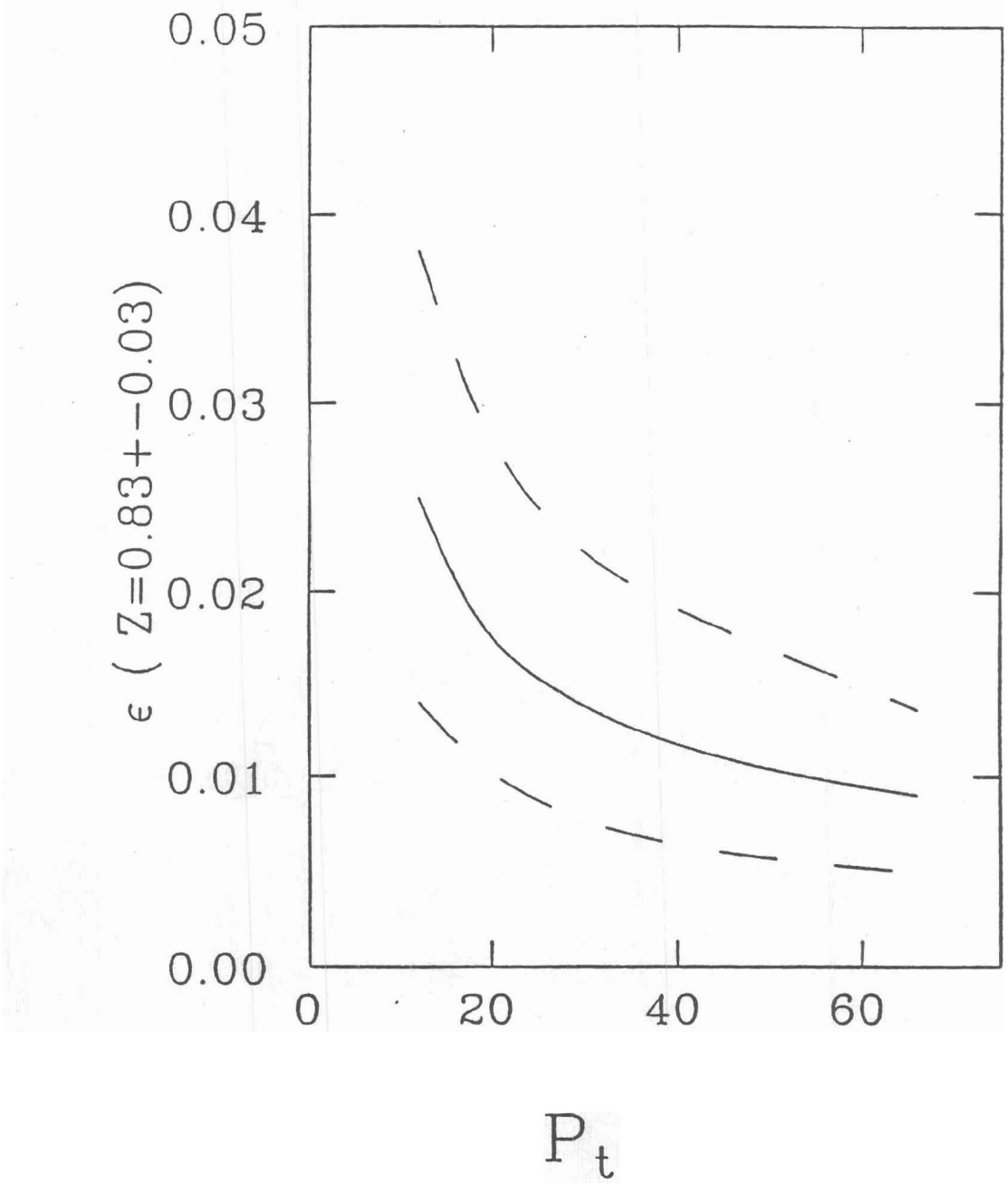

Figure 4: The input ϵ value for ISAJET b-jets to reproduce the $< z >$ value of 0.83 ± 0.03 as a function of b-quark p_T.

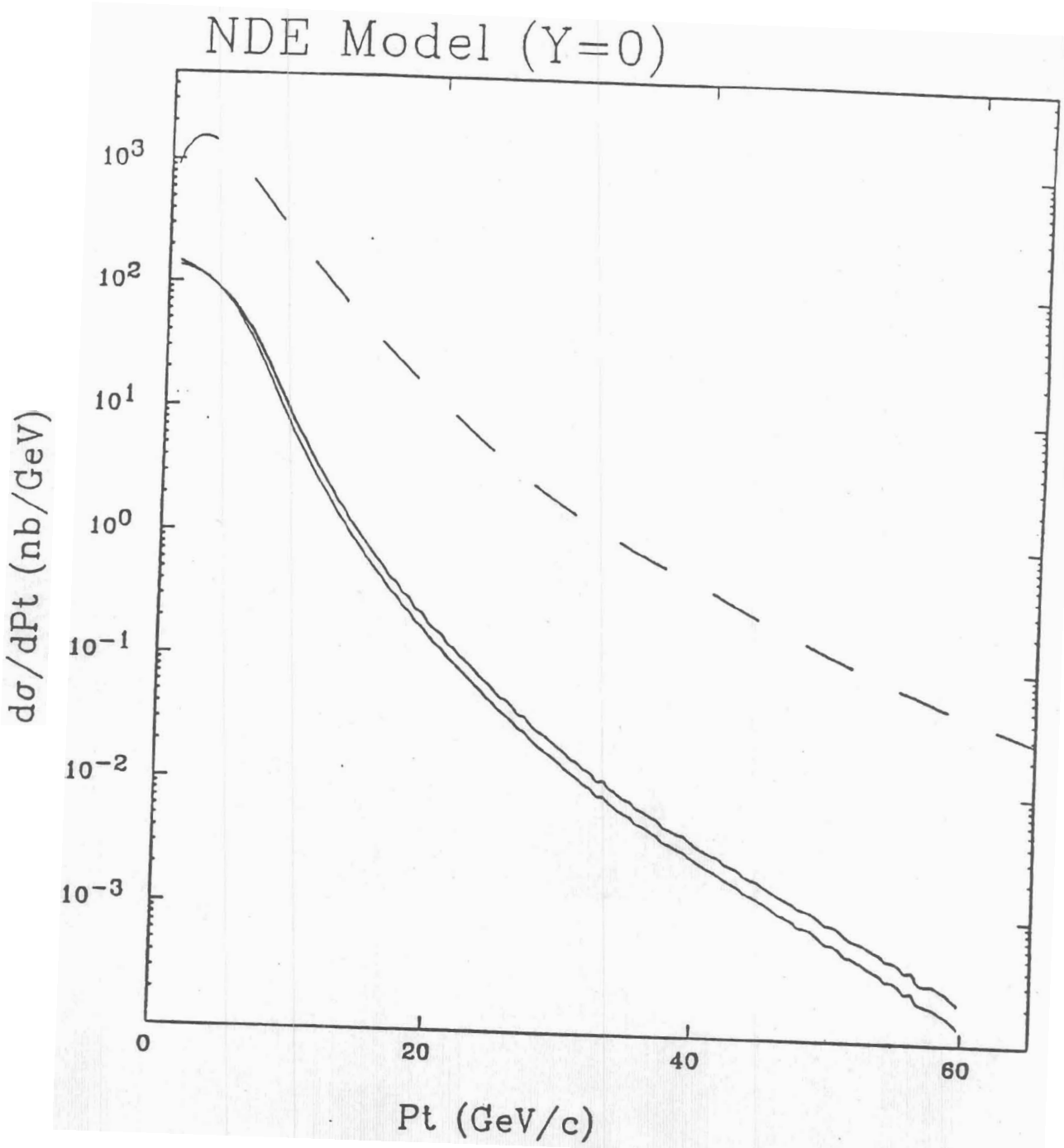

Figure 5: The analytic calculation of the electron spectrum for the $< z >$ values of 0.80 and 0.86, along with the assumed parent b-quark spectrum.

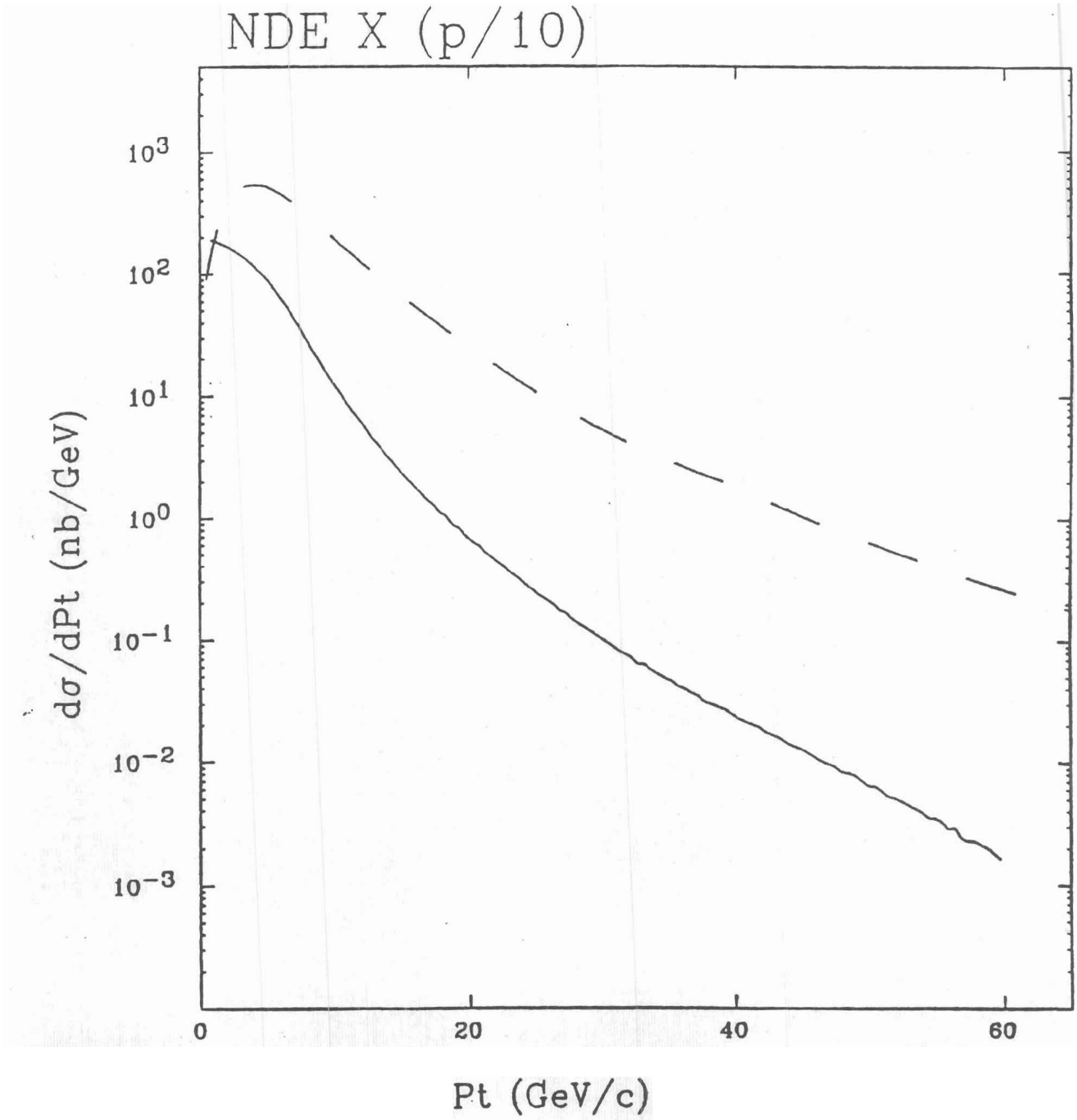

Figure 6: The electron spectrum for a harder b-quark p_T spectrum than the NDE model.

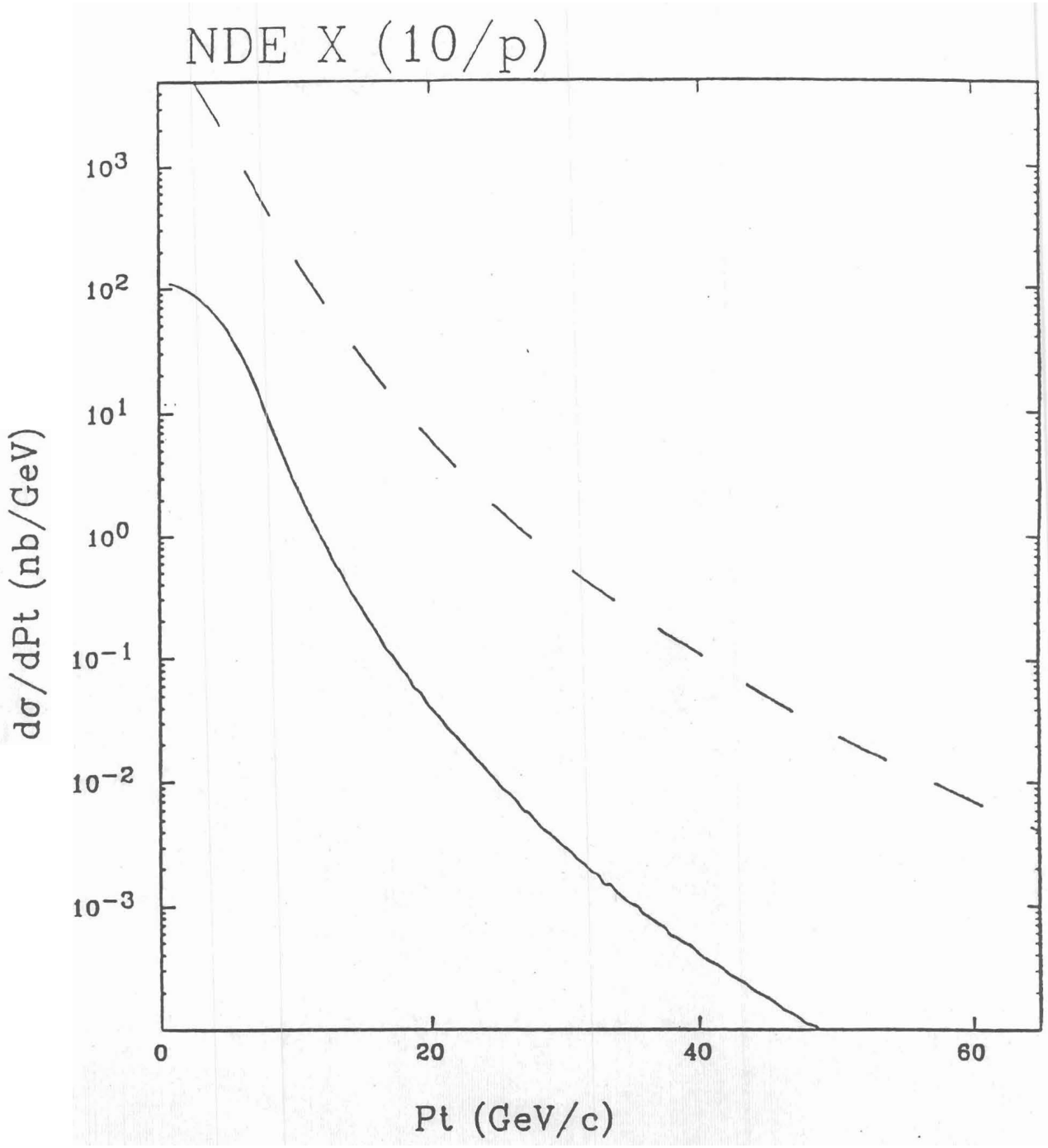

Figure 7: The electron spectrum for a softer b-quark p_T spectrum than the NDE model.

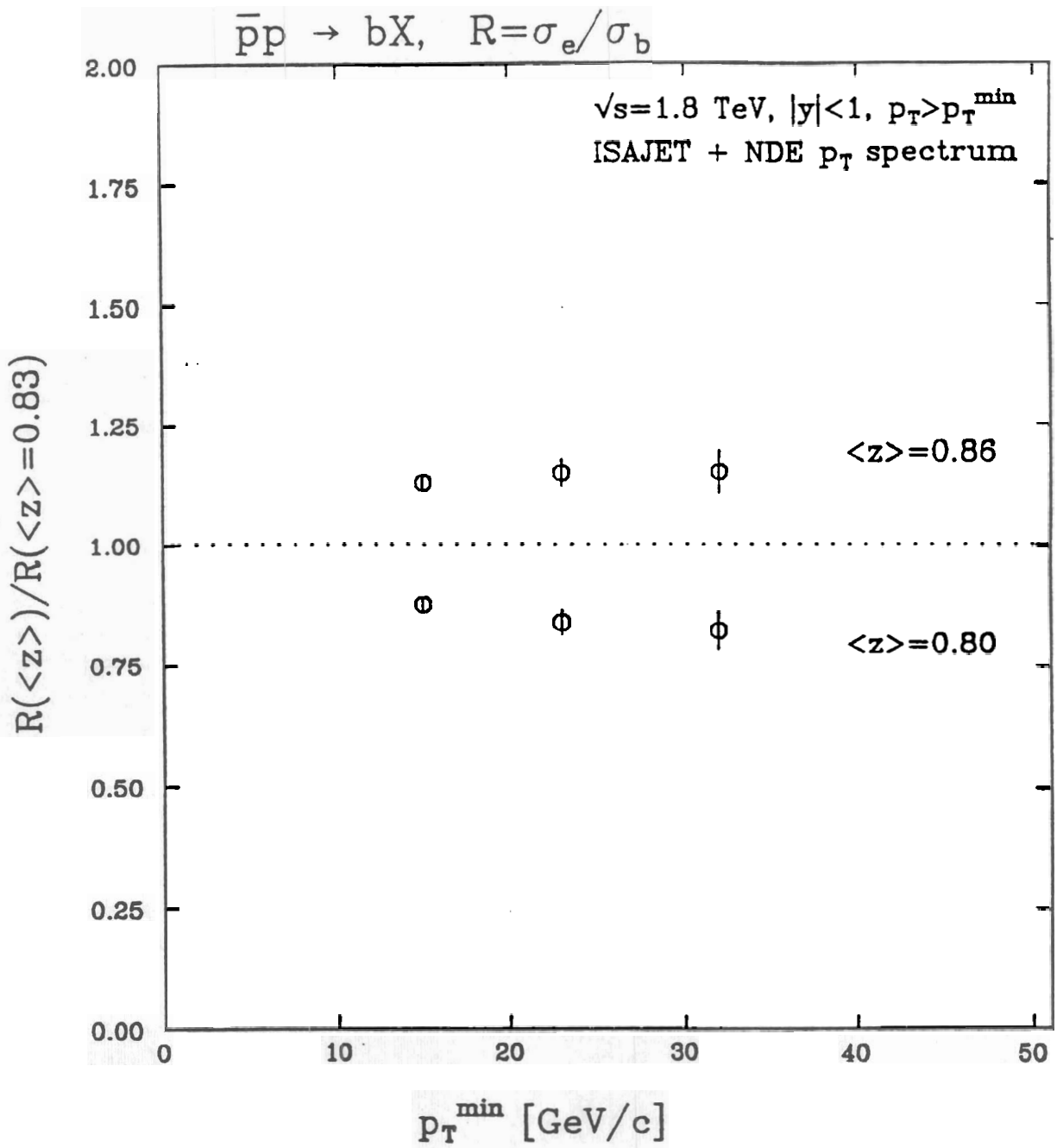

Figure 8: The ratio of the electron rate to the b-quark rate for three fragmentation values, normalized to the nominal fragmentation of $< z >= 0.83$.